
PLANE AND SPHERICAL
TRIGONOMETRY

LYMAN AND GODDARD

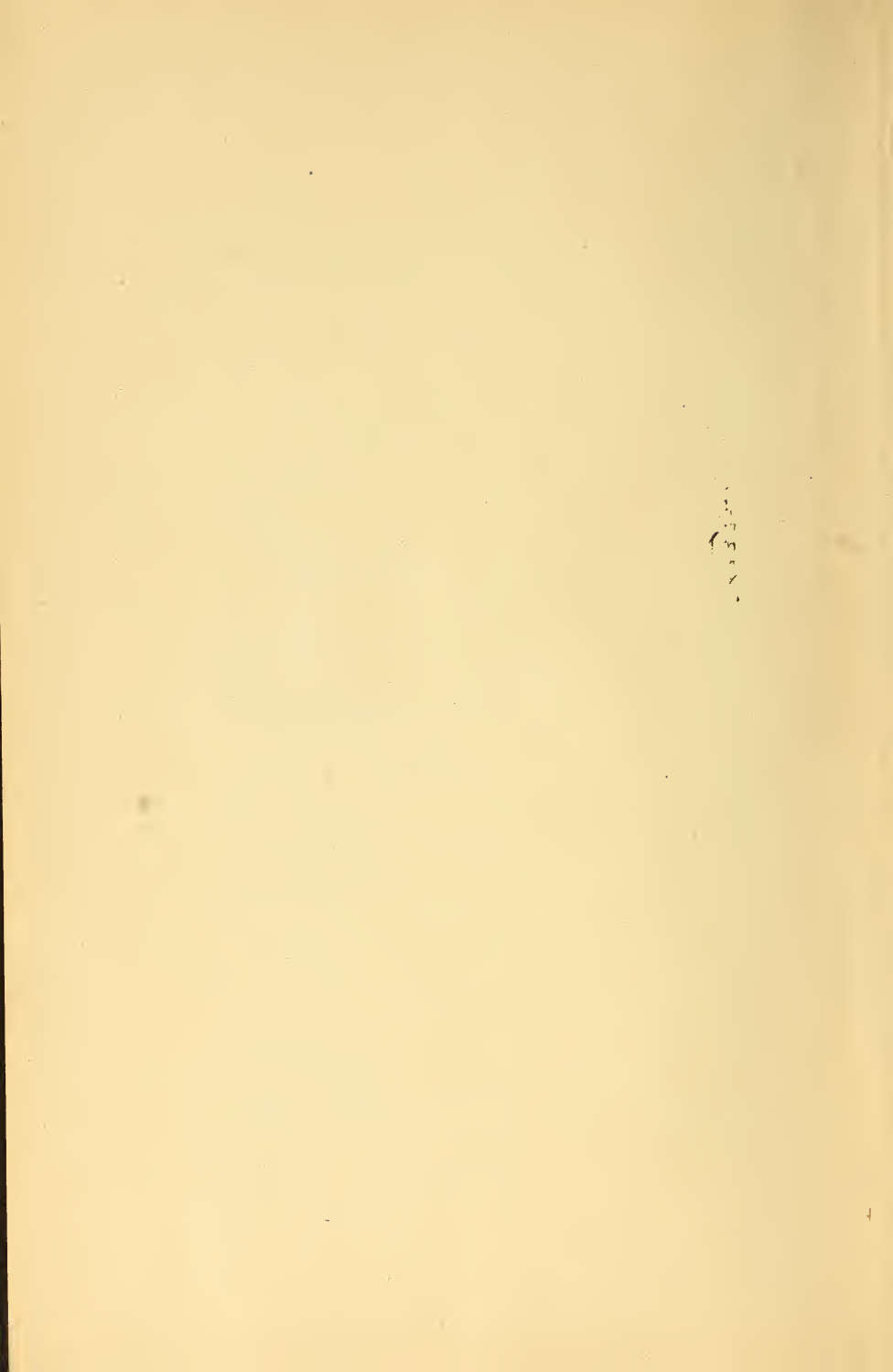
WITH TABLES

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PLANE AND SPHERICAL TRIGONOMETRY

BY

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AND

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Norwood Press
J. S. Cushing & Co. — Berwick & Smith
Norwood Mass. U.S.A.

EG 13 Dec. 23 12
Repr. new, H.B. Price 1 June 4

PREFACE.

MANY American text-books on trigonometry treat the solution of triangles quite fully; English text-books elaborate analytical trigonometry; but no book available seems to meet both needs adequately. To do that is the first aim of the present work, in the preparation of which nearly everything has been worked out and tested by the authors in their classes.

The work entered upon, other features demanded attention. For some unaccountable reason nearly all books, in proving the formulæ for functions of $\alpha \pm \beta$, treat the same line as both positive and negative, thus vitiating the proof; and proofs given for acute angles are (without further discussion) supposed to apply to all angles, or it is suggested that the student can draw other figures and show that the formulæ hold in all cases. As a matter of fact the average student cannot show anything of the kind; and if he could, the proof would still apply only to combinations of conditions the same as those in the figures actually drawn. These difficulties are avoided by so wording the proofs that the language applies to figures involving any angles, and to avoid drawing the indefinite number of figures necessary fully to establish the formulæ geometrically, the general case is proved algebraically (see page 58).

Inverse functions are introduced early, and used constantly. Wherever computations are introduced they are made by means of logarithms. The average student, using logarithms for a short time and only at the end of the subject, straightway forgets what manner of things they are. It is hoped, by dint of much practice, extended over as long a time as possible, to give the student a command of logarithms that will stay. The fundamental formulæ of trigonometry must be memorized. There is no substitute for this. For this purpose oral work is introduced, and there are frequent lists of review problems involving all principles and formulæ previously developed. These lists serve the

further purpose of throwing the student on his own resources, and compelling him to find in the problem itself, and not in any model solution, the key to its solution, thus developing power, instead of ability to imitate. To the same end, in the solution of triangles, divisions and subdivisions into cases are abandoned, and the student is thrown on his own judgment to determine which of the three possible sets of formulæ will lead to the solutions with the data given. Long experience justifies this as clearer and simpler. The use of checks is insisted upon in all computations.

For the usual course in plane trigonometry Chapters I-VII, omitting Arts. 26, 27, contain enough. Articles marked * (as Art. * 26) may be omitted unless the teacher finds time for them without neglecting the rest of the work. Classes that can accomplish more will find a most interesting field opened in the other chapters. More problems are provided than any student is expected to solve, in order that different selections may be assigned to different students, or to classes in different years. *Do not assign work too fast. Make sure the student has memorized and can use each preceding formula, before taking up new ones.*

No complete acknowledgment of help received could here be made. The authors are under obligation to many for general hints, and to several who, after going over the proof with care, have given valuable suggestions. The standard works of Levett and Davison, Hobson, Henrici and Treutlein, and others have been freely consulted, and while many of the problems have been prepared by the authors in their class-room work, they have not hesitated to take, from such standard collections as writers generally have drawn upon, any problems that seemed better adapted than others to the work. Quality has not been knowingly sacrificed to originality. Corrections and suggestions will be gladly received at any time.

E. A. L., YPSILANTI.

E. C. G., ANN ARBOR.

October, 1900.

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PLANE TRIGONOMETRY.



CHAPTER I.

ANGLES—MEASUREMENT OF ANGLES.

1. Angles. It is difficult, if not impossible, to define an angle. This difficulty may be avoided by telling how it is formed. *If a line revolve about one of its points, an angle is generated*, the magnitude of the angle depending on the amount of the rotation.

Thus, if one side of the angle θ , as OR , be originally in the position OX , and be revolved about the point O to the position in the figure, the angle XOR is generated.

OX is called the *initial* line, and any position of OR the *terminal* line of the angle formed. The angle θ is considered *positive* if generated by a counter-clockwise

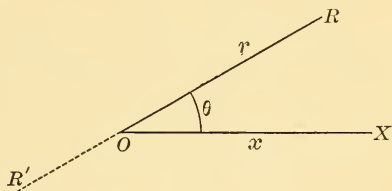


FIG. 1.

rotation of OR , and hence *negative* if generated by a clockwise rotation. The magnitude of θ depends on the amount of rotation of OR , and since the amount of such rotation may be unlimited, there is no limit to the possible magnitude of angles, for, evidently, the revolving line may reach the position OR by rotation through an acute angle θ , and, likewise, by rotation through once, twice, ..., n times 360° , plus the acute angle θ . So that XOR may mean the acute angle θ , $\theta + 360^\circ$, $\theta + 720^\circ$, ..., $\theta + n \cdot 360^\circ$.

In reading an angle, read first the initial line, then the terminal line. Thus in the figure the acute angle XOR , or rx , is a positive angle, and ROX , or rx , an equal negative angle.

Ex. 1. Show that if the initial lines for $\frac{1}{2}$, $\frac{3}{2}$, $\frac{25}{2}$, $-\frac{7}{2}$, right angles are the same, the terminal lines may coincide.

2. Name four other angles having the same initial and terminal lines as $\frac{1}{3}$ of a right angle; as $\frac{5}{4}$ of a right angle; as $\frac{3}{2}$ of a right angle.

2. Rectangular axes. Any plane surface may be divided by two perpendicular straight lines XX' and YY' into four portions, or *quadrants*.

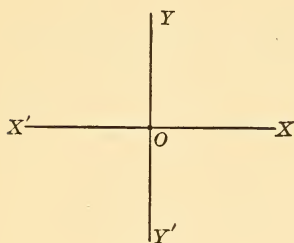


FIG. 2.

XX' is known as the *x-axis*, YY' as the *y-axis*, and the two together are called *axes of reference*. Their intersection O is the *origin*, and the four portions of the plane surface, XOY , YOX' , $X'OY'$, $Y'OX$, are called respectively the *first*, *second*, *third*, and *fourth quadrants*. The position of

any point in the plane is determined when we know its *distances* and *directions* from the axes.

3. Any direction may be considered positive. Then the opposite direction must be negative. Thus, if AB represents any positive line, BA is an equal negative line. Mathematicians usually consider *lines measured in the same direction as OX or OY (Fig. 2) as positive*. Then lines measured in the same direction as OX' or OY' must be negative.

The distance of any point from the *y-axis* is called the *abscissa*, its distance from the *x-axis* the *ordinate*, of that point; the two together are the *coördinates* of the point, usually denoted by the letters x and y respectively, and written (x, y) .

When taken with their proper signs, the coördinates define completely the position of the point. Thus, if the point P is $+a$ units from YY' , and $+b$ units from XX' , any convenient unit of length being chosen, the position of P is known. For we have only to measure a distance ON equal to a units along OX , and then from N measure a distance b units parallel to OY , and we arrive at the position of the point P , (a, b) . In like manner we may locate P' , $(-a, b)$, in the second quadrant, P'' , $(-a, -b)$, in the third quadrant, and P''' , $(a, -b)$, in the fourth quadrant.

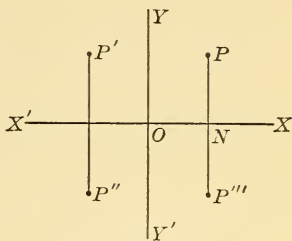


FIG. 3.

Ex. Locate $(2, -2)$; $(0, 0)$; $(-8, -7)$; $(0, 5)$; $(-2, 0)$; $(2, 2)$; (m, n) .

4. If OX is the initial line, θ is said to be an *angle of the first, second, third, or fourth quadrant*, according as its terminal line is in the first, second, third, or fourth quadrant. It is clear that as OR rotates its *quality* is in no way affected, and hence it is *in all positions considered positive*, and its extension through O , OR' , negative.

The student should notice that the initial line may take any position and revolve in either direction. While it is customary to consider the counter-clockwise rotation as forming a positive angle, yet the conditions of a figure may be such that a positive angle may be generated by a clockwise rotation.

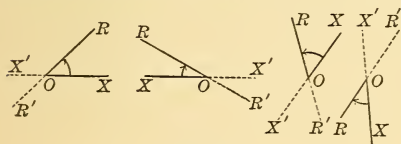


FIG. 4.

No confusion can result if the fact is clear that when an angle is read XOR , OX is considered a positive line revolving to the position OR . OX' and OR' then are negative lines in whatever directions drawn. These conceptions are mere matters of agreement, and the agreement may be determined in a particular case by the conditions of the problem quite as well as by such general agreements of mathematicians as those referred to in Arts. 3 and 4 above.

5. **Measurement.** All measurements are made in terms of some fixed standard adopted as a unit. This unit must

be of the same kind as the quantity measured. Thus, length is measured in terms of a unit length, surface in terms of a unit surface, weight in terms of a unit weight, value in terms of a unit value, an angle in terms of a unit angle.

The measure of a given quantity is the number of times it contains the unit selected.

Thus the area of a given surface in square feet is the number of times it contains the unit surface 1 sq. ft.; the length of a road in miles, the number of times it contains the unit length 1 mi.; the weight of a cargo of iron ore in tons, the number of times it contains the unit weight 1 ton; the value of an estate, the number of times it contains the unit value \$1.

The same quantity may have different measures, according to the unit chosen. So the measure of 80 acres, when the unit surface is 1 acre, is 80, when the unit surface is 1 sq. rd., is 12,800, when the unit surface is 1 sq. yd., is 387,200. What is its measure in square feet?

6. The essentials of a good unit of measure are:

1. *That it be invariable, i.e.* under all conditions bearing the same ratio to equal magnitudes.

2. *That it be convenient* for practical or theoretical purposes.

3. *That it be of the same kind as the quantity measured.*

7. Two systems of measuring angles are in use, the *sexagesimal* and the *circular*.

The *sexagesimal* system is used in most practical applications. The right angle, the unit of measure in geometry, though it is invariable, as a measure is too large for convenience. Accordingly it is divided into 90 equal parts, called *degrees*. The degree is divided into 60 *minutes*, and the minute into 60 *seconds*. Degrees, minutes, seconds, are indicated by the marks $^{\circ} ' ''$, as $36^{\circ} 20' 15''$.

The division of a right angle into hundredths, with subdivisions into hundredths, would be more convenient. The French have proposed such

a *centesimal* system, dividing the right angle into 100 grades, the grade into 100 minutes, and the minute into 100 seconds, marked $^{\circ}$ $'$ $''$, as $50^{\circ} 70' 28''$. The great labor involved in changing mathematical tables, instruments, and records of observation to the new system has prevented its adoption.

8. The *circular* system is important in theoretical considerations. It is based on the fact that for a given angle the ratio of the length of its arc to the length of the radius of that arc is constant, *i.e.* for a fixed angle the ratio *arc : radius* is the same no matter what the length of the radius. In the figure, for the angle θ ,

$$\frac{OA}{AA'} = \frac{OB}{BB'} = \frac{OC}{CC'} = \dots.$$

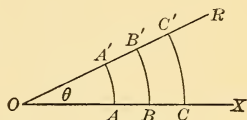


FIG. 5.

That this ratio of arc to radius for a fixed angle is constant follows from the established geometrical principles :

1. The circumference of any circle is 2π times its radius.
2. Angles at the centre are in the same ratio as their arcs.

The Radian. It follows that an angle whose arc is equal in length to the radius is a constant angle for all circles, since in four right angles, or the perigon, there are always 2π such angles. *This constant angle, whose arc is equal in length to the radius, is taken as the unit angle of circular measure, and is called the radian.* From the definition we have

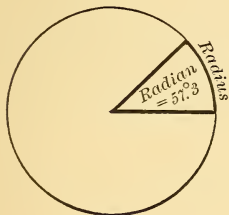


FIG. 6.

$$4 \text{ right angles} = 360^{\circ} = 2\pi \text{ radians,}$$

$$2 \text{ right angles} = 180^{\circ} = \pi \text{ radians,}$$

$$1 \text{ right angle} = 90^{\circ} = \frac{\pi}{2} \text{ radians.}$$

π is a numerical quantity, 3.14159+, and not an angle. When we speak of 180° as π , 90° as $\frac{\pi}{2}$, etc., we always mean π *radians*, $\frac{\pi}{2}$ *radians*, etc.

9. To change from one system of measurement to the other we use the relation,

$$2\pi \text{ radians} = 360^\circ.$$

$$\therefore 1 \text{ radian} = \frac{180^\circ}{\pi} = 57^\circ.2958-;$$

i.e. the radian is $57^\circ.3$, approximately.

Ex. 1. Express in radians $75^\circ 30'$.

$$75^\circ 30' = 75^\circ.5; 1 \text{ radian} = 57^\circ.3.$$

$$\therefore 75^\circ 30' = \frac{75.5}{57.3} = 1.317 \text{ radians.}$$

2. Express in degree measure 3.6 radians.

$$1 \text{ radian} = 57^\circ.3.$$

$$\therefore 3.6 \text{ radians} = 3.6 \times 57^\circ.3 = 206^\circ 16' 48''.$$

EXAMPLES.

1. Construct, approximately, the following angles: 50° , -20° , 90° , 179° , -135° , 400° , -380° , 1140° , $\frac{\pi}{4}$ radians, $\frac{\pi}{3}$ radians, $-\frac{\pi}{6}$ radians, 3π radians, $-\frac{3\pi}{4}$ radians, $\frac{12\pi}{5}$ radians. Of which quadrant is each angle?

2. What is the measure of:

(a) $\frac{5}{4}$ of a right angle, when 30° is the unit of measure?

(b) an acre, when a square whose side is 10 rds. is the unit?

(c) m miles, when y yards is the unit?

3. What is the unit of measure, when the measure of $2\frac{1}{2}$ miles is 50?

4. The Michigan Central R.R. is 535 miles long, and the Ann Arbor R.R. is 292 miles long. Express the length of the first in terms of the second as a unit.

5. What will be the measure of the radian when the right angle is taken for the unit? Of the right angle when the radian is the unit?

6. In which quadrant is 45° ? 10° ? -60° ? 145° ? 1145° ? -725° ? Express each in right angles; in radians.

7. Express in sexagesimal measure

$$\frac{\pi}{3}, \frac{\pi}{12}, 1, 6.28, \frac{1}{\pi}, \frac{7\pi}{3}, -\frac{4\pi}{3}, \text{ radians.}$$

8. Express in each system an interior angle of a regular hexagon; an exterior angle.

9. Find the distance in miles between two places on the earth's equator which are $11^{\circ} 15'$ apart. (The earth's radius is about 3963 miles.)

10. Find the length of an arc which subtends an angle of 4 radians at the centre of a circle of radius 12 ft. 3 in.

11. An arc 15 yds. long contains 3 radians. Find the radius of the circle.

12. Show that the hour and minute hands of a watch turn through angles of $30'$ and 6° respectively per minute; also find in degrees and in radians the angle turned through by the minute hand in 3 hrs. 20 mins.

13. Find the number of seconds in an arc of 1 mile on the equator; also the length in miles of an arc of $1'$ (1 knot).

14. Find to three decimal places the radius of a circle in which the arc of $71^{\circ} 36' 3''.6$ is 15 in. long.

15. Find the ratio of $\frac{\pi}{6}$ to 5° .

16. What is the shortest distance measured on the earth's surface from the equator to Ann Arbor, latitude $+42^{\circ} 16' 48''$?

17. The difference of two angles is 10° , and the circular measure of their sum is 2. Find the circular measure of each angle.

18. A water wheel of radius 6 ft. makes 30 revolutions per minute. Find the number of miles per hour travelled by a point on the rim.

CHAPTER II.

THE TRIGONOMETRIC FUNCTIONS.

10. Trigonometry, as the word indicates, was originally concerned with the measurement of triangles. It now includes the analytical treatment of certain *functions of angles*, as well as the solution of triangles by means of certain relations between the functions of the angles of those triangles.

11. Function. If one quantity depends upon another for its value, the first is called a *function* of the second. It always follows that the second quantity is also a function of the first; and, in general, functions are so related that if one is constant the other is constant, and if either varies in value, the other varies. This relation may be extended to any number of mutually dependent quantities.

Illustration. If a train moves at a rate of 30 miles per hour, the distance travelled is a function of the rate and time, the time is a function of the rate and distance, and the rate is a function of the time and distance.

Again, the circumference of a circle is a function of the radius, and the radius of the circumference, for so long as either is constant the other is constant, and if either changes in value, the other changes, since circumference and radius are connected by the relation $C = 2\pi R$.

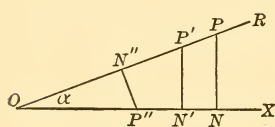


FIG. 7.

Once more, in the right triangle NOP , the ratio of any two sides is a function of the angle α , because all the right triangles of which α is one angle are similar, *i.e.* the ratio

of two corresponding sides is constant so long as α is constant, and varies if α varies.

Thus, the ratios

$$\frac{NP}{OP} = \frac{N'P'}{OP'} = \frac{N''P''}{OP''}$$

and

$$\frac{ON}{NP} = \frac{ON'}{N'P'} = \frac{ON''}{N''P''}, \text{ etc.,}$$

depend on α for their values, *i.e.* are functions of α .

12. The trigonometric functions. In trigonometry six functions of angles are usually employed, called the *trigonometric functions*.

By definition these functions are the six ratios between the sides of the triangle of reference of the given angle. The triangle of reference is formed by drawing from some point in the initial line, or the initial line produced, a perpendicular to that line meeting the terminal line of the angle.

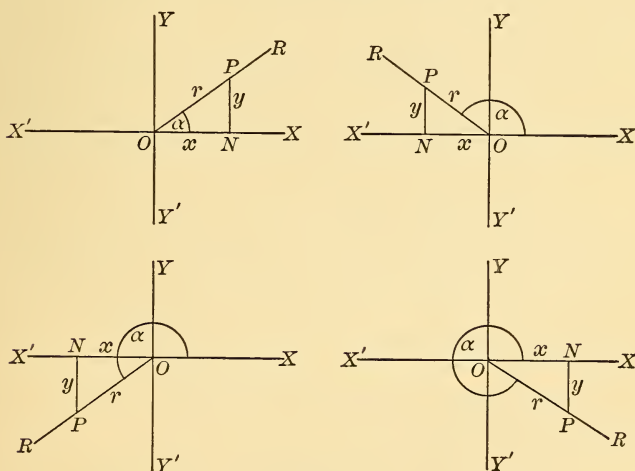


FIG. 8.

Let α be an angle of any quadrant. Each triangle of reference of α , NOP , is formed by drawing a perpendicular to OX , or OX produced, meeting the terminal line OR in P .

of an angle equals the *cotangent* of its complementary angle, and the *secant* of an angle equals the *cosecant* of its complementary angle.

2. Express each side of triangle ABC in terms of another side, and some function of an angle in all possible ways, as $a = b \tan A$, etc.

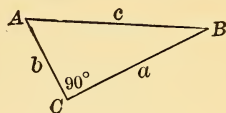


FIG. 10.

14. **Constancy of the trigonometric functions.** It is important to notice why these ratios are *functions of the angle*, i.e. are the same for equal angles and different for unequal angles. This is shown by the principles of similar triangles.



FIG. 11.

In each figure show that in all possible triangles of reference for α the ratios are the same, but in the triangles of reference for α and α' , respectively, the ratios are different.

The student must notice that $\sin \alpha$ is a *single symbol*. It is the *name of a number*, or *fraction*, belonging to the angle α ; and if it be at any time convenient, we may denote $\sin \alpha$ by a *single letter*, such as o , or x . Also, $\sin^2 \alpha$ is an abbreviation for $(\sin \alpha)^2$, i.e. for $(\sin \alpha) \times (\sin \alpha)$. Such abbreviations are used because they are convenient. Lock, *Elementary Trigonometry*.

15. **Fundamental relations.** From the definitions of Art. 13 the following reciprocal relations are apparent :

$$\sin \alpha = \frac{1}{\csc \alpha},$$

$$\csc \alpha = \frac{1}{\sin \alpha},$$

$$\cos \alpha = \frac{1}{\sec \alpha},$$

$$\sec \alpha = \frac{1}{\cos \alpha},$$

$$\tan \alpha = \frac{1}{\cot \alpha},$$

$$\cot \alpha = \frac{1}{\tan \alpha}.$$

Also from the definitions,

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha},$$

$$\cot \alpha = \frac{\cos \alpha}{\sin \alpha}.$$

From the right triangle NOP , page 9,

$$y^2 + x^2 = r^2;$$

whence (1) $\frac{y^2}{r^2} + \frac{x^2}{r^2} = 1,$

(2) $\frac{y^2}{x^2} + 1 = \frac{r^2}{x^2},$

(3) $1 + \frac{x^2}{y^2} = \frac{r^2}{y^2}.$

From (1) $\sin^2 \alpha + \cos^2 \alpha = 1$; $\sin \alpha = \sqrt{1 - \cos^2 \alpha}$; $\cos \alpha = ?$

(2) $\tan^2 \alpha + 1 = \sec^2 \alpha$; $\tan \alpha = \sqrt{\sec^2 \alpha - 1}$; $\sec \alpha = ?$

(3) $1 + \cot^2 \alpha = \csc^2 \alpha$; $\cot \alpha = \sqrt{\csc^2 \alpha - 1}$; $\csc \alpha = ?$

The foregoing definitions and fundamental relations are of the highest importance, and must be mastered at once. The student of trigonometry is helpless without perfect familiarity with them.

These relations are true for all values of α , positive or negative, but the signs of the functions are not in all cases positive, as appears from the fact that in the triangles of reference in Fig. 8 x and y are sometimes negative. The equations $\sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$, $\tan \alpha = \pm \sqrt{\sec^2 \alpha - 1}$, $\cot \alpha = \pm \sqrt{\csc^2 \alpha - 1}$, have the double sign $\pm$. Which sign is to be used in a given case depends on the quadrant in which α lies.

16. The relations of Art. 15 enable us to express any function in terms of any other, or when one function is given, to find all the others.

Ex. 1. To express the other functions in terms of tangent:

$$\sin \alpha = \frac{1}{\csc \alpha} = \frac{1}{\sqrt{1 + \cot^2 \alpha}} = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}; \quad \cot \alpha = \frac{1}{\tan \alpha};$$

$$\cos \alpha = \frac{1}{\sec \alpha} = \frac{1}{\sqrt{1 + \tan^2 \alpha}}; \quad \sec \alpha = \sqrt{1 + \tan^2 \alpha};$$

$$\tan \alpha = \tan \alpha; \quad \csc \alpha = \frac{\sqrt{1 + \tan^2 \alpha}}{\tan \alpha}.$$

In like manner determine the relations to complete the following table :

	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$	$\sec \alpha$	$\csc \alpha$
$\sin \alpha$			$\frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}$			
$\cos \alpha$			$\frac{1}{\sqrt{1 + \tan^2 \alpha}}$			
$\tan \alpha$			$\tan \alpha$			
$\cot \alpha$			$\frac{1}{\tan \alpha}$			
$\sec \alpha$			$\sqrt{1 + \tan^2 \alpha}$			
$\csc \alpha$			$\frac{\sqrt{1 + \tan^2 \alpha}}{\tan \alpha}$			

2. Given $\sin \alpha = \frac{3}{4}$; find the other functions.

$$\cos \alpha = \sqrt{1 - \frac{9}{16}} = \frac{1}{4}\sqrt{7}; \quad \tan \alpha = \frac{\frac{3}{4}}{\frac{1}{4}\sqrt{7}} = \frac{3}{\sqrt{7}} = \frac{3}{7}\sqrt{7};$$

$$\cot \alpha = \frac{1}{\frac{3}{7}\sqrt{7}} = \frac{1}{3}\sqrt{7}; \quad \sec \alpha = \frac{1}{\frac{1}{4}\sqrt{7}} = \frac{4}{\sqrt{7}} = \frac{4}{7}\sqrt{7}; \quad \csc \alpha = \frac{1}{\frac{3}{4}} = \frac{4}{3}.$$

3. Given $\tan \phi + \cot \phi = 2$; find $\sin \phi$.

$$\tan \phi + \frac{1}{\tan \phi} = 2, \quad \tan^2 \phi - 2 \tan \phi + 1 = 0, \quad \tan \phi = 1.$$

$$\therefore \sin \phi = \frac{\tan \phi}{\sqrt{1 + \tan^2 \phi}} = \frac{1}{\sqrt{2}}.$$

Or, expressing in terms of sine directly, $\frac{\sin \phi}{\cos \phi} + \frac{\cos \phi}{\sin \phi} = 2$,

$$\sin^2 \phi + \cos^2 \phi = 2 \sin \phi \cos \phi, \quad \sin^2 \phi - 2 \sin \phi \cos \phi + \cos^2 \phi = 0;$$

whence $\sin \phi - \cos \phi = 0$, $\sin \phi = \cos \phi$. $\therefore \sin \phi = \frac{1}{\sqrt{2}}$.

4. Prove $\sec^4 x - \sec^2 x = \tan^2 x + \tan^4 x$.

$$\sec^4 x - \sec^2 x = \sec^2 x (\sec^2 x - 1) = (1 + \tan^2 x) \tan^2 x = \tan^2 x + \tan^4 x.$$

5. Prove $\sin^6 y + \cos^6 y = 1 - 3 \sin^2 y \cos^2 y$.

$$\begin{aligned} \sin^6 y + \cos^6 y &= (\sin^2 y + \cos^2 y)(\sin^4 y - \sin^2 y \cos^2 y + \cos^4 y) \\ &= (\sin^2 y + \cos^2 y)^2 - 3 \sin^2 y \cos^2 y = 1 - 3 \sin^2 y \cos^2 y. \end{aligned}$$

6. Prove $\frac{\tan z}{1 - \cot z} + \frac{\cot z}{1 - \tan z} = \sec z \csc z + 1.$

$$\begin{aligned} \frac{\tan z}{1 - \cot z} + \frac{\cot z}{1 - \tan z} &= \frac{\frac{\sin z}{\cos z}}{1 - \frac{\cos z}{\sin z}} + \frac{\frac{\cos z}{\sin z}}{1 - \frac{\sin z}{\cos z}} \\ &= \frac{\sin^2 z}{\cos z (\sin z - \cos z)} + \frac{\cos^2 z}{\sin z (\cos z - \sin z)} \\ &= \frac{\sin^3 z - \cos^3 z}{\sin z \cos z (\sin z - \cos z)} = \frac{\sin^2 z + \sin z \cos z + \cos^2 z}{\sin z \cos z} \\ &= \frac{1 + \sin z \cos z}{\sin z \cos z} = \frac{1}{\sin z \cos z} + 1 = \sec z \csc z + 1. \end{aligned}$$

In solving problems like 3, 4, 5, and 6 above, it is usually safe, if no other step suggests itself, to express all other functions of one member in terms of sine and cosine. The resulting expression may then be reduced by the principles of algebra to the expression in the other member of the equation. For further suggestions as to the solution of trigonometric equations and identities see page 66.

EXAMPLES.

1. Find the values of all the functions of α , if $\sin \alpha = \frac{3}{5}$; if $\tan \alpha = \frac{4}{3}$; if $\sec \alpha = 2$; if $\cos \alpha = \frac{1}{3}\sqrt{3}$; if $\cot \alpha = \frac{1}{7}$; if $\csc \alpha = \sqrt{2}$.

2. Compute the functions of each acute angle in the right triangles whose sides are: (1) 3, 4, 5; (2) 8, 15, 17; (3) 480, 31, 481; (4) a, b, c ;

(5) $\frac{2xy}{x-y}, \frac{x^2+y^2}{x-y}, x+y.$

3. If $\cos \alpha = \frac{8}{17}$, find the value of $\frac{\sin \alpha + \tan \alpha}{\cos \alpha - \cot \alpha}.$

4. If $2 \cos \alpha = 2 - \sin \alpha$, find $\tan \alpha.$

5. If $\sec^2 \alpha \csc^2 \alpha - 4 = 0$, find $\cot \alpha.$

6. Solve for $\sin \beta$ in $13 \sin \beta + 5 \cos^2 \beta = 11.$

Prove

7. $\sin^4 \phi - \cos^4 \phi = 1 - 2 \cos^2 \phi.$

8. $(\sin \alpha + \cos \alpha)(\sin \alpha - \cos \alpha) = 2 \sin^2 \alpha - 1.$

9. $(\sec \alpha + \tan \alpha)(\sec \alpha - \tan \alpha) = 1.$

10. $\cos^2 \beta (\sec^2 \beta - 2 \sin^2 \beta) = \cos^4 \beta + \sin^4 \beta.$

11. $\tan v + \sec v = \frac{\cos v}{1 - \sin v}.$

12. $\frac{\sin w}{1 - \cos w} = \frac{1 + \cos w}{\sin w}.$

13. $(\sec \theta + 1)(1 - \cos \theta) = \tan^2 \theta \cos \theta.$

$$14. \sin^4 t - \sin^2 t = \cos^4 t - \cos^2 t.$$

$$15. \frac{\sin \beta}{1 - \sin \beta} + \frac{1 + \sin \beta}{\sin \beta} = \sec^2 \beta (\csc \beta + 1).$$

$$16. (\tan A + \cot A)^2 = \sec^2 A \csc^2 A.$$

$$17. \sec^2 x - \sin^2 x = \tan^2 x + \cos^2 x.$$

In the triangle ABC , right angled at C ,

$$18. \text{ Given } \cos A = \frac{8}{17}, BC = 45, \text{ find } \tan B, \text{ and } AB.$$

$$19. \text{ If } \cos A = \frac{m^2 - n^2}{m^2 + n^2}, \text{ and } AB = m^2 + n^2, \text{ find } AC \text{ and } BC.$$

$$20. \text{ If } AC = m + n, BC = m - n, \text{ find } \sin A, \cos B.$$

$$21. \text{ In examples 18, 19, 20, above, prove } \sin^2 A + \cos^2 A = 1; \\ 1 + \tan^2 A = \sec^2 A.$$

17. Functions of certain angles. The trigonometric functions are numerical quantities which may be determined for any angle. In general these values are taken from tables prepared for the purpose, but the principles already studied enable us to calculate the functions of the following angles.

18. Functions of 0° . If α be a very small angle, the value of y is very small, and decreases as α diminishes.

Clearly, when α approaches

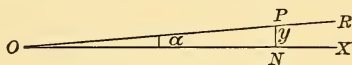


FIG. 12.

0° as a limit, y likewise approaches 0, and x approaches r , so that when $\alpha = 0^\circ$,

$$y = 0, \text{ and } x = r.$$

$$\therefore \sin 0^\circ = \frac{y}{r} = 0, \quad \cot 0^\circ = \frac{1}{\tan 0^\circ} = \infty,$$

$$\cos 0^\circ = \frac{x}{r} = 1, \quad \sec 0^\circ = \frac{1}{\cos 0^\circ} = 1,$$

$$\tan 0^\circ = \frac{y}{x} = 0, \quad \csc 0^\circ = \frac{1}{\sin 0^\circ} = \infty.$$

In the figure of Art. 18, by diminishing α it is clear that we can make y as small as we please, and by making α small enough, we can make the value of y less than any assignable quantity, however small, so that $\sin \alpha$ approaches as a limit 0. This is what we mean when we say $\sin 0^\circ = 0$. In like manner, it is evident that, by sufficiently diminishing α we can make $\cot \alpha$ greater than any assignable quantity. This we express by saying $\cot 0^\circ = \infty$.

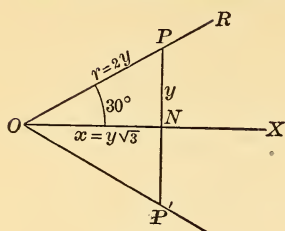
19. Functions of 30° .

FIG. 13.

Let NOP be the triangle of reference for an angle of 30° . Make triangle $NOP' = NOP$. Then POP' is an equilateral triangle (why?), and ON bisects PP' . Hence

$$PP' = r = 2y.$$

$$\text{Also } x = \sqrt{r^2 - y^2} = \sqrt{3}y = y\sqrt{3}.$$

$$\therefore \sin 30^\circ = \frac{y}{r} = \frac{y}{2y} = \frac{1}{2},$$

$$\csc 30^\circ = 2,$$

$$\cos 30^\circ = \frac{x}{r} = \frac{y\sqrt{3}}{2y} = \frac{1}{2}\sqrt{3},$$

$$\sec 30^\circ = \frac{2}{\sqrt{3}},$$

$$\tan 30^\circ = \frac{y}{x} = \frac{y}{y\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3},$$

$$\cot 30^\circ = \sqrt{3}.$$

20. Functions of 45° . Let NOP be the triangle of reference. If angle $NOP = 45^\circ$, $OPN = 45^\circ$.

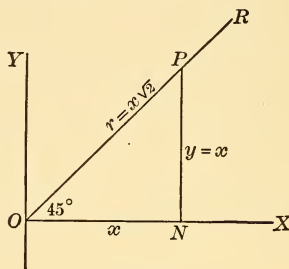


FIG. 14.

$$\therefore y = x, \text{ and } r = \sqrt{x^2 + y^2} = \sqrt{2}x = x\sqrt{2}.$$

Then

$$\sin 45^\circ = \frac{y}{r} = \frac{x}{x\sqrt{2}} = \frac{1}{2}\sqrt{2},$$

$$\cos 45^\circ = \frac{x}{r} = \frac{x}{x\sqrt{2}} = \frac{1}{2}\sqrt{2},$$

$$\tan 45^\circ = \frac{y}{x} = \frac{x}{x} = 1.$$

Find $\cot 45^\circ$, $\sec 45^\circ$, $\csc 45^\circ$.

21. Functions of 60° . The functions of 60° may be computed by means of the figure, or they may be written from the functions of the complement, or 30° . Let the student in both ways show that

$$\sin 60^\circ = \frac{1}{2}\sqrt{3}, \quad \cos 60^\circ = \frac{1}{2},$$

$$\tan 60^\circ = \sqrt{3}.$$

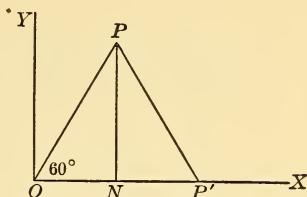


FIG. 15.

Compute also the other functions of 60° .

22. Functions of 90° . If α be an angle very near 90° , the value of x is very small, and decreases as α increases toward 90° . Clearly when α approaches 90° as a limit, x approaches 0, and y approaches r , so that when

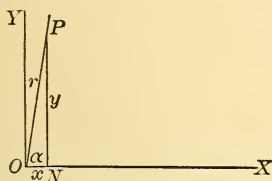


FIG. 16.

$$\alpha = 90^\circ, \quad x = 0, \quad y = r.$$

$$\therefore \sin 90^\circ = 1, \quad \cos 90^\circ = 0, \quad \tan 90^\circ = \infty.$$

Compute the other functions. Also find the functions of 90° from those of its complement, 0° .

23. It is of great convenience to the student to remember the functions of these angles. *They are easily found by recalling the relative values of the sides of the triangles of reference for the respective angles*, or the values of the other functions may readily be computed by means of the fundamental relations, if the values of the sine and cosine are remembered, as follows :

α	0°	30°	45°	60°	90°
sine	$\frac{1}{2}\sqrt{0}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{4}$
cosine	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{0}$

ORAL WORK.

1. Which is greater, $\sin 45^\circ$ or $\frac{1}{2} \sin 90^\circ$? $\sin 60^\circ$ or $2 \sin 30^\circ$?
2. From the functions of 60° , find those of 30° ; from the functions of 90° , those of 0° . Why are the functions of 45° equal to the co-functions of 45° ?
3. Given $\sin A = \frac{1}{3}$, find $\cos A$; $\tan A$.
4. Show that $\sin B \csc B = 1$; $\cos C \sec C = 1$; $\cot x \tan x = 1$.
5. Show that $\sec^2 \theta - \tan^2 \theta = \csc^2 \theta - \cot^2 \theta = \sin^2 \theta + \cos^2 \theta$.
6. Show that $\tan 30^\circ \tan 60^\circ = \cot 60^\circ \cot 30^\circ = \tan 45^\circ$.
7. Show that $\tan 60^\circ \sin^2 45^\circ = \cos 30^\circ \sin 90^\circ$.
8. Show that $\cos \alpha \tan \alpha = \sin \alpha$; $\sin \beta \cot \beta = \cos \beta$.
9. Show that $\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \cos 60^\circ = \frac{1}{2} \cos 0^\circ$.
10. Show that $(\tan y + \cot y) \sin y \cos y = 1$.

EXAMPLES.

1. Show that $\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ = \sin 90^\circ$.
2. Show that $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos 30^\circ$.
3. Show that $\sin 45^\circ \cos 0^\circ - \cos 45^\circ \sin 0^\circ = \cos 45^\circ$.
4. Show that $\cos^2 45^\circ - \sin^2 45^\circ = \cos 90^\circ$.
5. Show that $\frac{\tan 45^\circ + \tan 0^\circ}{1 - \tan 45^\circ \tan 0^\circ} = \tan 45^\circ$.

If $A = 60^\circ$, verify

6. $\sin \frac{A}{2} = \sqrt{\frac{1 - \cos A}{2}}$.
7. $\tan \frac{A}{2} = \sqrt{\frac{1 - \cos A}{1 + \cos A}}$.
8. $\cos A = 2 \cos^2 \frac{A}{2} - 1 = 1 - 2 \sin^2 \frac{A}{2}$.

If $\alpha = 0^\circ$, $\beta = 30^\circ$, $\gamma = 45^\circ$, $\delta = 60^\circ$, $\epsilon = 90^\circ$, find the values of

9. $\sin \beta + \cos \delta$.
10. $\cos \beta + \tan \delta$.
11. $\sin \beta \cos \delta + \cos \beta \sin \delta - \sin \epsilon$.
12. $(\sin \beta + \sin \epsilon)(\cos \alpha + \cos \delta) - 4 \sin \alpha (\cos \gamma + \sin \epsilon)$.

24. Variations in the trigonometric functions.

Signs. Thus far no account has been taken of the *signs of the functions*. By the definitions it appears that these depend on the signs of x , y , and r . Now r is always positive, and from the figures it is seen that x is positive in the first

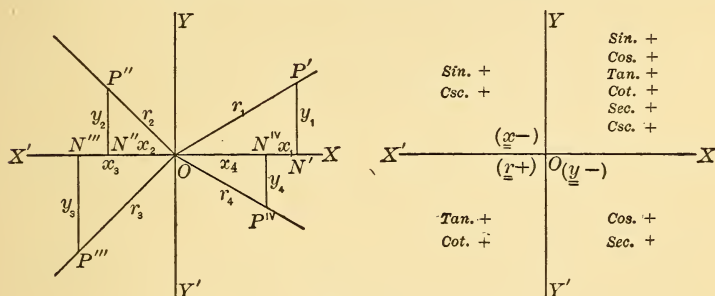


FIG. 17.

and fourth quadrants, and y is positive in the first and second. Hence

For an angle in the *first quadrant* all functions are *positive*, since x , y , r are *positive*.

In the *second quadrant* x alone is *negative*, so that those functions whose ratios involve x , viz. *cosine*, *tangent*, *cotangent*, *secant*, are *negative*; the others, *sine* and *cosecant*, are *positive*.

In the *third quadrant* x and y are both *negative*, so that those functions involving r , viz. *sine*, *cosine*, *secant*, *cosecant*, are *negative*; the others, *tangent* and *cotangent*, are *positive*.

In the *fourth quadrant* y is *negative*, so that *sine*, *tangent*, *cotangent*, *cosecant* are *negative*, and *cosine* and *secant*, *positive*.

Values. In the triangle of reference of any angle, the hypotenuse r is never less than x or y . Then if r be taken of any fixed length, as the angle varies, the base and perpendicular of the triangle of reference may each vary in length from 0 to r . Hence the ratios $\frac{x}{r}$ and $\frac{y}{r}$ can never be greater than 1, nor if x and y are negative, less than -1 ; and $\frac{r}{x}$, $\frac{r}{y}$

cannot have values between $+1$ and -1 . But the ratios $\frac{y}{x}$ and $\frac{x}{y}$ may vary without limit, *i.e.* from $+\infty$ to $-\infty$.

Therefore the possible values of the functions of an angle are:

sine and cosine between $+1$ and -1 ,

i.e. sine and cosine cannot be numerically greater than 1;

tangent and cotangent between $+\infty$ and $-\infty$,

i.e. tangent and cotangent may have any real value;

secant and cosecant between $+\infty$ and $+1$, and -1 and $-\infty$,

i.e. secant and cosecant may have any real values, except values between $+1$ and -1 .

These limits are indicated in the following figures. The student should carefully verify.

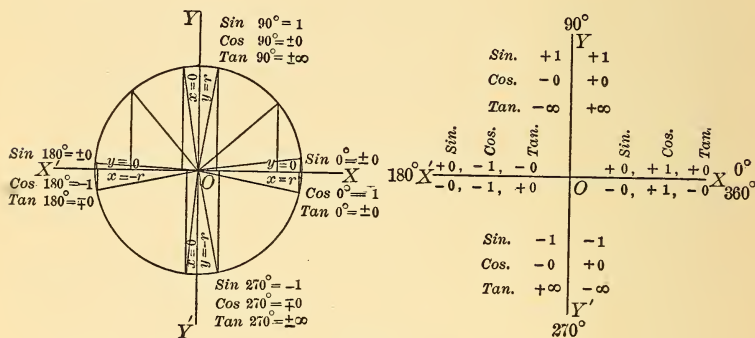


FIG. 18.

25. In tracing the changes in the values of the functions as α changes from 0° to 360° , consider the revolving line r as of fixed length. Then x and y may have any length between 0 and r .

Sine. At 0° , $\sin \alpha = \frac{y}{r} = \frac{0}{r} = 0$. As α increases through the first quadrant, y increases from 0 to r , whence $\frac{y}{r}$ increases from 0 to 1 . In passing to 180° $\sin \alpha$ decreases from 1 to 0 ,

since y decreases from r to 0. As α passes through 180° , y changes sign, and in the third quadrant decreases to negative r , so that $\sin \alpha$ decreases from 0 to -1 . In the fourth quadrant y increases from negative r to 0, and hence $\sin \alpha$ increases from -1 to 0.

Cosine depends on changing values of x . Show that, as α increases from 0° to 360° , $\cos \alpha$ varies in the four quadrants as follows: 1 to 0, 0 to -1 , -1 to 0, 0 to 1.

Tangent depends on changing values of both y and x .

At 0° , $y = 0$, $x = r$, at 180° , $y = 0$, $x = -r$,

at 90° , $x = 0$, $y = r$, at 270° , $x = 0$, $y = -r$.

Hence $\tan 0^\circ = \frac{y}{x} = \frac{0}{r} = 0$. As α passes to 90° , y increases

to r , and x decreases to 0, so that $\tan \alpha$ increases from 0 to ∞ . As α passes through 90° , x changes sign, so that $\tan \alpha$ changes from positive to negative by passing through ∞ . In the second quadrant x decreases to negative r , y to 0, and $\tan \alpha$ passes from $-\infty$ to 0. As α passes through 180° , $\tan \alpha$ changes from minus to plus by passing through 0, because at 180° y changes to minus. In the third quadrant $\tan \alpha$ passes from 0 to ∞ , changing sign at 270° by passing through ∞ , because at 270° x changes to plus. In the fourth quadrant $\tan \alpha$ passes from $-\infty$ to 0.

Cotangent. In like manner show that $\cot \alpha$ passes through the values ∞ to 0, 0 to $-\infty$, ∞ to 0, 0 to $-\infty$, as α passes from 0° to 360° .

Secant depends on x for its value. Noting the change in x as under cosine, we see that secant passes from 1 to ∞ , $-\infty$ to -1 , -1 to $-\infty$, ∞ to 1.

Cosecant passes through the values ∞ to 1, 1 to ∞ , $-\infty$ to -1 , -1 to $-\infty$.

The student should trace the changes in each function fully, as has been done for sine and tangent, giving the reasons at each step.

α	0° to 90°	90° to 180°	180° to 270°	270° to 360°
sin	0 to 1	1 to 0	- 0 to - 1	- 1 to - 0
cos	1 to 0	- 0 to - 1	- 1 to - 0	0 to 1
tan	0 to ∞	- ∞ to - 0	0 to ∞	- ∞ to - 0
cot	∞ to 0	- 0 to - ∞	∞ to 0	- 0 to - ∞
sec	1 to ∞	- ∞ to - 1	- 1 to - ∞	∞ to 1
csc	∞ to 1	1 to ∞	- ∞ to - 1	- 1 to - ∞

* **26. Graphic representation of functions.** These variations are clearly brought out by graphic representations of the functions. Two cases will be considered: I, when α is a constant angle; II, when α is a variable angle.

I. *When α is a constant angle.*

The trigonometric functions are ratios, pure numbers. By so choosing the triangle of reference that the denominator of the ratio is a side of unit length, the side forming the numerator of that ratio will be a geometrical representation of the value of that function, *e.g.* if in Fig. 19 $r=1$, then $\sin \alpha = \frac{y}{r} = \frac{y}{1} = y$. This may be done by making α a central angle in a circle of radius 1, and drawing triangles of reference as follows:

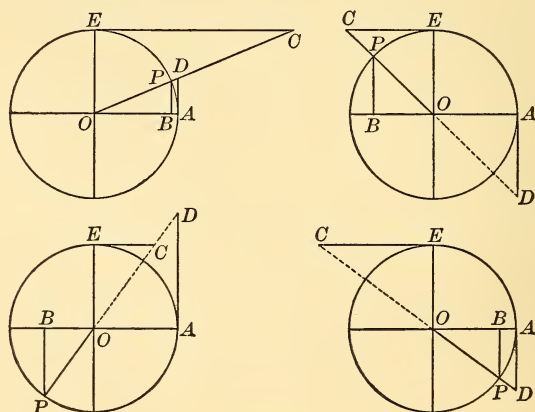


FIG. 19.

In all the figures $\angle AOP = \alpha$, and

$$\sin \alpha = \frac{BP}{OP} = \frac{BP}{1} = BP,$$

$$\cos \alpha = \frac{OB}{OP} = \frac{OB}{1} = OB,$$

$$\tan \alpha = \frac{BP}{OB} = \frac{AD}{OA} = \frac{AD}{1} = AD,$$

$$\cot \alpha = \frac{OA}{AD} = \frac{OE}{EC} = \frac{OE}{1} = EC,$$

$$\sec \alpha = \frac{OP}{OB} = \frac{OD}{OA} = \frac{OD}{1} = OD,$$

$$\csc \alpha = \frac{OP}{BP} = \frac{OC}{OE} = \frac{OC}{1} = OC.$$

It appears then that, by taking a radius 1,

sine is represented by the perpendicular to the initial line, drawn from that line to the terminus of the arc subtending the given angle;

cosine is represented by the line from the vertex of the angle to the foot of the sine;

tangent is represented by the geometrical tangent drawn from the origin of the arc to the terminal line, produced if necessary;

cotangent is represented by the geometrical tangent drawn from a point 90° from the origin of the arc to the terminal line, produced if necessary;

secant is represented by the terminal line, or the terminal line produced, from the origin to its intersection with the tangent line;

cosecant is represented by the terminal line, or the terminal line produced, from the origin to its intersection with the cotangent line.

These lines are not the functions, but in triangles drawn as explained their lengths are equal to the numerical values of the functions, and in this sense the lines may be said to represent the functions. It will be noticed also that their directions indicate the signs of the functions. Let the student by means of these representations verify the results of Arts. 24 and 25.

II. When α is a variable angle.

Take XX' and YY' as axes of reference, and let angle units be measured along the x -axis, and values of the functions parallel to the y -axis, as in Art. 3. We may write corresponding values of the angle and the functions thus:

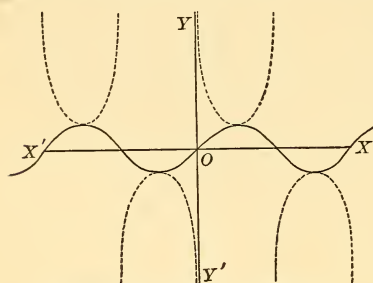
$$\alpha = 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ, 210^\circ, 225^\circ,$$

$$\sin \alpha = 0, \frac{1}{2}, \frac{1}{2}\sqrt{2}, \frac{1}{2}\sqrt{3}, 1, \frac{1}{2}\sqrt{3}, \frac{1}{2}\sqrt{2}, \frac{1}{2}, 0, -\frac{1}{2}, -\frac{1}{2}\sqrt{2},$$

$$\alpha = 240^\circ, 270^\circ, 300^\circ, 315^\circ, 330^\circ, 360^\circ, -30^\circ, -45^\circ, -60^\circ, -90^\circ, \text{etc.},$$

$$\sin \alpha = -\frac{1}{2}\sqrt{3}, -1, -\frac{1}{2}\sqrt{3}, -\frac{1}{2}\sqrt{2}, -\frac{1}{2}, 0, -\frac{1}{2}, -\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{3}, -1, \text{etc.}$$

These values will be sufficient to determine the form of the curve representing the function. By taking angles between



Curves of Sine and Cosecant.

Sine —————
Cosecant - - - - -

FIG. 20.

those above, and computing the values of the function, as given in mathematical tables, the form of the curve can be determined to any required degree of accuracy. Reducing the above fractions to decimals, it will be convenient to make the y -units large in comparison with the x -units. In the figure one x -unit represents 15° , and one y -unit 0.25.

Measuring the angle values along the x -axis, and from these points of division measuring the corresponding values of $\sin \alpha$ parallel to the y -axis, as in Art. 3, we have, approximately,

$$\begin{aligned}
 OX_1 = 30^\circ &= 2 \text{ units}, & OX_2 = 45^\circ &= 3 \text{ units}, \\
 X_1Y_1 = \frac{1}{2} &= 2 \text{ units}, & X_2Y_2 = 0.71 &= 2.84 \text{ units},
 \end{aligned}$$

$$\begin{aligned}
 OX_3 &= 60^\circ = 4 \text{ units, etc.,} \\
 X_3Y_3 &= 0.86 = 3.44 \text{ units, etc.}
 \end{aligned}$$

We have now only to draw through the points Y_1, Y_2, Y_3 , etc., thus determined, a continuous curve, and we have the *sine-curve* or *sinusoid*.

The dotted curve in the figure is the *cosecant curve*. Let the student compute values, as above, and draw the curve.

In like manner draw the *cosine* and *secant* curves, as follows :

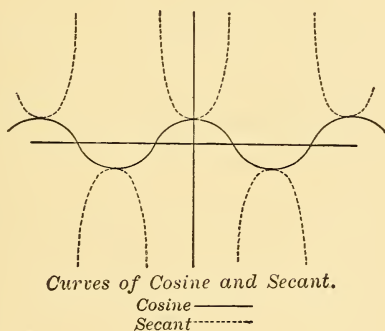


FIG. 21.

Tangent curve. Compute values for the angle α and for $\tan \alpha$, as before :

$$\begin{aligned}
 \alpha &= 0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ, 120^\circ, 135^\circ, 150^\circ, 180^\circ, 210^\circ, 225^\circ, 240^\circ, 270^\circ, \\
 \tan \alpha &= 0, \frac{1}{3}\sqrt{3}, 1, \sqrt{3}, \pm\infty, -\sqrt{3}, -1, -\frac{1}{3}\sqrt{3}, 0, \frac{1}{3}\sqrt{3}, 1, \sqrt{3}, \pm\infty,
 \end{aligned}$$

$$\begin{aligned}
 \alpha &= -30^\circ, -45^\circ, -60^\circ, -90^\circ, \text{ etc.,} \\
 \tan \alpha &= -\frac{1}{3}\sqrt{3}, -1, -\sqrt{3}, \pm\infty, \text{ etc.}
 \end{aligned}$$

Then lay off the values of α and of $\tan \alpha$ along the x , and parallel to the y -axis, respectively. It will be noted that,

as α approaches 90° , $\tan \alpha$ increases to ∞ , and when α passes 90° , $\tan \alpha$ is negative. Hence the value is measured parallel

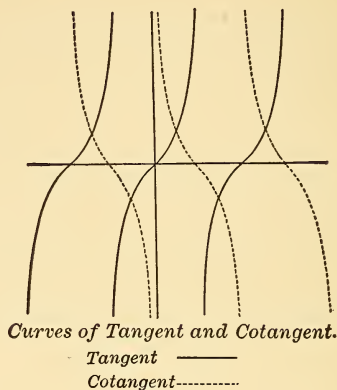


FIG. 22.

to the y -axis downward, thus giving a discontinuous curve, as in the figure.

* 27. The following principles are illustrated by the curves :

1. The sine and cosine are continuous for varying values of the angle, and lie within the limits $+1$ and -1 . Sine changes sign as the angle passes through 180° , 360° , ..., $n180^\circ$, while cosine changes sign as the angle passes through 90° , 270° , ..., $(2n+1)90^\circ$. Tangent and cotangent are discontinuous, the one as the angle approaches 90° , 270° , ..., $(2n+1)90^\circ$, the other as the angle approaches 180° , 360° , ..., $n180^\circ$, and each changes sign as the angle passes through these values. The limiting values of tangent and cotangent are $+\infty$ and $-\infty$.

2. A line parallel to the y -axis cuts any of the curves in but one point, showing that for any value of α there is but one value of any function of α . But a line parallel to the x -axis cuts any of the curves in an indefinite number of points, if at all, showing that for any value of the function there are an indefinite number of values, if any, of α .

3. The curves afford an excellent illustration of the variations in sign and value of the functions, as α varies from 0 to 360° , as discussed in Art. 25. Let the student trace these changes.

4. From the curves it is evident that the functions are *periodic*, i.e. each increase of the angle through 360° in the case of the sine and cosine, or through 180° in the case of the tangent and cotangent, produces a portion of the curve like that produced by the first variation of the angle within those limits.

5. The difference in rapidity of change of the functions at different values of α is important, and reference will be made to this in computations of triangles. (See Art. 64, Case III.) A glance at the curves shows that sine is changing in value rapidly at 0° , 180° , etc., while near 90° , 270° , etc., the rate of change is slow. But cosine has a slow rate of change at 0° , 180° , etc., and a rapid rate at 90° , 270° , etc. Tangent and cotangent change rapidly throughout.

Ex. Let the student discuss secant and cosecant curves.

ORAL WORK.

1. Express in radians 180° , 120° , 45° ; in degrees, $\frac{1}{7}$ radians, 2π , $\frac{2}{3}\pi$, $\frac{3}{2}\pi$.

2. If $\frac{1}{3}$ of a right angle be the unit, what is the measure of $\frac{1}{2}$ of a right angle? of 90° ? of 135° ?

3. Which is greater, $\cos 30^\circ$ or $\frac{1}{2} \cos 60^\circ$? $\tan \frac{\pi}{6}$ or $\cot \frac{\pi}{3}$? $\sin \frac{\pi}{4}$ or $\cos \frac{\pi}{4}$?

4. Express $\sin \alpha$ in terms of $\sec \alpha$; of $\tan \alpha$; $\tan \alpha$ in terms of $\cos \alpha$; of $\sec \alpha$.

5. Given $\sin \alpha = \frac{3}{5}$, find $\tan \alpha$. If $\tan \alpha = 1$, find $\sin \alpha$, $\csc \alpha$, $\cot \alpha$; also $\tan 2\alpha$, $\sin 2\alpha$, $\cos 2\alpha$.

6. If $\cos \alpha = \frac{1}{2}$, find $\sin \frac{\alpha}{2}$, $\tan \frac{\alpha}{2}$.

7. In what quadrant is angle t , if both $\sin t$ and $\cos t$ are minus? if $\sin t$ is plus and $\cos t$ minus? if $\tan t$ and $\cot t$ are both minus? if $\sin t$ and $\csc t$ are of the same sign? Why?

8. Of the numbers 3 , $\frac{4}{5}$, -5 , $-\frac{1}{3}$, a , $-b$, ∞ , 0 , which may be a value of $\sin p$? of $\sec p$? of $\tan p$? Why?

EXAMPLES.

1. If $\sin 26^\circ 40' = 0.44880$, find, correct to 0.00001, the cosine and tangent.

2. If $\tan \alpha = \sqrt{3}$, and $\cot \beta = \frac{1}{3}\sqrt{3}$, find $\sin \alpha \cos \beta - \cos \alpha \sin \beta$.

3. Evaluate $\frac{\sin 30^\circ \cot 30^\circ - \cos 60^\circ \tan 60^\circ}{\sin 90^\circ \cos 0^\circ}$.

Prove the identities:

4. $\tan A(1 - \cot^2 A) + \cot A(1 - \tan^2 A) = 0$.

5. $(\sin A + \sec A)^2 + (\cos A + \csc A)^2 = (1 + \sec A \csc A)^2$.

6. $\sin^2 x \cos x \csc x - \cos^3 x \csc x \sin^2 x + \cos^4 x \sec x \sin x = \sin^3 x \cos x + \cos^3 x \sin x$.

7. $\tan^2 w + \cot^2 w = \sec^2 w \csc^2 w - 2$.

8. $\sec^2 v + \cos^2 v = 2 + \tan^2 v \sin^2 v$.

9. $\cos^2 t + 1 = 2 \cos^3 t \sec t + \sin^2 t$.

10. $\csc^2 t - \sec^2 t = \cos^2 t \csc^2 t - \sin^2 t \sec^2 t$.

11. The sine of an angle is $\frac{m^2 - n^2}{m^2 + n^2}$; find the other functions.

12. If $\tan A + \sin A = m$, $\tan A - \sin A = n$, prove $m^2 - n^2 = 4\sqrt{mn}$.

Solve for one function of the angle involved the equations:

13. $\sin \theta + 2 \cos \theta = 1$.

16. $2 \sin^2 x + \cos x - 1 = 0$.

14. $\frac{\cos \alpha}{\tan \alpha} = \frac{3}{2}$.

17. $\sec^2 x - 7 \tan x - 9 = 0$.

15. $\sqrt{3} \csc^2 \theta = 4 \cot \theta$.

18. $3 \csc y + 10 \cot y - 35 = 0$.

19. $\sin^2 v - \frac{7}{3} \cos v - 1 = 0$.

20. $a \sec^2 w + b \tan w + c - a = 0$.

21. If $\frac{\sin A}{\sin B} = \sqrt{2}$, $\frac{\tan A}{\tan B} = \sqrt{3}$, find A and B .

22. Find to five decimal places the arc which subtends the angle of 1° at the centre of a circle whose radius is 4000 miles.

23. If $\csc A = \frac{2}{3}\sqrt{3}$, find the other functions, when A lies between $\frac{\pi}{2}$ and π .

24. In each of two triangles the angles are in G. P. The least angle of one of them is three times the least angle of the other, and the sum of the greatest angles is 240° . Find the circular measure of each of the angles.

CHAPTER III.

FUNCTIONS OF ANY ANGLE—INVERSE FUNCTIONS.

28. By an examination of the figure of Art. 24 it is seen that all the fundamental relations between the functions hold true for any value of α . The table of Art. 16 expresses the functions of α , whatever be its magnitude, in terms of each of the other functions of that angle if the $\pm$ sign be prefixed to the radicals.

The definitions of the trigonometric functions (Art. 12) apply to angles of any size and sign, but it is always possible to express the functions of any angle in terms of the functions of a *positive acute* angle.

The functions of any angle θ , greater than 360° , are the same as those of $\theta \pm n \cdot 360^\circ$, since θ and $\theta \pm n \cdot 360^\circ$ have the same triangle of reference. Thus the functions of 390° , or of 750° , are the same as the functions of $390^\circ - 360^\circ$, or of $750^\circ - 2 \cdot 360^\circ$, *i.e.* of 30° , as is at once seen by drawing a figure. So also the functions of -315° , or of -675° are the same as those of $-315^\circ + 360^\circ$, or of $-675^\circ + 2 \cdot 360^\circ$, *i.e.* of 45° .

For functions of angles less than 360° the relations of this chapter are important.

29. *To find the relations of the functions of $-\theta$, $90^\circ \pm \theta$, $180^\circ \pm \theta$, and $270^\circ \pm \theta$ to the functions of θ , θ being any angle.*

Four sets of figures are drawn, I for θ an acute angle, II for θ obtuse, III for θ an angle of the third quadrant, and IV for θ an angle of the fourth quadrant.

In every case generate the angles forming the compound angles separately, *i.e.* turn the revolving line first through

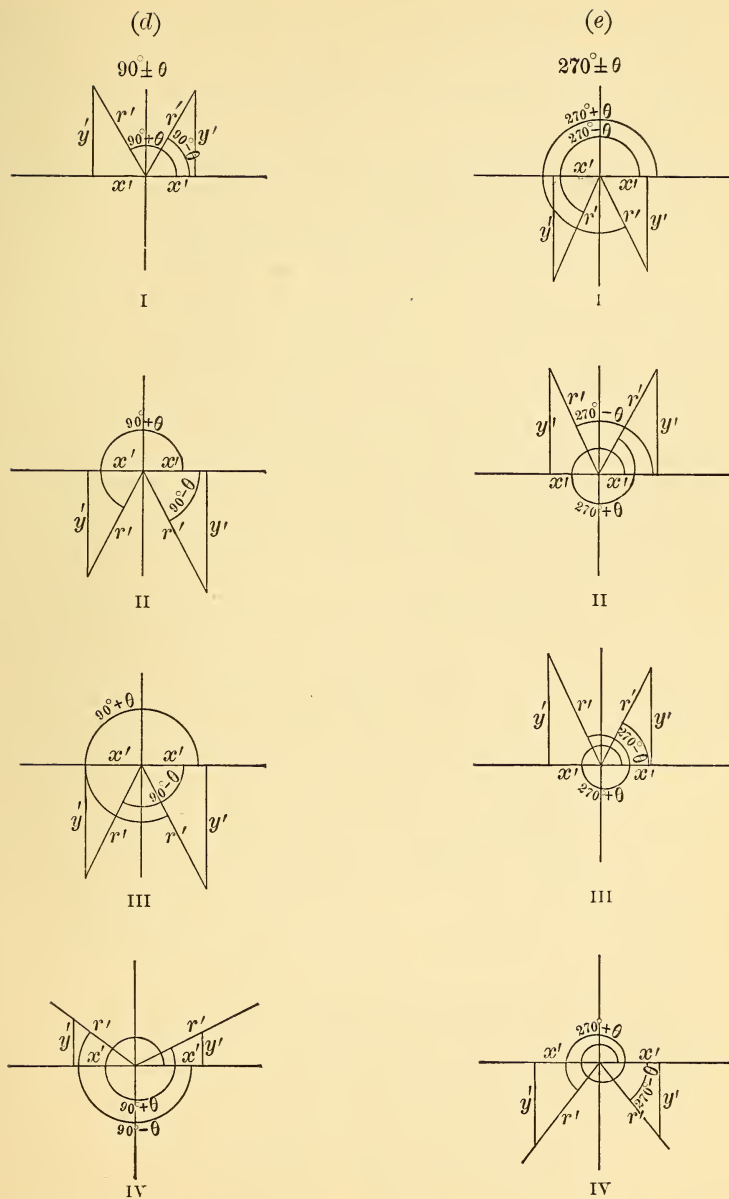


FIG. 23.

0° , 90° , 180° , or 270° , and then from this position through θ , or $-\theta$, as the case may be. Form the triangles of reference for (a) the angle θ , (b) $-\theta$, (c) $180^\circ \pm \theta$, (d) $90^\circ \pm \theta$, (e) $270^\circ \pm \theta$.

The triangles of reference (a), (b), (c), (d), and (e), in each of the four sets of figures, I, II, III, IV, are similar, being mutually equiangular, since all have a right angle and one acute angle equal each to each. Hence the sides x , y , r of the triangles (a) are homologous to x' , y' , r' of the corresponding triangles (b) and (c), but to y' , x' , r' , of the corresponding triangles (d) and (e). For the sides x of triangle (a) and x' of the triangles (b) and (c) are opposite equal angles, and hence are homologous, but the sides y' are opposite this same angle in triangles (d) and (e), and therefore sides y' of (d) and (e) are homologous to x of (a).

Attending to the signs of x and x' , y and y' in the similar triangles (a) and (b),

$$\sin(-\theta) = \frac{y'}{r'} = -\frac{y}{r} = -\sin \theta,$$

$$\cos(-\theta) = \frac{x'}{r'} = \frac{x}{r} = \cos \theta,$$

$$\tan(-\theta) = \frac{y'}{x'} = -\frac{y}{x} = -\tan \theta.$$

Also in the similar triangles (a) and (c),

$$\sin(180^\circ - \theta) = \frac{y'}{r'} = \frac{y}{r} = \sin \theta,$$

$$\cos(180^\circ - \theta) = \frac{x'}{r'} = -\frac{x}{r} = -\cos \theta,$$

$$\tan(180^\circ - \theta) = \frac{y'}{x'} = -\frac{y}{x} = -\tan \theta.$$

In like manner show that

$$\sin(180^\circ + \theta) = -\sin \theta,$$

$$\cos(180^\circ + \theta) = -\cos \theta,$$

$$\tan(180^\circ + \theta) = \tan \theta.$$

Again, in the similar triangles (*a*) and (*d*),

$$\sin (90^{\circ} + \theta) = \frac{y'}{r'} = \frac{x}{r} = \cos \theta,$$

$$\cos (90^{\circ} + \theta) = \frac{x'}{r'} = -\frac{y}{r} = -\sin \theta,$$

$$\tan (90^{\circ} + \theta) = \frac{y'}{x'} = -\frac{x}{y} = -\cot \theta.$$

Show that

$$\sin (90^{\circ} - \theta) = \cos \theta,$$

$$\cos (90^{\circ} - \theta) = \sin \theta,$$

$$\tan (90^{\circ} - \theta) = \cot \theta.$$

Finally, from the similar triangles (*a*) and (*e*), show that

$$\sin (270^{\circ} \pm \theta) = -\cos \theta,$$

$$\cos (270^{\circ} \pm \theta) = \pm \sin \theta,$$

$$\tan (270^{\circ} \pm \theta) = \mp \cot \theta.$$

From the reciprocal relations the student can at once write the corresponding relations for secant, cosecant, and cotangent.

30. Since in each of the four cases x' , y' of triangles (*b*) and (*c*) are homologous to x , y of triangle (*a*), while x' , y' of the triangles (*d*) and (*e*) are homologous to y , x of triangle (*a*), we may express the relations of the last article thus:

The functions of $\begin{cases} \pm \theta \\ 180^{\circ} \pm \theta \end{cases}$ correspond to the same functions of θ , while those of $\begin{cases} 90^{\circ} \pm \theta \\ 270^{\circ} \pm \theta \end{cases}$ correspond to the co-functions of θ , due attention being paid to the signs.

The student can readily determine the sign in any given case, whether θ be acute or obtuse, by considering in what quadrant the compound angle, $90^{\circ} \pm \theta$, $180^{\circ} \pm \theta$, etc., would

lie if θ were an acute angle, and prefixing to the corresponding functions of θ the signs of the respective functions for an angle in that quadrant. Thus $90^\circ + \theta$, if θ be acute, is an angle of the second quadrant, so that sine and cosecant are plus, the other functions minus. It will be seen that $\sin(90^\circ + \theta) = +\cos \theta$, $\cos(90^\circ + \theta) = -\sin \theta$, etc., and this will be true whatever be the magnitude of θ . It will assist in fixing in the memory these important relations to notice that when in the compound angle θ is measured from the y -axis, as in $90^\circ \pm \theta$, $270^\circ \pm \theta$, the functions of one angle correspond to the co-functions of the other, but when in the compound angle θ is measured from the x -axis, as in $\pm \theta$, $180^\circ \pm \theta$, then the functions of one angle correspond to the same functions of the other.

These relations, as has been noted in Art. 28, can be extended to angles greater than 360° , and it may be stated generally that

$$\text{function } \theta = \pm \text{function } (2n \cdot 90^\circ \pm \theta),$$

$$\text{function } \theta = \pm \text{co-function } [(2n + 1) 90^\circ \pm \theta].$$

Computation tables contain angles less than 90° only. The chief utility of the above relations will be the reduction of functions of angles greater than 90° to functions of acute angles. Thus, to find $\tan 130^\circ 20'$, look in the tables for $\cot 40^\circ 20'$, or for $\tan 49^\circ 40'$. Why?

Ex. 1. What angles less than 360° have the same numerical cosine as 20° ?

$$\cos 20^\circ = -\cos(180^\circ \pm 20^\circ) = \cos(360^\circ - 20^\circ).$$

$\therefore 200^\circ, 160^\circ, 340^\circ$ have the same cosine numerically as 20° .

2. Find the functions of 135° ; of 210° .

$$\sin 135^\circ = \sin(90^\circ + 45^\circ) = \cos 45^\circ = \frac{1}{2}\sqrt{2},$$

$$\cos 135^\circ = \cos(180^\circ - 45^\circ) = -\cos 45^\circ = -\frac{1}{2}\sqrt{2}, \text{ etc.}$$

$$\sin 210^\circ = \sin(180^\circ + 30^\circ) = -\sin 30^\circ = -\frac{1}{2}.$$

Let the student give the other functions for each angle.

ORAL WORK.

1. Determine the sine and tangent of each of the following angles: 30° , 120° , -30° , -60° , $\frac{5}{8}\pi$, $2\frac{2}{3}\pi$, -135° , $-\pi$.
2. Which is the greater, $\sin 30^\circ$ or $\sin(-30^\circ)$? $\tan 135^\circ$ or $\tan 45^\circ$? $\cos 60^\circ$ or $\cos(-60^\circ)$? $\sin 22^\circ 30'$ or $\cos 67^\circ 30'$?
3. What positive angle has the same tangent as $\frac{\pi}{3}$? the same sine as 50° ?
4. If $\tan \theta = -1$, find $\sin \theta$.
5. Find $\sin 510^\circ$, $\cos(-60^\circ)$, $\tan 150^\circ$.
6. Reduce in two ways to functions of a positive acute angle, $\cos 122^\circ$, $\tan 140^\circ 30'$, $\sin(-60^\circ)$.
7. Find all positive values of x , less than 360° , satisfying the following equations: $\cos x = \cos 45^\circ$, $\sin 2x = \sin 10^\circ$, $\tan 3x = \tan 60^\circ$, $\sin x = \sin 30^\circ$, $\tan x = \tan 135^\circ$.
8. What angles are determined when (a) sine and cosine are $+$? (b) cotangent and sine are $-$? (c) sine $+$ and cosine $-$? (d) cosine $-$ and cotangent $+$?

INVERSE FUNCTIONS.

31. That a is the sine of an angle θ may be expressed in two ways, viz., $\sin \theta = a$, or, inversely, $\theta = \sin^{-1} a$, the latter being read, *θ equals an angle whose sine is a* , or, more briefly, *θ is the anti-sine of a* .

The notation $\sin^{-1} a$, $\cos^{-1} a$, $\tan^{-1} a$, etc., is not a fortunate one, but is so generally accepted that a change is not probable. The symbol may have been suggested from the fact that if $ax = b$, then $x = a^{-1}b$, whence, by analogy, if $\sin \theta = a$, $\theta = \sin^{-1} a$. But the likeness is an analogy only, for there is no similarity in meaning. $\sin^{-1} a$ is an angle θ , where $\sin \theta = a$, and is entirely different from $(\sin a)^{-1} = \frac{1}{\sin a}$. In Europe the symbols are $\sin a$, $\arcsin a$, etc., are employed.

32. Principal value. We have found that in $\sin \theta = a$, for any value of θ , a can have but one value; but in $\theta = \sin^{-1} a$, for any value of a there are an indefinite number of values of θ (Art. 27, 2).

Thus, when $\sin \theta = a$, if $a = \frac{1}{2}$, θ may be 30° , 150° , 390° , 510° , -330° , etc., or, in general, $n\pi + (-1)^n 30^\circ$.

In the solution of problems involving inverse functions,

the numerically least of these angles, called the *principal value*, is always used; *i.e.* we understand that $\sin^{-1} a$, $\tan^{-1} a$, are angles between $+90^\circ$ and -90° , while the limits of $\cos^{-1} a$ are 0° and 180° .

Thus, $\sin^{-1} \frac{1}{2} = 30^\circ$, $\sin^{-1}(-\frac{1}{2}) = -30^\circ$, $\cos^{-1} \frac{1}{2} = 60^\circ$, $\cos^{-1}(-\frac{1}{2}) = 120^\circ$.

ORAL WORK.

How many degrees in each of the following angles? How many radians?

- | | |
|--|---------------------------|
| 1. $\cos^{-1} \frac{\sqrt{3}}{2}$? | 7. $\tan^{-1} \sqrt{3}$? |
| 2. $\tan^{-1} 1$? | 8. $\cos^{-1} 0$? |
| 3. $\cot^{-1}(-\sqrt{3})$? | 9. $\sin^{-1} 1$? |
| 4. $\sin^{-1}(-\frac{1}{2}\sqrt{2})$? | 10. $\tan^{-1} 0$? |
| 5. $\cos^{-1}(-\frac{1}{2}\sqrt{2})$? | 11. $\tan^{-1}(-1)$? |
| 6. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$? | 12. $\sin^{-1}(-1)$? |

Find the values of the functions:

- | | |
|---|---------------------------------|
| 13. $\sin(\tan^{-1} \frac{1}{3}\sqrt{3})$. | 19. $\cos(\sin^{-1} 0)$. |
| 14. $\tan(\cos^{-1} 1)$. | 20. $\sin(\cos^{-1}[-1])$. |
| 15. $\tan(\cot^{-1}[-\infty])$. | 21. $\cos(\cot^{-1}\sqrt{3})$. |
| 16. $\cos(\tan^{-1} \infty)$. | 22. $\tan(\sin^{-1}[-1])$. |
| 17. $\sin(\sin^{-1} \frac{1}{2}\sqrt{2})$. | 23. $\sin(\tan^{-1}[-1])$. |
| 18. $\tan(\tan^{-1} x)$. | |

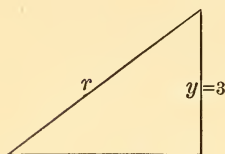


FIG. 24.

Ex. 1. Construct $\cot^{-1} \frac{4}{3}$.

Construct the right triangle xyr , so that $x = 4$, $y = 3$, whence angle $xyr = \cot^{-1} \frac{4}{3}$.

2. Find $\cos(\tan^{-1} \frac{8}{15})$.

Let $\theta = \tan^{-1} \frac{8}{15}$, whence

$$\tan \theta = \frac{8}{15}, \text{ and } \cos \theta = \frac{15}{17}.$$

$$\therefore \cos \theta = \cos(\tan^{-1} \frac{8}{15}) = \frac{15}{17}.$$

3. If $\theta = \csc^{-1} a$, prove $\theta = \cos^{-1} \frac{\sqrt{a^2-1}}{a}$.

$$\csc \theta = a; \therefore \sin \theta = \frac{1}{a},$$

$$\text{and } \cos \theta = \sqrt{1 - \frac{1}{a^2}} = \frac{\sqrt{a^2-1}}{a}, \text{ or } \theta = \cos^{-1} \frac{\sqrt{a^2-1}}{a}.$$

EXAMPLES.

1. Construct $\sin^{-1}\frac{2}{3}$, $\tan^{-1}\frac{5}{12}$, $\cos^{-1}(-\frac{1}{2})$.
2. Find $\tan(\sin^{-1}\frac{5}{13})$, $\sin(\tan^{-1}\frac{5}{12})$.
3. If $\theta = \sin^{-1}a$, prove $\theta = \tan^{-1}\frac{a}{\sqrt{1-a^2}}$.
4. Show that $\sin^{-1}a = 90^\circ - \cos^{-1}a$.
5. Prove $\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3} = \frac{\pi}{2}$.
6. Prove $\tan^{-1}\left(\sin\frac{\pi}{2}\right) = \cos^{-1}\frac{1}{2}\sqrt{2}$.
7. What angles, less than 360° , have the same tangent numerically as 10° ?
8. Given $\tan 143^\circ 22' = -0.74357$; find, correct to 0.00001, sine and cosine.
9. If $\cot^2(90^\circ + \beta) + \csc(90^\circ - \beta) - 1 = 0$, find $\tan \beta$.
10. Find all positive values of x , less than 360° , when $\sin x = \sin 22^\circ 30'$; when $\tan 2x = \tan 60^\circ$.
11. When is $\sin x = \frac{a^2 + b^2}{2ab}$ possible, and when impossible?
12. Verify $\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{\sqrt{3}}{2} + \tan^{-1}\sqrt{3} = \sin^{-1}\frac{\sqrt{3}}{2}$.
13. What values of x will satisfy $\sin^{-1}(x^2 - x) = 30^\circ$?
14. If $\tan^2 \theta - \sec^2 \alpha = 1$, prove $\sec \theta + \tan^3 \theta \csc \theta = (3 + \tan^2 \alpha)^{\frac{3}{2}}$.
15. Prove $\sin A(1 + \tan A) + \cos A(1 + \cot A) = \sec A + \csc A$.
16. Solve the simultaneous equations :

$$\sin^{-1}(2x + 3y) = 30^\circ \text{ and } 3x + 2y = 2.$$
17. Verify (a) $\tan 60^\circ = \sqrt{\frac{1 - \cos 120^\circ}{1 + \cos 120^\circ}}$.
 (b) $\cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$.
 (c) $2 \sin^2 60^\circ = 1 - \cos 120^\circ$.
18. Show that the cosine of the complement of $\frac{\pi}{6}$ equals the sine of the supplement of $\frac{\pi}{6}$.

REVIEW.

Before leaving a problem the student should review and master all principles involved.

1. Construct $\cos^{-1} \frac{8}{17}$; $\sin^{-1}(-\frac{3}{4})$; $\tan^{-1} 2$.
2. Find $\cos(\sin^{-1} \frac{3}{5})$; $\tan(\cos^{-1}[-\frac{1}{2}])$.
3. Prove $\cot^{-1} a = \cos^{-1} \frac{a}{\sqrt{1+a^2}}$.
4. Given $\alpha = \cot^{-1} \frac{3}{5}$, find $\tan \alpha + \sin(90^\circ + \alpha)$.
5. Find $\tan\left(\sin^{-1} \frac{1}{2} + \cos^{-1} \frac{\sqrt{3}}{2}\right)$.
6. State the fundamental relations between the trigonometric functions in terms of the inverse functions. Thus,

$$\sin^{-1} a = \csc^{-1} \frac{1}{a}, \quad \sin^{-1} a = \cos^{-1} \sqrt{1-a^2}, \text{ etc.}$$
7. Find all the angles, less than 360° , whose cosine equals $\sin 120^\circ$.
8. Given $\cot^{-1} 2.8449$, find the sine and cosine of the angle, correct to 0.0001.
9. If $\tan^2(180^\circ - \theta) - \sec(180^\circ + \theta) = 5$, find $\cos \theta$.
10. If $\sin \theta = \frac{2}{3}$, find $\frac{\tan^2 \theta + \cos^2 \theta}{\tan^2 \theta - \cos^2 \theta}$.
11. Is $\sin x - 2 \cos x + 3 \sin x - 6 = 0$ a possible equation?
12. Verify (a) $\sin 60^\circ = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$.
 (b) $2 \cos^2 60^\circ = 1 + \cos 120^\circ$.
 (c) $\cos 60^\circ - \cos 90^\circ = 2 \cos^2 30^\circ - 2 \cos^2 45^\circ$.
13. If $\sin x = \frac{a(a+2b)}{a^2 + 2ab + 2b^2}$, find $\sec x$ and $\tan x$.
14. Prove $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} + \frac{1 + \sin \theta + \cos \theta}{1 + \sin \theta - \cos \theta} = 2 \csc \theta$.
15. Prove
 $\cos 45^\circ + \cos 135^\circ + \cos 30^\circ + \cos 150^\circ - \cos 210^\circ + \cos 270^\circ = \sin 60^\circ$.
16. If $\tan \theta = \frac{b}{\sqrt{a^2 - b^2}}$, prove that

$$\sin \theta(1 + \tan \theta) + \cos \theta(1 + \cot \theta) - \sec \theta = \frac{a}{b}$$
17. Solve $\sin^2 x + \sin^2(x + 90^\circ) + \sin^2(x + 180^\circ) = 1$.

18. Given $\cos^2 \alpha = m \sin \alpha - n$, find $\sin \alpha$.
19. If $\sin^2 \beta = \frac{3}{2 \sec \beta}$, find β .
20. Given $\tan 238^\circ = 1.6$, find $\sin 148^\circ$.
21. Prove $\tan^{-1} m + \cot^{-1} m = 90^\circ$.
22. Find $\sin (\sin^{-1} p + \cos^{-1} p)$.
23. Solve $\cot^2 \theta (2 \csc \theta - 3) + 3 (\csc \theta - 1) = 0$.
24. Prove $\sin^2 \alpha \sec^2 \beta + \tan^2 \beta \cos^2 \alpha = \sin^2 \alpha + \tan^2 \beta$.
25. Prove $\cos^6 V + \sin^6 V = 1 - 3 \sin^2 V + 3 \sin^4 V$.
26. What values of A satisfy $\sin 2 A = \cos 3 A$?
27. If $\tan C = \frac{\sqrt{1-m^2}}{m}$, and $\tan D = \sqrt{\frac{1-\cos C}{1+\cos C}}$, find $\tan D$ in terms of m .
28. If $\sin x - \cos x + 4 \cos^2 x = 2$, find $\tan x$; $\sec x$.
29. Does the value of $\sec x$, derived from $\sec^2 x = \frac{1-2 \cos^2 x}{1-\cos^2 x}$, give a possible value of x ?
30. Prove

$$[\cot (90^\circ - A) - \tan (90^\circ + A)] [\sin (180^\circ - A) \sin (90^\circ + A)] = 1.$$
31. Prove $(1 + \sin A)^2 [\cot A + 2 \sec A (1 - \csc A)] + \csc A \cos^3 A = 0$.
32. Given $\sin x = m \sin y$, and $\tan x = n \tan y$, find $\cos x$ and $\cos y$.
33. Given $\cot 201^\circ = 2.6$, find $\cos 111^\circ$.
34. Find the value of

$$\cos^{-1} \frac{1}{2} + \sin^{-1} \frac{1}{2} \sqrt{2} + \csc^{-1}(-1) + \tan^{-1} 1 - 2 \cot^{-1} \sqrt{3}.$$
35. Solve $2 \cos^2 \theta + 11 \sin \theta - 7 = 0$.
36. Prove

$$\cos^2 B + \cos^2 (B + 90^\circ) + \cos^2 (B + 180^\circ) + \cos^2 (B + 270^\circ) = 2.$$

CHAPTER IV.

COMPUTATION TABLES.

33. Natural functions. It has been noted that the trigonometric functions of angles are *numbers*, but the values were found for only a few angles, viz. 0° , 30° , 45° , 60° , 90° , etc. In computations, however, it is necessary to know the values of the functions of any angle, and tables have been prepared giving the numerical values of the functions of all angles between 0° and 90° to every minute. In these tables the functions of any given angle, and conversely the angle corresponding to any given function, can be found to any required degree of accuracy; *e.g.* by looking in the tables we find $\sin 24^\circ 26' = 0.41363$, and also $1.6415 = \tan 58^\circ 39'$. These numbers are called the *natural functions*, as distinguished from their logarithms, which are called the *logarithmic functions* of the angles.

Ex. 1. Find from the tables of natural functions :

$$\sin 35^\circ 14'; \quad \cos 54^\circ 46'; \quad \tan 78^\circ 29'; \quad \cos 112^\circ 58'; \quad \sin 135^\circ.$$

2. Find the angles less than 180° corresponding to :

$$\sin^{-1} 0.37865; \quad \cos^{-1} 0.37865; \quad \tan^{-1} 0.58670; \quad \cos^{-1} 0.00291; \quad \sin^{-1} 0.99999.$$

34. Logarithms. The arithmetical processes of multiplication, division, involution, and evolution, are greatly abridged by the use of tables of logarithms of numbers and of the trigonometric ratios, which are numbers. The principles involved are illustrated in the following table :

Write in parallel columns a geometrical progression having the ratio 2, and an arithmetical progression having the difference 1, as follows :

G. P.	A. P.	
1	0	It will be perceived that the numbers in the second column are the indices of the powers of 2 producing the corresponding numbers in the first column, thus : $2^6 = 64$, $2^{11} = 2048$, $2^{18} = 262144$, etc. The use of such a table will be illustrated by examples.
2	1	
4	2	
8	3	
16	4	
32	5	Ex. 1. Multiply 8192 by 128.
64	6	From the table, $8192 = 2^{13}$, $128 = 2^7$. Then by
128	7	actual multiplication, $8192 \times 128 = 1048576$, or by the
256	8	law of indices, $2^{13} \times 2^7 = 2^{20} = 1048576$ (from table).
512	9	Notice that the simple operation of addition is substituted for multiplication by adding the numbers in
1024	10	the second column opposite the given factors in the
2048	11	first column. This sum corresponds to the number
4096	12	in the first column which is the required product.
8192	13	2. Divide 16384 by 512.
16384	14	
32768	15	$16384 \div 512 = 32$, which corresponds to the result
65536	16	obtained by use of the table, or $2^{14} \div 2^9 = 2^5 = 32$.
131072	17	The operation of subtraction takes the place of
262144	18	division.
524288	19	3. Find $\sqrt[6]{262144}$.
1048576	20	

$$\sqrt[6]{262144} = \sqrt[6]{2^{18}} = 2^{\frac{18}{6}} = 2^3 = 8.$$

In the table, 262144 is opposite 18. $18 \div 6 = 3$, which is opposite 8, the required root; *i.e.* simple division takes the place of the tedious process of evolution.

4. Cube 64.

6. Find $\sqrt[5]{32768}$.

5. Multiply 256 by 4096.

7. Divide 1048576 by 32768.

35. The above table can be made as complete as desired by continually inserting between successive numbers in the first column the geometrical mean, and between the opposite numbers in the second, the arithmetical mean, but in practice logarithms are computed by other methods. The numbers in the second column are called the *logarithms* of the numbers opposite in the first column. 2 is called the *base* of this system, so that the *logarithm of a number is the exponent by which the base is affected to produce the number*.

Thus, the logarithm of 512 to the base 2 is 9, since $2^9 = 512$.

Logarithms were invented by a Scotchman, John Napier, early in the seventeenth century, but his method of constructing tables was different from the above. See *Encyc. Brit.*, art. "*Logarithms*," for an exceedingly interesting account. De Morgan says that by the aid of logarithms the labor of computing has been reduced for the mathematician to about one-tenth part of the previous expense of time and labor, while Laplace has said that John Napier, by the invention of logarithms, lengthened the life of the astronomer by one-half.

Columns similar to those above might be formed with any other number as base. For practical purposes, however, 10 is always taken as the base of the system, called the *common system*, in distinction from the *natural system*, of which the base is 2.71828 ..., the value of the exponential series (*Higher Algebra*). The natural system is used in theoretical discussions. It follows that *common logarithms* are *indices, positive or negative, of the powers of 10*.

$$\begin{aligned}\text{Thus,} \quad 10^3 &= 1000; \text{ i.e. } \log 1000 = 3; \\ 10^{-2} &= \frac{1}{10^2} = 0.01; \text{ i.e. } \log 0.01 = -2.\end{aligned}$$

36. Characteristic and mantissa. Clearly most numbers are not integral powers of 10. Thus 300 is more than the second and less than the third power of 10, so that

$$\log 300 = 2 \text{ plus a decimal.}$$

Evidently the logarithms of numbers generally consist of an integral and a decimal part, called respectively the *characteristic* and the *mantissa* of the logarithms.

37. Characteristic law. The characteristic of the logarithm of a number is *independent* of the digits composing the number, but *depends* on the position of the decimal point, and *is found by counting the number of places the first significant figure in the number is removed from the units' place, being positive or negative according as the first significant*

figure is at the left or the right of units' place. This follows from the fact that common logarithms are indices of powers of 10, and that 10^n , n being a positive integer, contains $n + 1$ places, while 10^{-n} contains $n - 1$ zeros at the right of units' place. Thus in 146.043 the first significant figure is two places at the left of units' place; the characteristic of $\log 146.043$ is therefore 2. In 0.00379 the first significant digit is three places at the right of units' place, and the characteristic of $\log 0.00379$ is -3 .

To avoid the use of negative characteristics, such characteristics are increased by 10, and -10 is written after the logarithm. Thus, instead of $\log 0.00811 = \bar{3}.90902$, write $7.90902 - 10$. In practice the -10 is generally not written, but it must always be remembered and accounted for in the result.

Ex. Determine the characteristic of the logarithm of:

1; 46; 0.009; 14796.4; 230.001; $10^5 \times 76$; 0.525; 1.03; 0.000426.

38. Mantissa law. The mantissa of the logarithm of a number is *independent* of the position of the decimal point, but *depends* on the digits composing the number, *is always positive*, and *is found* in the tables.

For, moving the decimal point multiplies or divides a number by an integral power of 10, *i.e.* adds to or subtracts from the logarithm an integer, and hence does not affect the mantissa. Thus,

$$\log 225.67 = \log 225.67,$$

$$\log 2256.7 = \log 225.67 \times 10^1 = \log 225.67 + 1,$$

$$\log 22567.0 = \log 225.67 \times 10^2 = \log 225.67 + 2,$$

$$\log 22.567 = \log 225.67 \times 10^{-1} = \log 225.67 + (-1),$$

$$\log 0.22567 = \log 225.67 \times 10^{-3} = \log 225.67 + (-3),$$

so that the mantissæ of the logarithms of all numbers composed of the digits 22567 in that order are the same, .35347. Moving the decimal point affects the characteristic only. *The student must remember that the mantissa is always positive.*

Log 0.0022567 is never written $-3 + .35347$, but $\bar{3}.35347$, the minus sign being written above to indicate that the characteristic alone is negative. In computations negative characteristics are avoided by adding and subtracting 10, as has been explained.

39. We may now define the *logarithm of a number as the index of the power to which a fixed number, called the base, must be raised to produce the given number.*

Thus, $a^x = b$, and $x = \log_a b$ (where $\log_a b$ is read logarithm of b to the base a) are equivalent expressions. The relation between base, logarithm, and number is always

$$(\text{base})^{\log} = \text{number}.$$

To illustrate: $\log_2 8 = 3$ is the same as $2^3 = 8$; $\log_3 81 = 4$ and $3^4 = 81$ are equivalent expressions; and so are $\log_{10} 1000 = 3$ and $10^3 = 1000$, and $\log_{10} 0.001 = -3$ and $10^{-3} = 0.001$.

Find the value of :

$$\log_4 64; \log_5 125; \log_3 243; \log_a (a)^{\frac{4}{3}}; \log_{27} 3; \log_x 1.$$

40. From the definition it follows that the laws of indices apply to logarithms, and we have :

I. *The logarithm of a product equals the sum of the logarithms of the factors.*

II. *The logarithm of a quotient equals the logarithm of the dividend minus the logarithm of the divisor.*

III. *The logarithm of a power equals the index of the power times the logarithm of the number.*

IV. *The logarithm of a root equals the logarithm of the number divided by the index of the root.*

$$\text{For if} \quad a^x = n \text{ and } a^y = m,$$

$$\text{then} \quad n \times m = a^{x+y}, \quad \therefore \log nm = x + y = \log n + \log m;$$

$$\text{and} \quad n \div m = a^{x-y}, \quad \therefore \log \frac{n}{m} = x - y = \log n - \log m;$$

$$\text{also} \quad n^r = (a^x)^r = a^{rx}, \quad \therefore \log n^r = rx = r \times \log n;$$

$$\text{finally,} \quad \sqrt[r]{n} = \sqrt[r]{a^x} = a^{\frac{x}{r}}, \quad \therefore \log \sqrt[r]{n} = \frac{x}{r} = \frac{1}{r} \log n.$$

EXAMPLES.

Given $\log 2 = 0.30103$, $\log 3 = 0.47712$, $\log 5 = 0.69897$, find :

- | | | | |
|----------------|----------------------|-------------------------|--|
| 1. $\log 4$. | 4. $\log 9$. | 7. $\log 15^3$. | 10. $\log \sqrt{\frac{72}{25}}$. |
| 2. $\log 6$. | 5. $\log 25$. | 8. $\log \frac{8}{3}$. | 11. $\log \sqrt{\frac{9^2 \times 5^3}{2^4 \times 10}}$. |
| 3. $\log 10$. | 6. $\log \sqrt{3}$. | 9. $\log 15 \times 9$. | |

USE OF TABLES.

41. To find the logarithm of a number.

First. Find the characteristic, as in Art. 37.

Second. Find the mantissa in the tables, thus :

(a) When the number consists of not more than four figures.

In the column *N* of the tables find the first three figures, and in the row *N* the fourth figure of the number. The mantissa of the logarithm will be found in the row opposite the first three figures and in the column of the fourth figure.

Illustration. Find $\log 42.38$.

The characteristic is 1. (Why?)

In the table in column *N* find the figures 423, and on the same page in row *N* the figure 8. The last three figures of the mantissa, 716, lie at the intersection of column 8 and row 423. To make the tables more compact the first two figures of the mantissa, 62, are printed in column 0 only. Then $\log 42.38 = 1.62716$.

Find $\log 0.8734 = \bar{1}.94121$,
 $\log 3.5 = \log 3.500 = 0.54407$,
 $\log 36350 = 4.56050$.

(b) When the number consists of more than four figures.

Find the mantissa of the logarithm of the number composed of the first four figures as above. To correct for the remaining figures we *interpolate* by means of the *principle of proportional parts*, according to which it is assumed that, for differences small as compared with the numbers, the differences

between several numbers are proportional to the differences between their logarithms.

The theorem is only approximately correct, but its use leads to results accurate enough for ordinary computations.

Ex. 1. To find $\log 89.4562$.

As above, mantissa of $\log 894500 = 0.95158$,

mantissa of $\log 894600 = 0.95163$,

$\therefore \log 894600 - \log 894500 = 0.00005$, called the tabular difference.

Let $\log 894562 - \log 894500 = x$ hundred-thousandths.

Now, by the principle of proportional parts,

$$\frac{\log 894562 - \log 894500}{\log 894600 - \log 894500} = \frac{894562 - 894500}{894600 - 894500}$$

or $\frac{x}{5} = \frac{62}{100}$, whence $x = .62$ of $5 = 3.1$

$$\therefore \log 89.4562 = 1.95158 + 0.00003 = 1.95161,$$

all figures after the fifth place being rejected in five-place tables. If, however, the sixth place be 5 or more, it is the practice to add 1 to the figure in the fifth place. Thus, if $x = 0.0000456$, we should call it 0.00005, and add 5 to the mantissa.

2. Find $\log 537.0643$.

To interpolate we have $x : 9 = 643 : 1000$, i.e. $x = 5.787$;

$$\therefore \log 537.0643 = 2.72997 + 0.00006.$$

3. Find $\log 0.0168342 = \bar{2}.22619$.

4. Find $\log 39642.7 = 4.59816$.

42. To find the number corresponding to a given logarithm.

The characteristic of the logarithm determines the position of the decimal point (Art. 37).

(a) If the mantissa is in the tables, the required number is found at once.

Ex. 1. Find $\log^{-1} 1.94621$ (read, the number whose logarithm is 1.94621).

The mantissa is found in the tables at the intersection of row 883 and column 5.

$$\therefore \log^{-1} 1.94621 = 88.35,$$

the characteristic 1 showing that there are two integral places.

(b) If the exact mantissa of the given logarithm is not in the tables, the first four figures of the corresponding number are found, and to these are annexed figures found by interpolating by means of the principle of proportional parts, as follows :

Find the two successive mantissæ between which the given mantissa lies. Then, by the principle of proportional parts, the amount to be added to the four figures already found is such a part of 1 as the difference between the successive mantissæ is of the difference between the smaller of them and the given mantissa.

2. Find $\log^{-1} 1.43764$.

Mantissa of $\log 2740 = 0.43775$

of $\log 2739 = 0.43759$
Differences $\frac{1}{16}$

Mantissa of $\log$ required number = 0.43764

of $\log 2739 = 0.43759$
Differences $\frac{x}{5}$

By p. p. $x : 1 = 5 : 16$ and $x = \frac{5}{16} = 0.3125$.

Annexing these figures, $\log^{-1} 1.43764 = 27.3931+$.

3. Find $\log^{-1} \bar{1}.48762$.

The differences in logarithms are 14, 6.

$$\therefore x = \frac{6}{14} = .428+,$$

$$\text{and } \log^{-1} \bar{1}.48762 = 0.307343+.$$

4. Find $\log 891.59$; $\log 0.023$; $\log \frac{1}{2}$; $\log 0.1867$; $\log \sqrt{2}$.

5. Find $\log^{-1} 2.21042$; $\log^{-1} 0.55115$; $\log^{-1} \bar{1}.89003$.

43. Logarithms of trigonometric functions. These might be found by first taking from the tables the natural functions of the given angle, and then the logarithms of these numbers. It is more expeditious, however, to use tables showing directly the logarithms of the functions of angles less than 90° to every minute. Functions of angles greater than 90° are reduced to functions of angles less than 90° by

the formulæ of Art. 29. To make the work correct for seconds, or any fractional part of a minute, interpolation is necessary by the principle of proportional parts, thus :

Ex. 1. Find $\log \sin 28^\circ 32' 21''$.

In the table of logarithms of trigonometric functions, find 28° at the top of the page, and in the minute column at the left find $32'$. Then under $\log \sin$ column find $\log \sin 28^\circ 32' = 9.67913 - 10$

$$\log \sin 28^\circ 33' = 9.67936 - 10$$

Differences	1'	23
-------------	----	----

By p. p. $x : 23 = 21'' : 60''$, i.e. $x = \frac{21}{60} \times 23 = 8.05$.

$$\begin{aligned} \therefore \log \sin 28^\circ 32' 21'' &= 9.67913 + 0.00008 - 10 \\ &= 9.67921 - 10. \end{aligned}$$

Whenever functions of angles are less than unity, *i.e.* are decimals (as sine and cosine always are, except when equal to unity, and as tangent is for angles less than 45°), the characteristic of the logarithm will be negative, and, accordingly, 10 is always added in the tables, and it must be remembered that 10 is to be subtracted. Thus, in the example above, the characteristic of the logarithm is not 9, but $\bar{1}$, and the logarithm is not 9.67913, as written in the tables, but $9.67913 - 10$.

2. Find $\log \cos 67^\circ 27' 50''$.

In the table of logarithms at the foot of the page, find 67° , and in the minute column at the right, $27'$. Then computing the difference as above, $x = 25$.

But it must be noted that cosine decreases as the angle increases toward 90° . Hence, $\log \cos 67^\circ 27' 50''$ is less than $\log \cos 67^\circ 27'$, *i.e.* the difference 25 must be subtracted, so that

$$\begin{aligned} \log \cos 67^\circ 27' 50'' &= 9.58375 - 0.00025 - 10 \\ &= 9.58350 - 10. \end{aligned}$$

44. To find the angle when the logarithm is given, find the successive logarithms between which the given logarithm lies, compute by the principle of proportional parts the seconds, and add them to the less of the two angles corresponding to the successive logarithms. This will not necessarily be the angle corresponding to the less of the two logarithms; for, as has been seen, the number, and, therefore, the logarithm, may decrease as the angle increases.

Ex. 1. Find the angle whose $\log \tan$ is 9.88091.

$$\begin{array}{r}
 \log \tan 37^\circ 14' = 9.88079 - 10 \\
 \log \tan 37^\circ 15' = 9.88105 - 10 \\
 \hline
 \text{Differences} \quad 60'' \quad 26 \\
 \\
 \log \tan 37^\circ 14' = 9.88079 - 10 \\
 \log \tan \text{angle required} = 9.88091 - 10 \\
 \hline
 \text{Differences} \quad x'' \quad 12
 \end{array}$$

$\therefore x : 60 = 12 : 26$, or $x'' = \frac{12}{26} \times 60'' = 28''$, approximately, and the angle is $37^\circ 14' 28''$.

2. Find the angle whose $\log \cos = 9.82348$.

We find $x = \frac{6}{14} \times 60'' = 26''$, and the angle is $48^\circ 14' 26''$.

3. Show that $\log \cos 25^\circ 31' 20'' = 9.95541$;

$$\log \sin 110^\circ 25' 20'' = 9.97181;$$

$$\log \tan 49^\circ 52' 10'' = 0.07417.$$

4. Show that the angle whose $\log \tan$ is 9.92501 is $40^\circ 4' 39''$; whose $\log \sin$ is 9.88365 is $49^\circ 54' 18''$; whose $\log \cos$ is 9.50828 is $71^\circ 11' 49''$.

45. Cologarithms. In examples involving multiplications and divisions it is more convenient, if n is any divisor, to add $\log \frac{1}{n}$ than to subtract $\log n$. The logarithm of $\frac{1}{n}$ is called the cologarithm of n . Since

$$\log \frac{1}{n} = \log 1 - \log n = 0 - \log n,$$

it follows that $\text{colog } n = -\log n$, *i.e.* $\log n$ subtracted from zero. To avoid negative results, add and subtract 10.

Ex. 1. Find $\text{colog } 2963$.

$$\begin{array}{r}
 \log 1 = 10.00000 - 10 \\
 \log 2963 = 3.47173 \\
 \hline
 \therefore \text{colog } 2963 = 6.52827 - 10
 \end{array}$$

2. Find $\text{colog } \tan 16^\circ 17'$.

$$\begin{array}{r}
 \log 1 = 10.00000 - 10 \\
 \log \tan 16^\circ 17' = 9.46554 - 10 \\
 \hline
 \therefore \text{colog } \tan 16^\circ 17' = 0.53446
 \end{array}$$

By means of the definitions of the trigonometric functions, the parts of a right triangle may be computed if any two parts, one of them being a side, are given. Thus,

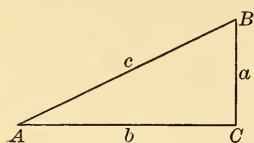


FIG. 25.

given a and A in the rt. triangle ABC .

Then $c = a \div \sin A$, $b = a \div \tan A$,
and $B = 90^\circ - A$.

Again, if a and b are given, then

$\tan A = \frac{a}{b}$, $c = a \div \sin A$, and $B = 90^\circ - A$.

3. Given $c = 25.643$, $B = 37^\circ 25' 20''$, compute the other parts.

$$A = 90^\circ - 37^\circ 25' 20'' = 52^\circ 34' 40''.$$

$$a = c \cos B.$$

$$b = a \tan B.$$

$$\log c = 1.40897$$

$$\log a = 1.30889$$

$$\log \cos B = \underline{9.89992}$$

$$\log \tan B = \underline{9.88376}$$

$$\log a = 1.30889$$

$$\log b = \underline{1.19265}$$

$$\therefore a = 20.365.$$

$$\therefore b = 15.583.$$

$$\text{Check: } c^2 = a^2 + b^2 = 20.365^2 + 15.583^2 = 657.57 = 25.643^2.$$

4. Given $b = 0.356$, $B = 63^\circ 28' 40''$, compute the other parts.

$$A = 26^\circ 31' 20''.$$

$$c = \frac{b}{\sin B}.$$

$$a = \frac{b}{\tan B}.$$

$$\log b = 9.55145$$

$$\log b = 9.55145$$

$$\text{colog } \sin B = \underline{9.04829}$$

$$\text{colog } \tan B = \underline{9.69816}$$

$$\log c = 9.59974$$

$$\log a = 9.24961$$

$$c = 0.3979$$

$$a = 0.1777$$

$$\text{Check: } c^2 - a^2 = 0.1583 - 0.03157 = 0.12673 = b^2.$$

EXAMPLES.

Compute the other parts:

1. Given $a = 9.325$, $A = 43^\circ 22' 35''$.

2. Given $c = 240.32$, $a = 174.6$.

3. Given $B = 76^\circ 14' 23''$, $a = 147.53$.

4. Given $a = 2789.42$, $b = 4632.19$.

5. Given $c = 0.0213$, $A = 23^\circ 14''$.

6. Given $b = 2$, $c = 3$.

CHAPTER V.

APPLICATIONS.

46. Many problems in measurements of heights and distances may be solved by applying the preceding principles. By means of instruments certain distances and angles may be measured, and from the data thus determined other distances and angles computed. The most common instruments are the *chain*, the *transit*, and the *compass*.

The *chain* is used to measure distances. Two kinds are in use, the *engineer's chain* and the *Gunter's chain*. They each contain 100 links, each link in the engineer's chain being 12 inches long, and in the Gunter's 7.92 inches.

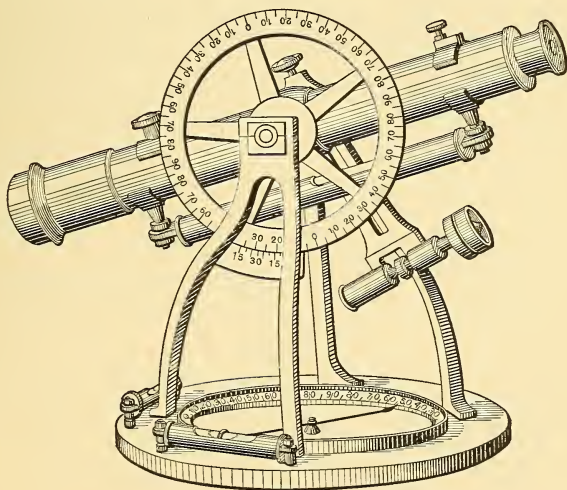


FIG. 26.

The *transit* is the instrument most used to measure horizontal angles, and with certain attachments to measure vertical angles. The figure shows the form of the instrument.

The *mariner's compass* is used to determine the directions, or *bearings*, of objects at sea. Each quadrant is divided into 8 parts, making the 32 points of the compass, so that each point contains $11^{\circ} 15'$.

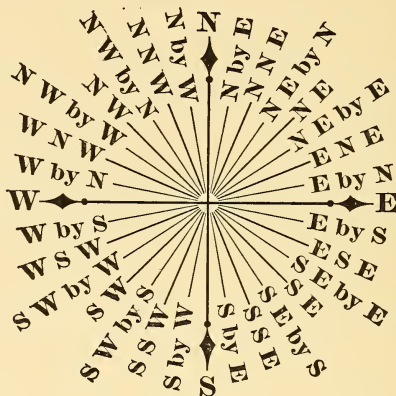


FIG. 27.

47. The angle between the horizontal plane and the line of vision from the eye to the object is called the *angle of elevation*, or of *depression*, according as the object is above or below the observer.

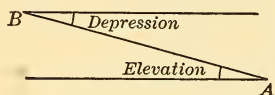


FIG. 28.

It is evident that the elevation angle of *B*, as seen from *A*, is equal to the depression angle of *A*, as seen from *B*, so that in the solution of examples the two angles are interchangeable.

PROBLEMS.

48. Some of the more common problems met with in practice are illustrated by the following:

To find the height of an object when the foot is accessible.

The distance *BC*, and the elevation angle *B* are measured, and *x* is determined from the relation $x = BC \tan B$.

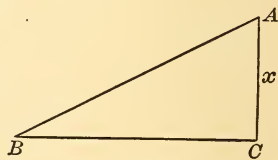


FIG. 29.

Ex. 1. The elevation angle of a cliff measured from a point 300 ft. from its base is found to be 30° . How high is the cliff?

$$BC = 300, B = 30^\circ.$$

Then
$$x = 300 \cdot \tan 30^\circ = 300 \cdot \frac{1}{\sqrt{3}} = 100\sqrt{3}.$$

2. From a point 175 ft. from the foot of a tree the elevation of the top is found to be $27^\circ 19'$. Find the height of the tree.

The problem may be solved by the use of natural functions, or of logarithms. The work should be arranged for the solution before the tables are opened. Let the student complete.

$$BC = 175. \quad B = 27^\circ 19'.$$

Then
$$x = BC \tan B.$$

Or by natural functions,

$$\log BC =$$

$$BC = 175$$

$$\log \tan B =$$

$$\tan B = 0.5165$$

$$\log x =$$

$$\therefore x = 90.3875.$$

$$\therefore x = 90.39.$$

To find the height of an object when the foot is inaccessible.

Measure BB' , θ and θ' .

Then
$$x = \frac{BC}{\cot \theta} = \frac{BB' + B'C}{\cot \theta}.$$

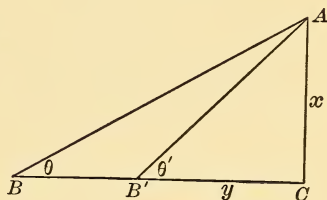


FIG. 30.

But $B'C = x \cot \theta'$, whence substituting,

$$x = \frac{BB'}{\cot \theta - \cot \theta'},$$

which is best solved by the use of the natural functions of θ and θ' .

3. Measured from a certain point at its base the elevation of the peak of a mountain is 60° . At a distance of one mile directly from this point the elevation is 30° . Find the height of the mountain.

$$BB' = 5280 \text{ ft.}, \quad \theta = 30^\circ, \quad \theta' = 60^\circ.$$

$$x = \frac{y + 5280}{\cot 30^\circ}. \quad \text{But } y = x \cot 60^\circ.$$

$$\therefore x = \frac{5280}{\cot 30^\circ - \cot 60^\circ} = 4572.48 \text{ ft.}$$

In surveying it is often necessary to make measurements across a stream or other obstacle too wide to be spanned by a single chain.

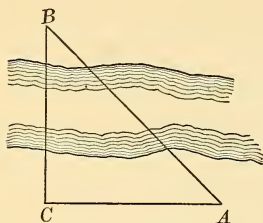


FIG. 31.

To find the distance from *C* to a point *B* on the opposite side of a stream.

At *C* measure a right angle, and take *CA* a convenient distance. Measure angle *A*, then

$$BC = CA \cdot \tan A.$$

4. Find *CB* when angle *A* = $47^{\circ} 16'$, and *CA* = 250 ft.

5. From a point due south of a kite its elevation is found to be $42^{\circ} 30'$; from a point 20 yds. due west from this point the elevation is $36^{\circ} 24'$. How high is the kite above the ground?

$$AB = x \cdot \cot 42^{\circ} 30',$$

$$AC = x \cdot \cot 36^{\circ} 24',$$

$$AC^2 - AB^2 = BC^2 = 400.$$

$$\therefore x^2 (\cot^2 36^{\circ} 24' - \cot^2 42^{\circ} 30') = 400,$$

whence

$$x^2 = \frac{400}{.6489}, \text{ and } x = \frac{20}{.805} = 24.84 \text{ yds.}$$

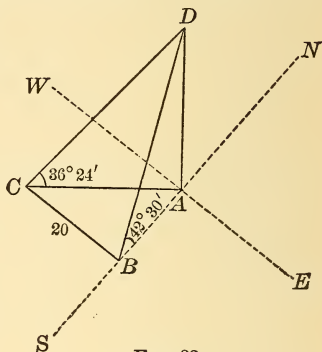


FIG. 32.

EXAMPLES.

1. What is the altitude of the sun when a tree 71.5 ft. high casts a shadow 37.75 ft. long?

2. What is the height of a balloon directly over Ann Arbor when its elevation at Ypsilanti, 8 miles away, is $10^{\circ} 15'$?

3. The Washington monument is 555 ft. high. How far apart are two observers who, from points due east, see the top of the monument at elevations of $23^{\circ} 20'$ and $47^{\circ} 30'$, respectively?

4. A mountain peak is observed from the base and top of a tower 200 ft. high. The elevation angles being $25^{\circ} 30'$ and $23^{\circ} 15'$, respectively, compute the height of the mountain above the base of the tower.

5. From a point in the street between two buildings the elevation angles of the tops of the buildings are 30° and 60° . On moving across

the street 20 ft. toward the first building the elevation angles are found to be each 45° . Find the width of the street and the height of each building.

6. From the peak of a mountain two towns are observed due south. The first is seen at a depression of $48^\circ 40'$, and the second, 8 miles farther away and in the same horizontal plane, at a depression of $20^\circ 50'$. What is the height of the mountain above the plane?

7. A building 145 ft. long is observed from a point directly in front of one corner. The length of the building subtends $\tan^{-1} 3$, and the height $\tan^{-1} 2$. Find the height.

8. An inaccessible object is observed to lie due N.E. After the observer has moved S.E. 2 miles, the object lies N.N.E. Find the distance of the object from each point of observation.

9. Assuming the earth to be a sphere with a radius of 3963 miles, find the height of a lighthouse just visible from a point 15 miles distant at sea.

10. The angle of elevation of a tower 120 ft. high due north of an observer was 35° ; what will be its angle of elevation from a point due west from the first point of observation 250 ft.? Also the distance of the observer from the base of the tower in each position?

11. A railway 5 miles long has a uniform grade of $2^\circ 30'$; find the rise per mile. What is the grade when the road rises 70 ft. in one mile?

(The grade depends on the tangent of the angle.)

12. The foot of a ladder is in the street at a point 30 ft. from the line of a building, and just reaches a window $22\frac{1}{2}$ ft. above the ground. By turning the ladder over it just reaches a window 36 ft. above the ground on the other side of the street. Find the breadth of the street.

13. From a point 200 ft. from the base of the Forefathers' monument at Plymouth, the base and summit of the statue of Faith are at an elevation of $12^\circ 40' 48''$ and $22^\circ 2' 53''$, respectively; find the height of the statue and of the pedestal on which it stands.

14. At a distance of 100 ft. measured in a horizontal plane from the foot of a tower, a flagstaff standing on the top of the tower subtends an angle of 8° , while the tower subtends an angle of $42^\circ 20'$. Find the length of the flagstaff.

15. The length of a string attached to a kite is 300 ft. The kite's elevation is $56^\circ 6'$. Find the height of the kite.

16. From two rocks at sea level, 50 ft. apart, the top of a cliff is observed in the same vertical plane with the rocks. The angles of elevation of the cliff from the two rocks are $24^\circ 40'$ and $32^\circ 30'$. What is the height of the cliff above the sea?

CHAPTER VI.

GENERAL FORMULÆ—TRIGONOMETRIC EQUATIONS AND IDENTITIES.

49. Thus far functions of single angles only have been considered. Relations will now be developed to express functions of angles which are sums, differences, multiples, or sub-multiples of single angles in terms of the functions of the single angles from which they are formed.

First it will be shown that,

$$\sin (\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta,$$

$$\cos (\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta.$$

$$\tan (\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}.$$

The following cases must be considered :

1. $\alpha, \beta, \alpha + \beta$ acute angles.
2. α, β , acute, but $\alpha + \beta$ an obtuse angle.
3. Either α , or β , or both, of any magnitude, positive or negative.

The figures apply to cases 1 and 2.

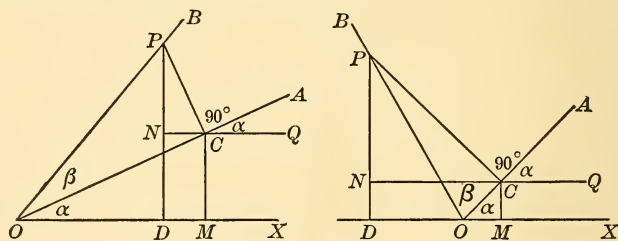


FIG. 33.

Let the terminal line revolve through the angle α , and then through the angle β , to the position OB , so that angle

$XOB = \alpha + \beta$. Through any point P in OB draw perpendiculars to the sides of α , DP and CP , and through C draw a perpendicular and a parallel to OX , MC and NC .

Then the angle $QCA = \alpha$ (why?), and CNP is the triangle of reference for angle $QCP = 90^\circ + \alpha$.

CNP is sometimes treated as the triangle of reference for angle CPN . The fallacy of this appears when we develop $\cos(\alpha + \beta)$, in which PC would be treated as both plus and minus.

$$\text{Now } \sin(\alpha + \beta) = \sin XOB = \frac{DP}{OP} = \frac{MC}{OP} + \frac{NP}{OP},$$

or expressing in trigonometric ratios,

$$\begin{aligned} &= \frac{MC}{OC} \cdot \frac{OC}{OP} + \frac{NP}{CP} \cdot \frac{CP}{OP} \\ &= \sin \alpha \cos \beta + \sin(90^\circ + \alpha) \sin \beta. \end{aligned}$$

Hence, since $\sin(90^\circ + \alpha) = \cos \alpha$, we have

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta.$$

In like manner

$$\cos(\alpha + \beta) = \cos XOB = \frac{OD}{OP} = \frac{OM}{OP} + \frac{CN}{OP},$$

or expressing in trigonometric ratios,

$$\begin{aligned} &= \frac{OM}{OC} \cdot \frac{OC}{OP} + \frac{CN}{CP} \cdot \frac{CP}{OP} \\ &= \cos \alpha \cos \beta + \cos(90^\circ + \alpha) \sin \beta. \end{aligned}$$

And since $\cos(90^\circ + \alpha) = -\sin \alpha$, we have

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta.$$

It will be noted that the wording of the demonstration applies to both figures, the only difference being that when $\alpha + \beta$ is obtuse OD is negative. CN is negative in each figure.

50. In the case, when α , or β , or both, are of any magnitude, positive or negative, figures may be constructed as before described by *drawing through any point in the terminal line of β a perpendicular to each side of α , and through the foot of the perpendicular on the terminal line of α a perpendicular and a parallel to the initial line of α* . Noting negative lines,

the demonstrations already given will be found to apply for all values of α and β .

To make the proof complete by this method would require an unlimited number of figures, *e.g.* we might take α obtuse, both α and β obtuse, either or both greater than 180° , or than 360° , or negative angles, etc.

Instead of this, however, the generality of the proposition is more readily shown algebraically, as follows :

Let $\alpha' = 90^\circ + \alpha$ be any obtuse angle, and α, β , acute angles.

Then

$$\begin{aligned}\sin(\alpha' + \beta) &= \sin(90^\circ + \alpha + \beta) = \cos(\alpha + \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ &= \sin(90^\circ + \alpha) \cos \beta + \cos(90^\circ + \alpha) \sin \beta \text{ (why?) } \\ &= \sin \alpha' \cos \beta + \cos \alpha' \sin \beta.\end{aligned}$$

In like manner, considering any obtuse angle $\beta' = 90^\circ + \beta$, it can be shown that

$$\sin(\alpha' + \beta') = \sin \alpha' \cos \beta' + \cos \alpha' \sin \beta'.$$

Show that $\cos(\alpha' + \beta') = \cos \alpha' \cos \beta' - \sin \alpha' \sin \beta'$.

By further substitutions, *e.g.* $\alpha'' = 90^\circ \pm \alpha'$, $\beta'' = 90^\circ \pm \beta'$, etc., it is clear that the above relations hold for all values, positive or negative, of the angles α and β .

Since α and β may have any values, we may put $-\beta$ for β , and $\sin(\alpha + [-\beta])$

$$\begin{aligned}&= \sin(\alpha - \beta) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta) \\ &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \text{ (why?)}.\end{aligned}$$

$$\begin{aligned}\text{Also } \cos(\alpha - \beta) &= \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha \sin \beta.\end{aligned}$$

Finally,

$$\begin{aligned}\tan(\alpha \pm \beta) &= \frac{\sin(\alpha \pm \beta)}{\cos(\alpha \pm \beta)} = \frac{\sin \alpha \cos \beta \pm \cos \alpha \sin \beta}{\cos \alpha \cos \beta \mp \sin \alpha \sin \beta} \\ &= \frac{\frac{\sin \alpha \cos \beta}{\cos \alpha \cos \beta} \pm \frac{\cos \alpha \sin \beta}{\cos \alpha \cos \beta}}{\frac{\cos \alpha \cos \beta}{\cos \alpha \cos \beta} \mp \frac{\sin \alpha \sin \beta}{\cos \alpha \cos \beta}} = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}.\end{aligned}$$

ORAL WORK.

By the above formulæ develop :

- | | |
|---------------------------------|---|
| 1. $\sin (2 A + 3 B)$. | 7. $\sin 90^{\circ} = \sin (45^{\circ} + 45^{\circ})$. |
| 2. $\cos (90^{\circ} - B)$. | 8. $\cos 90^{\circ}$. |
| 3. $\tan (45^{\circ} + \phi)$. | 9. $\tan 90^{\circ}$. |
| 4. $\sin 2 A = \sin (A + A)$. | 10. $\sin (90^{\circ} + \beta + \gamma)$. |
| 5. $\cos 2 \theta$. | 11. $\cos (270^{\circ} - m - n)$. |
| 6. $\tan (180^{\circ} + C)$. | 12. $\tan (90^{\circ} + m + n)$. |

Ex. 1. Find $\sin 75^{\circ}$.

$$\begin{aligned}\sin 75^{\circ} &= \sin (45^{\circ} + 30^{\circ}) = \sin 45^{\circ} \cos 30^{\circ} + \cos 45^{\circ} \sin 30^{\circ} \\ &= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{1 + \sqrt{3}}{2\sqrt{2}} = 0.9659.\end{aligned}$$

2. Find $\tan 15^{\circ}$.

$$\begin{aligned}\tan 15^{\circ} &= \tan (45^{\circ} - 30^{\circ}) = \frac{\tan 45^{\circ} - \tan 30^{\circ}}{1 + \tan 45^{\circ} \tan 30^{\circ}} \\ &= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 2 - \sqrt{3} = 0.2679.\end{aligned}$$

3. Prove $\frac{\sin 3 A}{\sin A} - \frac{\cos 3 A}{\cos A} = 2$.

$$\begin{aligned}\text{Combining, } \frac{\sin 3 A \cos A - \cos 3 A \sin A}{\sin A \cos A} &= \frac{\sin (3 A - A)}{\sin A \cos A} \\ &= \frac{\sin 2 A}{\sin A \cos A} = \frac{\sin (A + A)}{\sin A \cos A} = \frac{\sin A \cos A + \cos A \sin A}{\sin A \cos A} = 2.\end{aligned}$$

4. Prove $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$.

$$\text{Let } \alpha = \tan^{-1} a, \beta = \tan^{-1} b, \gamma = \tan^{-1} \frac{a+b}{1-ab}.$$

$$\text{Hence, } \tan \alpha = a, \tan \beta = b, \tan \gamma = \frac{a+b}{1-ab}.$$

$$\text{Then } \alpha + \beta = \gamma, \text{ and hence } \tan (\alpha + \beta) = \tan \gamma.$$

$$\text{Expanding, } \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \tan \gamma.$$

$$\text{Substituting, } \frac{a+b}{1-ab} = \frac{a+b}{1-ab}.$$

EXAMPLES.

1. Find $\cos 15^\circ$, $\tan 75^\circ$.
 2. Prove $\cot(\alpha \pm \beta) = \frac{\cot \alpha \cot \beta \mp 1}{\cot \beta \pm \cot \alpha}$.
 3. Prove geometrically $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$,
and $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$,
given
 - (a) α acute, β obtuse;
 - (b) α, β , obtuse;
 - (c) α, β , either, or both, negative angles.
 4. Prove geometrically $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$.
- Verify the formula by assigning values to α and β , and finding the values of the functions from the tables of natural tangents.
5. Prove $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$.
 6. Show that $\tan \alpha + \tan \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cos \beta}$.
 7. Given $\tan \alpha = \frac{1}{2}$, $\tan \beta = \frac{3}{4}$, find $\sin(\alpha + \beta)$.
 8. Given $\sin 280^\circ = s$, find $\sin 170^\circ$.
 9. If $\alpha = 67^\circ 22'$, $\beta = 128^\circ 40'$, by use of the tables of natural functions verify the formulæ on page 56.
 10. Prove $\tan^{-1} \frac{\sqrt{x} + \sqrt{a}}{1 - \sqrt{ax}} = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{a}$.
 11. Prove $\tan^{-1} \frac{2x - b}{b\sqrt{3}} + \tan^{-1} \frac{2b - x}{x\sqrt{3}} = \tan^{-1} \sqrt{3}$.
 12. Prove $\sec^{-1} \frac{a}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a}$.
 13. If $\alpha + \beta = \omega$, prove $\cos^2 \alpha + \cos^2 \beta - 2 \cos \alpha \cos \beta \cos \omega = \sin^2 \omega$.
 14. Solve $\frac{1}{2} \sin \theta = 1 - \cos \theta$.
 15. Prove $\sin(A + B) \cos A - \cos(A + B) \sin A = \sin B$.
 16. Prove $\cos(A + B) \cos(A - B) + \sin(A + B) \sin(A - B) = \cos 2B$.
 17. Prove $\sin(2\alpha - \beta) \cos(\alpha - 2\beta)$
 $- \cos(2\alpha - \beta) \sin(\alpha - 2\beta) = \sin(\alpha + \beta)$.
 18. Prove $\sin(n-1)\alpha \cos(n+1)\alpha + \cos(n-1)\alpha \sin(n+1)\alpha = \sin 2n\alpha$.
 19. Prove $\sin(135^\circ - \theta) + \cos(135^\circ + \theta) = 0$.

$$20. \text{ Prove } 1 - \tan^2 \alpha \tan^2 \beta = \frac{\cos^2 \beta - \sin^2 \alpha}{\cos^2 \alpha \cos^2 \beta}.$$

$$21. \text{ Prove } \frac{\tan \alpha + \tan \beta}{\cot \alpha + \cot \beta} = \tan \alpha \tan \beta.$$

$$22. \tan^2 \left(\frac{\pi}{4} - \alpha \right) = \frac{1 - 2 \sin \alpha \cos \alpha}{1 + 2 \sin \alpha \cos \alpha}.$$

51. The following formulæ are very important and should be carefully memorized. They enable us to change sums and differences to products, *i.e.* to displace terms by factors.

$$\sin \theta + \sin \phi = 2 \sin \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2},$$

$$\sin \theta - \sin \phi = 2 \cos \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2},$$

$$\cos \theta + \cos \phi = 2 \cos \frac{\theta + \phi}{2} \cos \frac{\theta - \phi}{2},$$

$$\cos \theta - \cos \phi = -2 \sin \frac{\theta + \phi}{2} \sin \frac{\theta - \phi}{2}.$$

Since	$\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta,$	
and	$\sin (\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta,$	
then	$\sin (\alpha + \beta) + \sin (\alpha - \beta) = 2 \sin \alpha \cos \beta,$	(1)
and	$\sin (\alpha + \beta) - \sin (\alpha - \beta) = 2 \cos \alpha \sin \beta.$	(2)

Also since	$\cos (\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta,$	
and	$\cos (\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta,$	
then	$\cos (\alpha + \beta) + \cos (\alpha - \beta) = 2 \cos \alpha \cos \beta,$	(3)
and	$\cos (\alpha + \beta) - \cos (\alpha - \beta) = -2 \sin \alpha \sin \beta.$	(4)

Put	$\alpha + \beta = \theta$
and	$\alpha - \beta = \phi$
	$\frac{2\alpha}{2} = \theta + \phi, \text{ and } \alpha = \frac{\theta + \phi}{2},$
	$2\beta = \theta - \phi, \text{ and } \beta = \frac{\theta - \phi}{2}.$

Substituting in (1), (2), (3), (4), we have the above formulæ.

EXAMPLES.

1. Prove $\frac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta} = \tan \frac{3\theta}{2}$.

By formulæ of last article the first member becomes

$$\frac{2 \sin \frac{3\theta}{2} \cos \frac{\theta}{2}}{2 \cos \frac{3\theta}{2} \cos \frac{\theta}{2}} = \tan \frac{3\theta}{2}.$$

2. Prove $\frac{\sin \alpha + 2 \sin 3\alpha + \sin 5\alpha}{\sin 3\alpha + 2 \sin 5\alpha + \sin 7\alpha} = \frac{\sin 3\alpha}{\sin 5\alpha}$.

$$\begin{aligned} \frac{(\sin \alpha + \sin 5\alpha) + 2 \sin 3\alpha}{(\sin 3\alpha + \sin 7\alpha) + 2 \sin 5\alpha} &= \frac{2 \sin 3\alpha \cos 2\alpha + 2 \sin 3\alpha}{2 \sin 5\alpha \cos 2\alpha + 2 \sin 5\alpha} \\ &= \frac{(\cos 2\alpha + 1) \sin 3\alpha}{(\cos 2\alpha + 1) \sin 5\alpha} = \frac{\sin 3\alpha}{\sin 5\alpha}. \end{aligned}$$

3. Prove $\frac{\sin(4A - 2B) + \sin(4B - 2A)}{\cos(4A - 2B) + \cos(4B - 2A)} = \tan(A + B)$.

$$\begin{aligned} \frac{2 \sin \frac{4A - 2B + 4B - 2A}{2} \cos \frac{4A - 2B - 4B + 2A}{2}}{2 \cos \frac{4A - 2B + 4B - 2A}{2} \cos \frac{4A - 2B - 4B + 2A}{2}} \\ = \frac{\sin(A + B)}{\cos(A + B)} = \tan(A + B). \end{aligned}$$

4. Prove $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ = 0$.

$$2 \cos \frac{50^\circ + 70^\circ}{2} \sin \frac{50^\circ - 70^\circ}{2} = 2 \cos 60^\circ \sin(-10^\circ) = -\sin 10^\circ.$$

5. Prove $\frac{\cos 2\alpha \cos 3\alpha - \cos 2\alpha \cos 7\alpha + \cos \alpha \cos 10\alpha}{\sin 4\alpha \sin 3\alpha - \sin 2\alpha \sin 5\alpha + \sin 4\alpha \sin 7\alpha} = \cot 6\alpha \cot 5\alpha$.

By (3) and (4), p. 61,

$$\begin{aligned} \frac{\cos 5\alpha + \cos \alpha - \cos 9\alpha - \cos 5\alpha + \cos 11\alpha + \cos 9\alpha}{\cos \alpha - \cos 7\alpha - \cos 3\alpha + \cos 7\alpha + \cos 3\alpha - \cos 11\alpha} \\ = \frac{\cos \alpha + \cos 11\alpha}{\cos \alpha - \cos 11\alpha} = \frac{2 \cos 6\alpha \cos 5\alpha}{2 \sin 6\alpha \sin 5\alpha} = \cot 6\alpha \cot 5\alpha. \end{aligned}$$

ORAL WORK.

By the formulæ of Art. 51 transform :

6. $\cos 5\alpha + \cos \alpha$.

8. $2 \sin 3\theta \cos \theta$.

7. $\cos \alpha - \cos 5\alpha$.

9. $\sin 2\alpha - \sin 4\alpha$.

10. $\cos 9\theta \cos 2\theta$.

16. $\cos (30^\circ + 2\phi) \sin (30^\circ - \phi)$.

11. $\sin \theta + \sin \frac{\theta}{2}$.

17. $\sin (2r + s) + \sin (2r - s)$.

12. $\sin 75^\circ \sin 15^\circ$.

18. $\cos (2\beta - \alpha) - \cos 3\alpha$.

13. $\cos 7p - \cos 2p$.

19. $\sin 36^\circ + \sin 54^\circ$.

14. $\cos (2p + 3q) \sin (2p - 3q)$.

20. $\cos 60^\circ + \cos 20^\circ$.

15. $\sin \frac{3t}{2} - \sin \frac{t}{2}$.

21. $\sin 30^\circ + \cos 30^\circ$.

Prove: 22. $\frac{\sin \alpha + \sin \beta}{\sin \alpha - \sin \beta} = \tan \frac{\alpha + \beta}{2} \cot \frac{\alpha - \beta}{2}$.

23. $\frac{\cos \alpha + \cos \beta}{\cos \beta - \cos \alpha} = \cot \frac{\alpha + \beta}{2} \cot \frac{\alpha - \beta}{2}$.

24. $\frac{\sin x + \sin y}{\cos x + \cos y} = \tan \frac{x + y}{2}$.

25. $\frac{\sin x - \sin y}{\cos x - \cos y} = -\cot \frac{x + y}{2}$.

26. $\cos 55^\circ + \sin 25^\circ = \sin 85^\circ$.

Simplify: 27. $\frac{\sin B + \sin 2B + \sin 3B}{\cos B + \cos 2B + \cos 3B}$.

28. $\frac{\sin C - \sin 4C + \sin 7C - \sin 10C}{\cos C - \cos 4C + \cos 7C - \cos 10C}$.

52. *Functions of an angle in terms of those of the half angle.*

If in $\sin (\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$, $\alpha = \beta$,

then $\sin (\alpha + \alpha) = \sin 2\alpha = 2 \sin \alpha \cos \alpha$.

In like manner

$$\begin{aligned} \cos (\alpha + \alpha) &= \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha ; \end{aligned}$$

and

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}.$$

ORAL WORK.

Ex. Express in terms of functions of half the given angles :

1. $\sin 4 \alpha$.

4. $\cos x$.

6. $\sin (2 p - q)$.

2. $\cos 3 p$.

5. $\sin \frac{\beta}{2}$.

7. $\cos (30^\circ + 2 \phi)$.

3. $\tan 5 t$.

8. $\sin (x + y)$.

9. From the functions of 30° find those of 60° ; from the functions of 45° , those of 90° .

53. *Functions of an angle in terms of those of twice the angle.*

By Art. 52, $\cos \alpha = 1 - 2 \sin^2 \frac{\alpha}{2} = 2 \cos^2 \frac{\alpha}{2} - 1$.

$$\therefore 2 \sin^2 \frac{\alpha}{2} = 1 - \cos \alpha, \quad \text{and} \quad 2 \cos^2 \frac{\alpha}{2} = 1 + \cos \alpha.$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}; \quad \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}.$$

$$\therefore \tan \frac{\alpha}{2} = \frac{\sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2}} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}.$$

Explain the significance of the $\pm$ sign before the radicals.

Express in terms of the double angle the functions of 120° ; 50° ; 90° , with proper signs prefixed.

Ex. 1. Express in terms of functions of twice the given angles each of the functions in Examples 1-8 above.

2. From the functions of 45° find those of $22^\circ 30'$; from the functions of 36° , those of 18° (see tables of natural functions).

3. Find the corresponding functions of twice and of half each of the following angles, and verify results by the tables of natural functions :

Given

$$\sin 26^\circ 42' = 0.4493,$$

$$\tan 62^\circ 24' = 1.9128,$$

$$\cos 21^\circ 34' = 0.9300.$$

4. Prove $\tan^{-1} \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{x}{2}$.

5. $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1 - x^2}$.

6. If A, B, C are angles of a triangle, prove

$$\sin A + \sin C + \sin B = 4 \cos \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.$$

7. If $\cos^2 \alpha + \cos^2 2\alpha + \cos^2 3\alpha = 1$, then

$$\cos \alpha \cos 2\alpha \cos 3\alpha = 0.$$

8. Prove $\cot A - \cot 2A = \csc 2A$.

9. Prove
$$\frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} = \left[\frac{1 - \tan \frac{\phi}{2}}{1 + \tan \frac{\phi}{2}} \right]^2.$$

10.
$$\frac{\tan \alpha}{\tan(\alpha + \phi)} = 1 - \frac{2 \sin \phi}{\sin(2\alpha + \phi) + \sin \phi}.$$

11. If $y = \tan^{-1} \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}$, prove $x^2 = \sin 2y$.

12. Prove $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x} + \tan^{-1} \frac{2x}{1-x^2} = \frac{5}{2} \tan^{-1} x$.

13. If $y = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$, prove $x = \tan y$.

14. Prove $\cos^2 \alpha + \cos^2 \beta - 1 = \cos(\alpha + \beta) \cos(\alpha - \beta)$.

15. Prove $\sqrt{(\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2} = 2 \sin \frac{\alpha - \beta}{2}$.

16. Prove $\sin^{-1} \sqrt{\frac{x}{a+x}} = \tan^{-1} \sqrt{\frac{x}{a}} = \frac{1}{2} \cos^{-1} \frac{a-x}{a+x}$.

17. Prove $\cos^2 \theta - \cos^2 \phi = \sin(\phi + \theta) \sin(\phi - \theta)$.

18. Prove $\tan A + \tan(A + 120^\circ) + \tan(A - 120^\circ) = 3 \tan 3A$.

19. Prove $\tan \alpha - \tan \frac{\alpha}{2} = \tan \frac{\alpha}{2} \sec \alpha$.

20. $3 \tan^{-1} a = \tan^{-1} \frac{3a - a^3}{1 - 3a^2}$.

21. $\cos^2 3A (\tan^2 3A - \tan^2 A) = 8 \sin^2 A \cos 2A$.

22. $1 + \cos 2(A - B) \cos 2B = \cos^2 A + \cos^2(A - 2B)$.

23. $\cot^2\left(\frac{\pi}{4} + \frac{\theta}{2}\right) = \frac{2 \csc 2\theta - \sec \theta}{2 \csc 2\theta + \sec \theta}$.

TRIGONOMETRIC EQUATIONS AND IDENTITIES.

54. Identities. It was shown in Chapter I that

$$\sin^2 \theta + \cos^2 \theta = 1$$

is true for all values of θ , and in Chapter VI, that

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

is true for all values of α and β . It may be shown that

$$\frac{\sin 2A}{1 + \cos 2A} = \tan A$$

is true for all values of A , thus:

$$\begin{aligned} \frac{\sin 2A}{1 + \cos 2A} &= \frac{2 \sin A \cos A}{1 + 2 \cos^2 A - 1} \quad (\text{by trigonometric transformation}) \\ &= \frac{\sin A}{\cos A} \quad (\text{by algebraic transformation}) \\ &= \tan A \quad (\text{by trigonometric transformation}). \end{aligned}$$

Such expressions are called *trigonometric identities*. *They are true for all values of the angles involved.*

55. Equations. The expression

$$2 \cos^2 \alpha - 3 \cos \alpha + 1 = 0$$

is true for but two values of $\cos \alpha$, viz. $\cos \alpha = \frac{1}{2}$ and 1, *i.e.* the expression is true for $\alpha = 0^\circ, 60^\circ, 300^\circ$, and for no other positive angles less than 360° . Such expressions are called *trigonometric equations*. *They are true only for particular values of the angles involved.*

56. Method of attack. The transformations necessary at any step in the proof of identities, or the solution of equations, are either *trigonometric*, or *algebraic*; *i.e.* in proving an identity, or solving an equation, the student must choose at each step to apply either some principles of algebra, or some trigonometric relations. If at any step no algebraic operation seems advantageous, then usually the expression

should be simplified by endeavoring to state the *different functions* involved in terms of a *single function* of the angle, or if there are *multiple angles*, to reduce all to functions of a *single angle*.

$$\text{Transformations} \left\{ \begin{array}{l} \text{Algebraic} \\ \text{Trigonometric, } \left\{ \begin{array}{l} \text{Single function} \\ \text{to change to a } \left\{ \begin{array}{l} \text{Single angle} \end{array} \right. \end{array} \right. \end{array} \right.$$

No other transformations are needed, and the student will be greatly assisted by remembering that the ready solution of a trigonometric problem consists in wisely choosing at each step between the possible algebraic and trigonometric transformations. Problems involving trigonometric functions will in general be simplified by expressing them entirely in terms of sine and cosine.

EXAMPLES.

1. Prove $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = 2.$

By algebra, $\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = \frac{\sin 3A \cos A - \cos 3A \sin A}{\sin A \cos A}$

by trigonometry,
$$\begin{aligned} &= \frac{\sin (3A - A)}{\sin A \cos A} = \frac{\sin 2A}{\sin A \cos A} \\ &= \frac{2 \sin A \cos A}{\sin A \cos A} = 2. \end{aligned}$$

Or, by trigonometry,

$$\frac{\sin 3A}{\sin A} - \frac{\cos 3A}{\cos A} = \frac{3 \sin A - 4 \sin^3 A}{\sin A} - \frac{4 \cos^3 A - 3 \cos A}{\cos A}$$

by algebra,
$$\begin{aligned} &= 3 - 4 \sin^2 A - 4 \cos^2 A + 3 \\ &= 6 - 4(\sin^2 A + \cos^2 A) = 2. \end{aligned}$$

2. Prove $\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}.$

No algebraic operation simplifies. Two trigonometric changes are needed. 1. To change the functions to a single function, sine or cosine. 2. To change the angles to a single angle, $8A$, $4A$, or $2A$.

By trigonometry and algebra,

$$\frac{\frac{1 - \cos 8\theta}{\cos 8\theta}}{\frac{1 - \cos 4\theta}{\cos 4\theta}} = \frac{\frac{\sin 8\theta}{\cos 8\theta}}{\frac{\sin 2\theta}{\cos 2\theta}};$$

by algebra,
$$\frac{\cos 4\theta(1 - \cos 8\theta)}{1 - \cos 4\theta} = \frac{\sin 8\theta \cos 2\theta}{\sin 2\theta};$$

by trigonometry,

$$\frac{\cos 4\theta(1 - 1 + 2 \sin^2 4\theta)}{1 - 1 + 2 \sin^2 2\theta} = \frac{2 \sin 4\theta \cos 4\theta \cos 2\theta}{\sin 2\theta};$$

by algebra,
$$\frac{\sin 4\theta}{\sin 2\theta} = 2 \cos 2\theta;$$

and
$$\sin 4\theta = 2 \sin 2\theta \cos 2\theta,$$

which is a trigonometric identity.

3. Solve $2 \cos^2 \theta + 3 \sin \theta = 0$.

By trigonometry, $2(1 - \sin^2 \theta) + 3 \sin \theta = 0$,

a quadratic equation in $\sin \theta$.

By algebra, $2 \sin^2 \theta - 3 \sin \theta - 2 = 0$,

and $(\sin \theta - 2)(2 \sin \theta + 1) = 0$.

$$\therefore \sin \theta = 2, \text{ or } -\frac{1}{2}. \text{ Verify.}$$

The value 2 must be rejected. Why?

$\therefore \theta = 210^\circ$, and 330° are the only positive values less than 360° that satisfy the equation.

4. Solve $\sec \theta - \tan \theta = 2$.

Here $\tan \theta = -0.75$, $\therefore$ from the tables of natural functions,

$$\theta = 143^\circ 7' 48'', \text{ or } 323^\circ 7' 48''.$$

Find $\sec \theta$, and verify.

5. Solve $2 \sin \theta \sin 3\theta - \sin^2 2\theta = 0$.

By trigonometry, $\cos 2\theta - \cos 4\theta - \sin^2 2\theta = 0$,

also $\cos 2\theta - \cos^2 2\theta + \sin^2 2\theta - \sin^2 2\theta = 0$.

By algebra, $\cos 2\theta(1 - \cos 2\theta) = 0$.

$$\therefore \cos 2\theta = 0 \text{ or } 1,$$

and $2\theta = 90^\circ, 270^\circ, 0^\circ, \text{ or } 360^\circ,$

whence $\theta = 45^\circ, 135^\circ, 0^\circ, \text{ or } 180^\circ$. Verify.

Or, by trigonometry,

$$2 \sin \theta (3 \sin \theta - 4 \sin^3 \theta) - 4 \sin^2 \theta \cos^2 \theta = 0;$$

by trigonometry and algebra,

$$6 \sin^2 \theta - 8 \sin^4 \theta - 4 \sin^2 \theta + 4 \sin^4 \theta = 0;$$

by algebra,

$$2 \sin^2 \theta - 4 \sin^4 \theta = 0,$$

and

$$2 \sin^2 \theta (1 - 2 \sin^2 \theta) = 0.$$

$$\therefore \sin \theta = 0, \text{ or } \pm \sqrt{\frac{1}{2}},$$

and

$$\theta = 0^\circ, 180^\circ, 45^\circ, 135^\circ, 225^\circ, \text{ or } 315^\circ.$$

The last two values do not appear in the first solution, because only angles less than 360° are considered, and the solution there gave values of 2θ , which in the last two cases would be 450° and 630° .

Solve: 6. $\tan \theta = \cot \theta$.

8. $2 \cos 2\theta - 2 \sin \theta = 1$.

7. $\sin^2 \theta + \cos \theta = 1$.

9. $\sin 2\theta \cos \theta = \sin \theta$.

Prove: 10. $2 \cot 2A = \cot A - \tan A$.

11. $\cos 2x + \cos 2y = 2 \cos(x+y) \cos(x-y)$.

12. $(\cos \alpha + \sin \alpha)^2 = 1 + \sin 2\alpha$.

57. Simultaneous trigonometric equations.

13. Solve $\cos(x+y) + \cos(x-y) = 2$,

$$\sin \frac{x}{2} + \sin \frac{y}{2} = 0.$$

By trigonometry,

$$\cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y = 2,$$

so that

$$\cos x \cos y = 1;$$

also,

$$\sqrt{\frac{1 - \cos x}{2}} + \sqrt{\frac{1 - \cos y}{2}} = 0,$$

and

$$\therefore \cos x = \cos y.$$

Substituting,

$$\cos^2 x = 1,$$

$$\cos x = \pm 1.$$

$$\therefore x = 0^\circ, \text{ or } 180^\circ,$$

and

$$y = x = 0^\circ, \text{ or } 180^\circ. \text{ Verify.}$$

14. Solve for
- R
- and
- F
- .

$$W - F \sin i - R \cos i = 0,$$

$$W + F \cos i - R \sin i = 0.$$

To eliminate F ,

$$W \cos i - F \sin i \cos i - R \cos^2 i = 0,$$

$$W \sin i + F \cos i \sin i - R \sin^2 i = 0.$$

$$\text{Adding,} \quad W(\sin i + \cos i) - R(\sin^2 i + \cos^2 i) = 0.$$

$$\therefore R = W(\sin i + \cos i).$$

$$\text{Substituting,} \quad W - F \sin i - W(\sin i + \cos i) \cos i = 0$$

$$\therefore F = \frac{W - W(\sin i + \cos i) \cos i}{\sin i}.$$

If $W = 3$ tons, and $i = 22^\circ 30'$, compute F and R .

$$R = 3(0.3827 + 0.9239) = 3.9198.$$

$$F = \frac{3 - 3(0.3827 + 0.9239)0.9239}{0.3827} = -1.624.$$

Solve:

15. $472 \cot \theta - 263 \cot \phi = 490, \quad 307 \cot \theta - 379 \cot \phi = 0.$

16. $\sin 2x + 1 = \cos x + 2 \sin x.$

17. $\cos^2 \theta + \sin \theta = 1.$

18. If $2h(\cos^2 \theta - \sin^2 \theta) - 2a \sin \theta \cos \theta + 2b \sin \theta \cos \theta = 0$, prove

$$\theta = \frac{1}{2} \tan^{-1} \frac{2h}{a-b}.$$

Prove:

19. $\tan y = (1 + \sec y) \tan \frac{y}{2}.$

20. $2 \cot^{-1} x = \csc^{-1} \frac{1+x^2}{2x}.$

21. $\sin(\phi + 45^\circ) + \sin(\phi + 135^\circ) = \sqrt{2} \cos \phi.$

22. $\frac{\cos v + \cos 3v}{\cos 3v + \cos 5v} = \frac{1}{2 \cos 2v - \sec 2v}.$

23. $\cos 3x - \sin 3x = (\cos x + \sin x)(1 - 2 \sin 2x).$

Solve:

24. $\sin 2\theta + \sin \theta = \cos 2\theta + \cos \theta.$

25. $4 \cos(\theta + 60^\circ) - \sqrt{2} = \sqrt{6} - 4 \cos(\theta + 30^\circ).$

26. $\cot 2\theta = \tan \theta - 1.$

27. $\cos \theta + \cos 2\theta + \cos 3\theta = 0.$

$$28. \sin 2x + \sqrt{3} \cos 2x = 1.$$

$$29. 3 \tan^2 p + 8 \cos^2 p = 7.$$

30. Determine for what relative values of P and W the following equation is true:

$$\cos^2 \frac{y}{2} - \frac{P}{W} \cos \frac{y}{2} - \frac{1}{2} = 0.$$

31. Compute N from the equation $N + \frac{W}{3} \cos \alpha - \frac{W}{3} \sin \alpha - W \cos \alpha = 0$, when $W = 2000$ pounds and α satisfies the equation $2 \sin \alpha = 1 + \cos \alpha$.

$$32. \sin \theta - \tan \phi (\cos \theta + \sin \theta) = \cos \theta, \sin \theta - \tan \phi \cos \theta = 1.$$

Prove:

$$33. \cot(t + 15^\circ) - \tan(t - 15^\circ) = \frac{4 \cos 2t}{2 \sin 2t + 1}.$$

$$34. \sin^{-1} \frac{3}{5} - \sin^{-1} \frac{5}{13} = \sin^{-1} \frac{16}{65}.$$

$$35. \tan\left(\frac{\pi}{4} + \frac{\omega}{2}\right) = \sqrt{\frac{1 + \sin \omega}{1 - \sin \omega}}.$$

$$36. 2 \sin^{-1} \frac{1}{2} = \cos^{-1} \frac{1}{2}.$$

37. If $\sin A$ is a geometric mean between $\sin B$ and $\cos B$, prove $\cos 2A = 2 \sin(45^\circ - B) \cos(45^\circ + B)$.

$$38. \text{ Prove } \sin(\alpha + \beta + \gamma) = \sin \alpha \cos \beta \cos \gamma + \cos \alpha \sin \beta \cos \gamma \\ + \cos \alpha \cos \beta \sin \gamma - \sin \alpha \sin \beta \sin \gamma.$$

Also find $\cos(\alpha + \beta + \gamma)$.

$$39. \text{ Prove } \tan(\alpha + \beta + \gamma) = \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \beta - \tan \beta \tan \gamma - \tan \gamma \tan \alpha}.$$

If α , β , and γ are angles of a triangle, prove

$$40. \tan \alpha + \tan \beta + \tan \gamma = \tan \alpha \tan \beta \tan \gamma.$$

$$41. \cot \frac{\alpha}{2} + \cot \frac{\beta}{2} + \cot \frac{\gamma}{2} = \cot \frac{\alpha}{2} \cot \frac{\beta}{2} \cot \frac{\gamma}{2}.$$

If $\alpha + \beta + \gamma = 90^\circ$, prove

$$42. \tan \alpha \tan \beta + \tan \beta \tan \gamma + \tan \gamma \tan \alpha = 1.$$

Prove:

$$43. \sin n\alpha = 2 \sin(n-1)\alpha \cos \alpha - \sin(n-2)\alpha.$$

$$44. \cos n\alpha = 2 \cos(n-1)\alpha \cos \alpha - \cos(n-2)\alpha.$$

$$45. \tan n\alpha = \frac{\tan(n-1)\alpha + \tan \alpha}{1 - \tan(n-1)\alpha \tan \alpha}.$$

CHAPTER VII.

TRIANGLES.

58. In geometry it has been shown that a triangle is determined, except in the ambiguous case, if there are given any three independent parts, as follows :

- I. Two angles and a side.
- II. Two sides and an angle,
 (a) the angle being included by the given sides,
 (b) the angle being opposite one of the given sides (ambiguous case).
- III. Three sides.

The angles of a triangle are not three *independent* parts, since they are connected by the relation $A + B + C = 180^\circ$.

The three angles of a triangle will be designated A, B, C , the sides opposite, a, b, c .

But the principles of geometry do not enable us to *compute* the unknown parts. This is accomplished by the following laws of trigonometry :

- I. *Law of Sines*, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.
- II. *Law of Tangents*, $\frac{\tan \frac{1}{2}(A - B)}{\tan \frac{1}{2}(A + B)} = \frac{a - b}{a + b}$, etc.
- III. *Law of Cosines*, $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$, etc.

59. Law of Sines. *In any triangle the sides are proportional to the sines of the angles opposite.*

Let ABC be any triangle, p the perpendicular from B on ac . In I (Fig. 34), C is an *acute*, in II, an *obtuse*, in III,

a *right* angle. The demonstration applies to each triangle, but in II, $\sin ACB = \sin DCB$ (why?); in III, $\sin C = 1$ (why?).

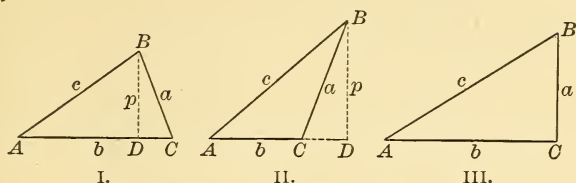


FIG. 34.

$$\text{Now} \quad \sin A = \frac{p}{c}, \quad \therefore p = c \sin A.$$

$$\sin C = \frac{p}{a}, \quad \therefore p = a \sin C.$$

$$\text{Equating values of } p, \quad c \sin A = a \sin C,$$

$$\text{or,} \quad \frac{\sin A}{a} = \frac{\sin C}{c}.$$

By dropping a perpendicular from A , or C , on a , or c , show that

$$\frac{\sin B}{b} = \frac{\sin C}{c}, \text{ or } \frac{\sin A}{a} = \frac{\sin B}{b},$$

$$\text{whence} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

60. Law of Tangents. *The tangent of half the difference of two angles of a triangle is to the tangent of half their sum, as the difference of the sides opposite is to their sum.*

$$\text{By Art. 59,} \quad \frac{a}{b} = \frac{\sin A}{\sin B}.$$

By composition and division,

$$\begin{aligned} \frac{a-b}{a+b} &= \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{2 \cos \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)}{2 \sin \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)} \\ &= \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}; \end{aligned}$$

$$\text{or,} \quad \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)} = \frac{a-b}{a+b}.$$

61. Law of Cosines. *The cosine of any angle of a triangle is equal to the quotient of the sum of the squares of the adjacent sides less the square of the opposite side, divided by twice the product of the adjacent sides.*

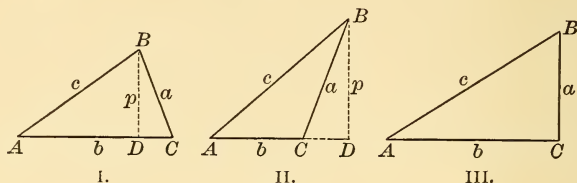


FIG. 34.

$$\begin{aligned} \text{In each figure} \quad a^2 &= p^2 + DC^2 \\ &= c^2 - AD^2 + (b - AD)^2 \end{aligned}$$

(in Fig. 34, II, DC is negative; in III, zero)

$$\begin{aligned} &= c^2 - AD^2 + b^2 - 2b \cdot AD + AD^2 \\ &= b^2 + c^2 - 2b \cdot AD. \end{aligned}$$

But

$$AD = c \cos A, \quad \therefore a^2 = b^2 + c^2 - 2bc \cos A;$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

$$\text{Prove that} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac},$$

$$\text{and} \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}.$$

62. Though these formulæ may be used for the solution of the triangle, they are not adapted to the use of logarithms (why?). Hence we derive the following:

$$\text{Since} \quad \cos A = 2 \cos^2 \frac{A}{2} - 1 = 1 - 2 \sin^2 \frac{A}{2},$$

we have

$$2 \cos^2 \frac{A}{2} = 1 + \cos A, \text{ and } 2 \sin^2 \frac{A}{2} = 1 - \cos A.$$

From the latter

$$\begin{aligned} 2 \sin^2 \frac{A}{2} &= 1 - \frac{b^2 + c^2 - a^2}{2bc} = \frac{2bc - b^2 - c^2 + a^2}{2bc} \\ &= \frac{a^2 - (b - c)^2}{2bc} = \frac{(a - b + c)(a + b - c)}{2bc}. \end{aligned}$$

Let $a + b + c = 2s$, then $a + b - c = a + b + c - 2c = 2s - 2c$;

i.e. $a + b - c = 2(s - c).$

In like manner, $a - b + c = 2(s - b).$

$$-a + b + c = 2(s - a).$$

Substituting, $2 \sin^2 \frac{A}{2} = \frac{2(s - b) \cdot 2(s - c)}{2bc}.$

$$\therefore \sin \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{bc}}.$$

Show that $\sin \frac{B}{2} = ?$

also $\sin \frac{C}{2} = ?$

From $2 \cos^2 \frac{A}{2} = 1 + \cos A,$

show that $\cos \frac{A}{2} = \sqrt{\frac{s(s - a)}{bc}},$

also $\cos \frac{B}{2} = ?$

and $\cos \frac{C}{2} = ?$

Also derive the formulæ

$$\tan \frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}},$$

$$\tan \frac{B}{2} = ?$$

$$\tan \frac{C}{2} = ?$$

63. Area of the triangle. In the figures of Art. 59 the area of the triangle $ABC = \Delta = \frac{1}{2}pb$.

$$\text{But } p = c \sin A. \quad \therefore \Delta = \frac{1}{2}bc \sin A. \quad (\text{i})$$

$$\text{Again, by law of sines, } b = \frac{c \sin B}{\sin C}.$$

$$\begin{aligned} \text{Substituting,} \quad \Delta &= \frac{c^2 \sin A \sin B}{2 \sin C} \\ &= \frac{c^2 \sin A \sin B}{2 \sin(A+B)} \quad (\text{why?}). \quad (\text{ii}) \end{aligned}$$

Finally, since $\sin A = 2 \sin \frac{A}{2} \cos \frac{A}{2}$, we have from (i)

$$\Delta = \frac{1}{2}bc \cdot 2 \sin \frac{A}{2} \cos \frac{A}{2} = bc \sqrt{\frac{s(s-a)(s-b)(s-c)}{bc \cdot bc}}$$

$$\text{or} \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}. \quad (\text{iii})$$

Find Δ ; (1) Given $a = 10$, $b = 12$, $C = 45^\circ$.

(2) Given $a = 4$, $b = 5$, $c = 6$.

(3) Given $a = 2$, $B = 45^\circ$, $C = 60^\circ$.

SOLUTION OF TRIANGLES.

64. For the solution of triangles we have the following formulæ, which should be carefully memorized:

$$\text{I. } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

$$\text{II. } \tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B).$$

$$\text{III. } \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}, \text{ or } \cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}},$$

$$\text{or } \tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

$$\text{IV. } \Delta = \frac{1}{2}bc \sin A = \frac{c^2 \sin A \sin B}{2 \sin(A+B)} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Which of the above formulæ shall be used in the solution of a given triangle must be determined by examining the parts known, as will appear in Art. 69. It is always possible to express each of the unknown parts in terms of three known parts.

In solving triangles such as Case I, Art. 58, the law of sines applies; for, if the given side is not opposite either given angle, the third angle of the triangle is found from the relation $A + B + C = 180^\circ$, and then three of the four quantities in $\frac{\sin A}{a} = \frac{\sin B}{b}$ being known, the solution gives the fourth.

In Case II (*b*) the law of sines applies, but in II (*a*) two only of the four quantities in $\frac{\sin A}{a} = \frac{\sin B}{b}$ are known. Therefore, we resort to the formula

$$\tan \frac{1}{2}(A - B) = \frac{a - b}{a + b} \tan \frac{1}{2}(A + B),$$

in which all the factors of the second member are known.

In Case III, $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ is clearly applicable, and is *preferred* to the formulæ for $\sin \frac{A}{2}$ and $\cos \frac{A}{2}$; for, first, it is more accurate since tangent varies in magnitude from 0 to ∞ , while sine and cosine lie between 0 and 1. (See Art. 27, 5.)

Let the student satisfy himself on this point by finding, correct to seconds, the angle whose logarithmic sine is 9.99992, and whose logarithmic tangent is 1.71668. Does the first determine the angle? Does the second?

And, second, it is more convenient, since in the complete solution of the triangle by $\sin \frac{A}{2}$ *six* logarithms must be taken from the table, by $\cos \frac{A}{2}$ *seven*, and by $\tan \frac{A}{2}$ but *four*.

The right triangle may be solved as a special case by the law of sines, since $\sin C = 1$.

65. Ambiguous case. In geometry it was proved that a triangle having two sides and an angle opposite one of them of given magnitude is not always determined. The marks of the undetermined or ambiguous triangle are :

1. *The parts given are two sides and an angle opposite one.*
2. *The given angle is acute.*
3. *The side opposite this angle is less than the other given side.*

When these marks are all present, the number of solutions must be tested in one of two ways :

(a) From the figure it is apparent that there will be *no solution* when the side opposite is less than the perpendicular p ; *one solution* when side a equals p ; and *two solutions* when a is greater than p .

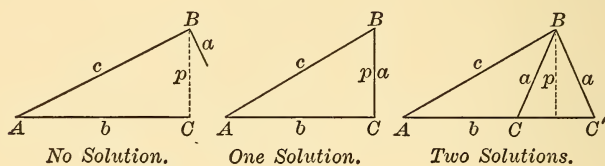


FIG. 35.

And since $\sin A = \frac{p}{c}$, it follows that there will be *no solution*, *one solution*, *two solutions*, according as $\sin A \begin{matrix} \geq \\ < \end{matrix} \frac{a}{c}$.

(b) A good test is found in solving by means of logarithms ; and there will be *no solutions*, *one solution*, *two solutions*, according as $\log \sin C$ proves to be *impossible*, *zero*, *possible*, i.e. as $\log \sin C$ is positive, zero, or negative. This results from the fact that sine cannot be greater than unity, whence $\log$ sine must have a negative characteristic, or be zero.

66. In computations *time* and *accuracy* assume more than usual importance. *Time* will be saved by an orderly arrangement of the formulæ for the complete solution, before opening the book of logarithms, thus :

Given A, B, a . Solve completely.

$$C = 180^\circ - (A + B), \quad b = \frac{a \sin B}{\sin A}, \quad c = \frac{a \sin C}{\sin A}, \quad \Delta = \frac{1}{2} ab \sin C.$$

180°	$\log a =$	$\log a =$
$A + B =$	$\log \sin B =$	$\log \sin C =$
$\therefore C =$	$\text{colog} \sin A =$	$\text{colog} \sin A =$
	$\log b =$	$\log c =$
	$\therefore b =$	$\therefore c =$

Check:

$\log a =$	$\log (s - b) =$
$\log b =$	$\log (s - c) =$
$\log \sin C =$	$\text{colog } s =$
$\text{colog } 2 =$	$\text{colog} (s - a) =$
$\log \Delta =$	$2 \overline{) \quad \quad \quad}$
$\therefore \Delta =$	$\log \tan \frac{A}{2} =$
	$\therefore A =$

67. *Accuracy* must be secured by checks on the work at every step; *e.g.* in adding columns of logarithms, first add up, and then check by adding down. Too much care cannot be given to verification in the simple operations of addition, subtraction, multiplication, and division. A final check should be made by using other formulæ involving the parts in a different way, as in the check above. As far as possible the parts originally given should be used throughout in the solution, so that an error in computing one part may not affect later computations.

68. The formulæ should always be *solved for the unknown part before using*, and it should be noted whether the solution gives one value, or more than one, for each part; *e.g.* the same value of $\sin B$ belongs to two supplementary angles, one or both of which may be possible, as in the ambiguous case.

69. Write formulæ for the complete solution of the following triangles, showing whether you find no solution, one solution, two or more solutions, in each case, with reasons for your conclusion:

	<i>a</i>	<i>b</i>	<i>c</i>	<i>A</i>	<i>B</i>	<i>C</i>
1.				81° 26' 28"	44° 11' 20"	54° 22' 12"
2.		78.54		63° 18' 20"		41° 30' 18"
3.		135.82	26.89	53° 28' 30"		
4.	0.75	0.85	0.95			
5.	243		562			36° 15' 40"
6.		38.75	25.92			63° 50' 10"
7.	0.058			78° 15'	33° 46'	
8.	2986		1493			30°
9.		48	50		26° 15'	

MODEL SOLUTIONS.

1. Given $a = 0.785$, $b = 0.85$, $c = 0.633$. Solve completely.

$$\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}, \quad \tan \frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}, \quad \tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.$$

$$\text{Check: } A + B + C = 180^\circ. \quad \Delta = \sqrt{s(s-a)(s-b)(s-c)}.$$

$a = 0.735$	$\log(s-b) = 9.45332$	$\log(s-a) = 9.54283$
$b = 0.85$	$\log(s-c) = 9.69984$	$\log(s-c) = 9.69984$
$c = 0.633$	$\text{colog } s = 9.94539$	$\text{colog } s = 9.94539$
$2) 2.268$	$\text{colog}(s-a) = 0.45717$	$\text{colog}(s-b) = 0.54668$
$s = 1.134$	$2) 19.55572$	$2) 19.73474$
$s-a = 0.349$	$\log \tan \frac{1}{2} A = 9.77786$	$\log \tan \frac{1}{2} B = 9.86737$
$s-b = 0.284$	$\frac{1}{2} A = 30^\circ 56' 49''$	$\frac{1}{2} B = 36^\circ 23' 2''$
$s-c = 0.501$	$A = 61^\circ 53' 38''$	$B = 72^\circ 46' 4''$

<i>Check:</i>	$\log(s-a) = 9.54283$	$\log s = 0.05461$
$A = 61^\circ 53' 38''$	$\log(s-b) = 9.45332$	$\log(s-a) = 9.54283$
$B = 72^\circ 46' 4''$	$\text{colog } s = 9.94539$	$\log(s-b) = 9.45332$
$C = 45^\circ 20' 20''$	$\text{colog}(s-c) = 0.30016$	$\log(s-c) = 9.69984$
$180^\circ 0' 2''$	$2) 19.24170$	$2) 18.75060$
	$\log \tan \frac{1}{2} C = 9.62085$	$\log \Delta = 9.37530$
	$\frac{1}{2} C = 22^\circ 40' 10''$	$\Delta = 0.2373$
	$C = 45^\circ 20' 20''$	

Solve: (1) Given $a = 30$, $b = 40$, $c = 50$.

(2) Given $a = 2159$, $b = 1431.6$, $c = 914.8$.

(3) Given $a = 78.54$, $b = 32.56$, $c = 48.9$.

2. Given $A = 57^\circ 23' 12''$, $C = 68^\circ 15' 30''$, $c = 832.56$. Solve completely.

$$a = \frac{c \sin A}{\sin C}, \quad b = \frac{c \sin B}{\sin C}, \quad \Delta = \frac{1}{2} bc \sin A.$$

$$B = 180^\circ - (A + C) \\ = 54^\circ 21' 18''.$$

$$\text{Check: } \tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}.$$

$\log c = 2.92042$	$\log c = 2.92042$	$\log b = 2.86236$
$\log \sin A = 9.92548$	$\log \sin B = 9.90990$	$\log c = 2.92042$
$\text{colog } \sin C = 0.03204$	$\text{colog } \sin C = 0.03204$	$\log \sin A = 9.92548$
$\log a = 2.87794$	$\log b = 2.86236$	$\log 2 \Delta = 5.70826$

$$a = 754.98 \quad b = 728.38 \quad \Delta = \frac{510811}{2} = 255405.5$$

$\text{Check: } a = 754.98$	$s - a = 402.98$	$\log (s - b) = 2.63304$
$b = 728.38$	$s - b = 429.58$	$\log (s - c) = 2.51242$
$c = 832.56$	$s - c = 325.40$	$\text{colog } s = 6.93634$
$2) 2315.92$		$\text{colog } (s - a) = 7.39471$
$s = 1157.96$		$2) 19.47651$

$$\log \tan \frac{1}{2} A = 9.73826 \\ \frac{1}{2} A = 28^\circ 41' 38'' \\ A = 57^\circ 23' 16''$$

Solve:

(1) Given $a = 215.73$, $B = 92^\circ 15'$, $C = 28^\circ 14'$.

(2) Given $b = 0.827$, $A = 78^\circ 14' 20''$, $B = 63^\circ 42' 30''$.

(3) Given $b = 7.54$, $c = 6.93$, $B = 54^\circ 28' 40''$.

3. Given $a = 25.384$, $c = 52.925$, $B = 28^\circ 32' 20''$. Solve completely.

(Why not use the same formulæ as in Example 1, or 2?)

$$\tan \frac{C - A}{2} = \frac{c - a}{c + a} \tan \frac{C + A}{2}, \quad b = \frac{c \sin B}{\sin C}, \quad \Delta = \frac{1}{2} ac \sin B.$$

$$180^\circ - B = C + A = 151^\circ 27' 40''.$$

$$\therefore \frac{1}{2} (C + A) = 75^\circ 43' 50''.$$

$$\text{Check: } b = \frac{a \sin B}{\sin A}.$$

$$c = 52.925 \quad \log (c - a) = 1.43998 \quad \therefore \frac{1}{2} (C - A) = 54^\circ 7' 38''$$

$$a = 25.384 \quad \text{colog } (c + a) = 8.10619 \quad \frac{1}{2} (C + A) = 75^\circ 43' 50''$$

$$c + a = 78.309 \quad \log \tan \frac{1}{2} (C + A) = 0.59460 \quad \text{adding, } C = 129^\circ 51' 28''$$

$$c - a = 27.541 \quad \log \tan \frac{1}{2} (C - A) = 0.14077 \quad \text{subtracting, } A = 21^\circ 36' 12''$$

$\log c = 1.72366$		$\log a = 1.40456$
$\log \sin B = 9.67921$		$\log c = 1.72366$
$\text{colog } \sin C = 0.11484$	$\text{Check: } \log a = 1.40456$	$\log \sin B = 9.67921$
$\log b = 1.51771$	$\log \sin B = 9.67921$	$\log 2 \Delta = 2.70743$
	$\text{colog } \sin A = 0.43395$	$\Delta = \frac{509.83}{2} = 254.965$
$b = 32.939$	$\log b = 1.51772$	

Solve: (1) Given $a = 0.325$, $c = 0.426$, $B = 48^\circ 50' 10''$.

(2) Given $b = 4291$, $c = 3194$, $A = 73^\circ 24' 50''$.

(3) Given $b = 5.38$, $c = 12.45$, $A = 62^\circ 14' 40''$.

4. *Ambiguous cases.* Since the required angle is found in terms of its sine, and since $\sin \alpha = \sin(180^\circ - \alpha)$, it follows that there may be two values of α , one in the first, and the other in the second quadrant, their sum being 180° . In the following examples the student should note that all the marks of the ambiguous case are present. The solutions will show the treatment of the ambiguous triangle having no solution, one solution, two solutions.

(a) Given $b = 70$, $c = 40$, $C = 47^\circ 32' 10''$. Solve. Why ambiguous?

$$\sin B = \frac{b \sin C}{c}$$

$$\log b = 1.84510$$

$$\log \sin C = 9.86788$$

$$\text{colog } c = \underline{8.39794}$$

$$\log \sin B = 0.11092$$

$\therefore B$ is impossible, and there is no solution. Why?
Show the same by $\sin C > \frac{c}{b}$.

(b) Given $a = 1.5$, $c = 1.7$, $A = 61^\circ 55' 38''$. Solve.

$$\sin C = \frac{c \sin A}{a}$$

$$\log c = 0.23045$$

$$\log \sin A = 9.94564$$

$$\text{colog } a = \underline{9.82391}$$

$$\log \sin C = 0.00000$$

$$C = 90^\circ$$

and there is one solution. Why? Show the same by $\sin A = \frac{a}{c}$. Solve for the remaining parts and check the work.

(c) Given $a = 0.235$, $b = 0.189$, $B = 36^\circ 28' 20''$. Solve.

$$\sin A = \frac{a \sin B}{b},$$

$$c = \frac{b \sin C}{\sin B}.$$

$\log a = 9.37107$	$\log b = 9.27646$	9.27646
$\log \sin B = 9.77411$	$\log \sin C = 9.99772$	or 9.28774
$\text{colog } b = 0.72354$	$\text{colog } \sin B = 0.22589$	0.22589
$\log \sin A = 9.86872$	$\log c = 9.50007$	or 8.79009
$A = 47^\circ 39' 25''$	$c = 0.31628$	or 0.06167
or $132^\circ 20' 35''$.		

$$\therefore C = 95^\circ 52' 15'' \text{ or } 11^\circ 11' 5''.$$

Solve for Δ , and check. Show the same by $\sin B < \frac{b}{a}$.

Solve :

(1) Given $b = 216.4$, $c = 593.2$, $B = 98^\circ 15'$.

(2) Given $a = 22$, $b = 75$, $B = 32^\circ 20'$.

(3) Given $a = 0.353$, $c = 0.295$, $A = 46^\circ 15' 20''$.

(4) Given $a = 293.445$, $b = 450$, $A = 40^\circ 42'$.

(5) Given $b = 531.03$, $c = 629.20$, $B = 34^\circ 28' 16''$.

Solve completely, given :

	a	b	c	A	B	C
1.	50	60				$78^\circ 27' 47''$
2.		10	11			$93^\circ 35'$
3.	4	5	6			
4.			10	$109^\circ 28' 16''$	$38^\circ 56' 54''$	
5.	40	51		$49^\circ 28' 32''$		
6.	352.25	513.27	482.68			
7.	0.573	0.394		$112^\circ 4'$		
8.	107.087			$56^\circ 15'$	$48^\circ 35'$	
9.			$\sqrt{2}$	117°	45°	
10.	197.63	246.35		$34^\circ 27'$		
11.	4090	3850	3811			
12.	3795				$73^\circ 15' 15''$	$42^\circ 18' 30''$
13.		234.7	185.4	$84^\circ 36'$		
14.		26.234	22.6925		$49^\circ 8' 24''$	
15.	273	136		$72^\circ 25' 13''$		

APPLICATIONS.

70. Measurements of heights and distances often lead to the solution of oblique triangles. With this exception, the methods of Chapter V apply, as will be illustrated in the following problems.

The *bearing* of a line is the angle it makes with a north and south line, as determined by the magnetic needle of the mariner's compass. If the bearing does not correspond to any of the points of the compass, it is usual to express it thus: N. 40° W., meaning that the line bears from N. 40° toward W.

EXAMPLES.

1. When the altitude of the sun is 48° , a pole standing on a slope inclined to the horizon at an angle of 15° casts a shadow directly down the slope 44.3 ft. How high is the pole?

2. A tree standing on a mountain side rising at an angle of $18^\circ 30'$ breaks 32 ft. from the foot. The top strikes down the slope of the mountain 28 ft. from the foot of the tree. Find the height of the tree.

3. From one corner of a triangular lot the other corners are found to be 120 ft. E. by N., and 150 ft. S. by W. Find the area of the lot, and the length of the fence required to enclose it.

4. A surveyor observed two inaccessible headlands, A and B. A was W. by N. and B, N.E. He went 20 miles N., when they were S.W. and S. by E. How far was A from B?

5. The bearings of two objects from a ship were N. by W. and N.E. by N. After sailing E. 11 miles, they were in the same line W.N.W. Find the distance between them.

6. From the top and bottom of a vertical column the elevation angles of the summit of a tower 225 ft. high and standing on the same horizontal plane are 45° and 55° . Find the height of the column.

7. An observer in a balloon 1 mile high observes the depression angle of an object on the ground to be $35^\circ 20'$. After ascending vertically and uniformly for 10 mins., he observes the depression angle of the same object to be $55^\circ 40'$. Find the rate of ascent of the balloon in miles per hour.

8. A statue 10 ft. high standing on a column subtends, at a point 100 ft. from the base of the column and in the same horizontal plane, the same angle as that subtended by a man 6 ft. high, standing at the foot of the column. Find the height of the column.

9. From a balloon at an elevation of 4 miles the dip of the horizon is $2^\circ 33' 40''$. Required the earth's radius.

10. Two ships sail from Boston, one S.E. 50 miles, the other N.E. by E. 60 miles. Find the bearing and distance of the second ship from the first.

11. The sides of a valley are two parallel ridges sloping at an angle of 30° . A man walks 200 yds. up one slope and observes the angle of elevation of the other ridge to be 15° . Show that the height of the observed ridge is 273.2 yds.

12. To determine the height of a mountain, a north and south base line 1000 yds. long is measured; from one end of the base line the summit bears E. 10° N., and is at an altitude of $13^\circ 14'$. From the other end it bears E. $46^\circ 30'$ N. Find the height of the mountain.

13. The shadow of a cloud at noon is cast on a spot 1600 ft. due west of an observer. At the same instant he finds that the cloud is at an elevation of 23° in a direction W. 14° S. Find the height of the cloud and the altitude of the sun.

14. From the base of a mountain the elevation of its summit is $54^\circ 20'$. From a point 3000 ft. toward the summit up a plane rising at an angle of $25^\circ 30'$ the elevation angle is $68^\circ 42'$. Find the height of the mountain.

15. From two observations on the same meridian, and $92^\circ 14'$ apart, the zenith angles of the moon are observed to be $44^\circ 54' 21''$ and $48^\circ 42' 57''$. Calling the earth's radius 3956.2 miles, find the distance to the moon.

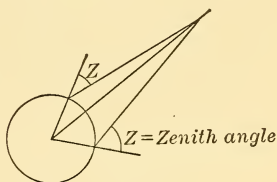


FIG. 36.

16. The distances from a point to three objects are 1130, 1850, 1456, and the angles subtended by the distances between the three objects are respectively $102^\circ 10'$, 142° , and $115^\circ 50'$. Find the distances between the three objects.

17. From a ship A running N.E. 6 mi. an hour direct to a port distant 35 miles, another ship B is seen steering toward the same port, its bearing from A being E.S.E., and distance 12 miles. After keeping on their courses $1\frac{1}{2}$ hrs., B is seen to bear from A due E. Find B's course and rate of sailing.

18. From the mast of a ship 64 ft. high the light of a lighthouse is just visible when 30 miles distant. Find the height of the lighthouse, the earth's radius being 3956.2 miles.

19. From a ship two lighthouses are observed due N.E. After sailing 20 miles E. by S., the lighthouses bear N.N.W. and N. by E. Find the distance between the lighthouses.

20. A lighthouse is seen N. 20° E. from a vessel sailing S. 25° E. A mile further on it appears due N. Determine its distance at the last observation.

EXAMPLES FOR REVIEW.

In connection with each problem the student should review all principles involved. The following list of problems will then furnish a thorough review of the book. In solving equations, find all values of the unknown angle less than 360° that satisfy the equation.

1. If $\tan \alpha = \frac{1}{7}$, $\tan \beta = \frac{1}{2}$, show that $\tan (\beta - 2\alpha) = \frac{2}{11}$.

2. Prove $\tan \alpha + \cot \alpha = 2 \csc 2\alpha$.

3. From the identities $\sin^2 \frac{A}{2} + \cos^2 \frac{A}{2} = 1$, and $2 \sin \frac{A}{2} \cos \frac{A}{2} = \sin A$,

prove $2 \sin \frac{A}{2} = \pm \sqrt{1 + \sin A} \pm \sqrt{1 - \sin A}$,

and $2 \cos \frac{A}{2} = \pm \sqrt{1 + \sin A} \mp \sqrt{1 - \sin A}$.

4. Remove the ambiguous signs in Ex. 3 when A is in turn an angle of each quadrant.

5. A wall 20 feet high bears S. $59^\circ 5'$ E.; find the width of its shadow on a horizontal plane when the sun is due S. and at an altitude of 60° .

6. Solve $\sin x + \sin 2x + \sin 3x = 1 + \cos x + \cos 2x$.

7. Prove $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$.

8. If $A = 60^\circ$, $B = 45^\circ$, $C = 30^\circ$, evaluate

$$\frac{\tan A + \tan B + \tan C}{\tan A \tan B + \tan B \tan C + \tan C \tan A}.$$

9. Prove $\frac{\cos(A+B) \cos C}{\cos(A+C) \cos B} = \frac{1 - \tan A \tan B}{1 - \tan A \tan C}$.

10. Solve completely the triangle whose known parts are $b = 2.35$, $c = 1.96$, $C = 38^\circ 45' 4$.

11. Find the functions of 18° , 36° , 54° , 72° .

Let $x = 18^\circ$. Then $2x = 36^\circ$, $3x = 54^\circ$, and $2x + 3x = 90^\circ$.

12. If $\cot \alpha = \frac{p}{q}$, find the value of

$$\sin \alpha + \cos \alpha + \tan \alpha + \cot \alpha + \sec \alpha + \csc \alpha.$$

13. Prove $\frac{\sin 3\alpha \sin 2\beta - \sin 3\beta \sin 2\alpha}{\sin 2\alpha \sin \beta - \sin 2\beta \sin \alpha} = 1 + 4 \cos \alpha \cos \beta$.

14. From a ship sailing due N., two lighthouses bear N.E. and N.N.E., respectively; after sailing 20 miles they are observed to bear due E. Find the distance between the lighthouses.

15. Solve $1 - 2 \sin x = \sin 3x$.

16. Prove $\sin^{-1} \sqrt{\frac{a}{a+b}} = \tan^{-1} \sqrt{\frac{a}{b}}$.

17. If $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$, then $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$.

18. Solve completely the triangle ABC , given $a = 0.256$, $b = 0.387$, $C = 102^\circ 20' 5$.

19. Prove $\tan(30^\circ + \alpha) \tan(30^\circ - \alpha) = \frac{2 \cos 2\alpha - 1}{2 \cos 2\alpha + 1}$.

20. Solve $\tan(45^\circ - \theta) + \tan(45^\circ + \theta) = 4$.

21. Prove $\sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta = \sin^2 \alpha - \sin^2 \beta$.

22. Prove $\cos^2 \alpha \cos^2 \beta - \sin^2 \alpha \sin^2 \beta = \cos^2 \alpha - \sin^2 \beta$.

23. A man standing due S. of a water tower 150 feet high finds its elevation to be $72^\circ 30'$; he walks due W. to A street, where the elevation is $44^\circ 50'$; proceeding in the same direction one block to B street, he finds the elevation to be $22^\circ 30'$. What is the length of the block between A and B streets?

24. Prove $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

25. If $P = 60^\circ$, $Q = 45^\circ$, $R = 30^\circ$, evaluate

$$\frac{\sin P \cos Q + \tan P \cos Q}{\sin P \cos P + \cot P \cot R}$$

26. If $\cos(90^\circ + \alpha) = -\frac{3}{4}$, evaluate $3 \cos 2\alpha + 4 \sin 2\alpha$.

27. If $\sin B + \sin C = m$, $\cos B + \cos C = n$, show that $\tan \frac{B+C}{2} = \frac{m}{n}$.

28. Show that $\sin 2\beta$ can never be greater than $2 \sin \beta$.

29. Prove $\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} = \tan^{-1} \frac{56}{33}$.

30. Solve $\cot^{-1} x + \sin^{-1} \frac{1}{5} \sqrt{5} = \frac{\pi}{4}$.

31. Solve $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.

32. A man standing between two towers, 200 feet from the base of the higher, which is 90 feet high, observes their altitudes to be the same; 70 feet nearer the shorter tower he finds the altitude of one is twice that of the other. Find the height of the shorter tower, and his original distance from it.

33. Solve $\cos 3\beta + 8 \cos^3 \beta = 0$.

34. Solve $\cot m - \tan (180^\circ + m) = \sec m + \sec (90^\circ - m)$.

35. Solve $\frac{1 - \tan t}{1 + \tan t} = 2 \cos 2t$.

36. Prove $\cot A + \cot B = \frac{\sin(A+B)}{\sin A \sin B}$.

37. Prove $\cot P - \cot Q = -\frac{\sin(P-Q)}{\sin P \sin Q}$.

38. In the triangle ABC prove

$$a = b \sin C + c \sin B,$$

$$b = c \sin A + a \sin C,$$

$$c = a \sin B + b \sin A.$$

39. Solve completely the triangle, given

$$a = 927.56, b = 648.25, c = 738.42.$$

40. Prove $\cos^2 \alpha - \sin(30^\circ + \alpha) \sin(30^\circ - \alpha) = \frac{3}{4}$.

41. Prove $\tan 3x \tan x = \frac{\cos 2x - \cos 4x}{\cos 2x + \cos 4x}$.

42. Simplify $\cos(270^\circ + \alpha) + \sin(180^\circ + \alpha) + \cos(90^\circ + \alpha)$.

43. Simplify $\tan(270^\circ - \theta) - \tan(90^\circ + \theta) + \tan(270^\circ + \theta)$.

44. Solve $\cos 3\phi - \cos 2\phi + \cos \phi = 0$.

45. Solve $\cos A + \cos 3A + \cos 5A + \cos 7A = 0$.

46. The topmast of a yacht from a point on the deck subtends the same angle α , that the part below it does. Show that if the topmast be a feet high, the length of the part below it is $a \cos 2\alpha$.

47. A horizontal line AB is measured 400 yards long. From a point in AB a balloon ascends vertically till its elevation angles at A and B are $64^\circ 15'$ and $48^\circ 20'$, respectively. Find the height of the balloon.

48. If $\cos \phi = n \sin \alpha$, and $\cot \phi = \frac{\sin \alpha}{\tan \beta}$, prove $\cos \beta = \frac{n}{\sqrt{1 + n^2 \cos^2 \alpha}}$.

49. Find $\cos 3\alpha$, when $\tan 2\alpha = -\frac{3}{4}$.

50. Solve completely the triangle, given $a = 0.296$, $B = 28^\circ 47' 3$, $C = 84^\circ 25'$.

51. Evaluate $\sin 300^\circ + \cos 240^\circ + \tan 225^\circ$.

52. Evaluate $\sec \frac{2\pi}{3} - \csc \frac{5\pi}{3} + \tan \frac{4\pi}{3}$.

53. If $\tan \theta = \frac{\sin \alpha \cos \gamma - \sin \beta \sin \gamma}{\cos \alpha \cos \gamma - \cos \beta \sin \gamma}$

and $\tan \phi = \frac{\sin \alpha \sin \gamma - \sin \beta \cos \gamma}{\cos \alpha \sin \gamma - \cos \beta \cos \gamma}$,

show that $\tan(\theta + \phi) = \tan(\alpha + \beta)$.

54. If $\tan 466^\circ 15' 38'' = -\frac{2}{7}$, find the sine and cosine of $233^\circ 7' 49''$.

55. Prove $\frac{\csc \alpha - \cot \alpha}{\sec \alpha + \tan \alpha} = \frac{\sec \alpha - \tan \alpha}{\csc \alpha + \cot \alpha}$.

56. Prove $\frac{\cos(\alpha - 3\beta) - \cos(3\alpha - \beta)}{\sin 2\alpha + \sin 2\beta} = 2 \sin(\alpha - \beta)$.

57. Prove $\sin 80^\circ = \sin 40^\circ + \sin 20^\circ$.

58. Prove $\cos 20^\circ = \cos 40^\circ + \cos 80^\circ$.

59. Prove $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239} = \frac{\pi}{4}$.

60. From the deck of a ship a rock bears N.N.W. After the ship has sailed 10 miles E.N.E., the rock bears due W. Find its distance from the ship at each observation.

61. Find the length of an arc of 80° in a circle of 4 feet radius.

62. Given $\tan \theta = \frac{4}{3}$, $\tan \phi = \frac{5}{12}$, evaluate $\sin(\theta + \phi) + \cos(\theta - \phi)$.

63. If $\tan \theta = 2 \tan \phi$, show that $\sin(\theta + \phi) = 3 \sin(\theta - \phi)$.

64. Prove $\cos(\alpha + \beta) \cos(\alpha - \beta) + \sin(\alpha + \beta) \sin(\alpha - \beta) = \frac{1 - \tan^2 \beta}{1 + \tan^2 \beta}$.

65. Solve $4 \cos 2\theta + 3 \cos \theta = 1$.

66. Solve $3 \sin \alpha = 2 \sin(60^\circ - \alpha)$.

67. Prove $(\sin \alpha - \csc \alpha)^2 - (\tan \alpha - \cot \alpha)^2 + (\cos \alpha - \sec \alpha)^2 = 1$.

68. Prove $2(\sin^6 \alpha + \cos^6 \alpha) + 1 = 3(\sin^4 \alpha + \cos^4 \alpha)$.

69. Prove $\csc 2\beta + \cot 4\beta = \cot \beta - \csc 4\beta$.

70. If $\tan p = \frac{5}{12}$, $\cos 2q = \frac{527}{625}$, then $\csc \frac{p-q}{2} = 5\sqrt{13}$.

71. Solve completely the triangle, given

$$a = 0.0654, \quad b = 0.092, \quad B = 38^\circ 40' 4.$$

72. Solve completely the triangle, given

$$b = 10, \quad c = 26, \quad B = 22^\circ 37'.$$

73. A railway train is travelling along a curve of $\frac{1}{2}$ mile radius at the rate of 25 miles per hour. Through what angle (in circular measure) will it turn in half a minute?

74. Express the following angles in circular measure :

$$63^\circ, \quad 4^\circ 30', \quad 6^\circ 12' 36''.$$

75. Express the following angles in sexagesimal measure :

$$\frac{\pi}{6}, \quad \frac{3\pi}{8}, \quad \frac{17\pi}{64}.$$

76. If A, B, C are angles of a triangle, prove

$$\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.$$

77. Prove $\sin 2x + \sin 2y + \sin 2z = 4 \sin x \sin y \sin z$, when x, y, z are the angles of a triangle.

78. Prove $\sec \alpha = 1 + \tan \alpha \tan \frac{\alpha}{2}$.

79. Prove $\sin^2(\alpha + \beta) - \sin^2(\alpha - \beta) = \sin 2\alpha \sin 2\beta$.

80. Prove $\cos^2(\alpha + \beta) - \sin^2(\alpha - \beta) = \cos 2\alpha \cos 2\beta$.

81. Prove $\frac{\sin 19p + \sin 17p}{\sin 10p + \sin 8p} = 2 \cos 9p$.

82. Consider with reference to their ambiguity the triangles whose known parts are :

$$(a) \quad a = 2743, \quad b = 6452, \quad B = 43^\circ 15';$$

$$(b) \quad a = 0.3854, \quad c = 0.2942, \quad C = 38^\circ 20';$$

$$(c) \quad b = 5, \quad c = 53, \quad B = 15^\circ 22';$$

$$(d) \quad a = 20, \quad b = 90, \quad A = 63^\circ 28'.5.$$

83. From a ship at sea a lighthouse is observed to bear S.E. After the ship sailed N.E. 6 miles the bearing of the lighthouse is S. $27^\circ 30'$ E. Find the distance of the lighthouse at each time of observation.

84. Prove $\frac{\sin(\theta + 3\phi) + \sin(3\theta + \phi)}{\sin 2\theta + \sin 2\phi} = 2 \cos(\theta + \phi)$.

85. Prove $\cos 15^\circ - \sin 15^\circ = \frac{1}{\sqrt{2}}$.

86. Show that $\cos(\alpha + \beta) \cos(\alpha - \beta) = \cos^2 \alpha - \sin^2 \beta$
 $= \cos^2 \beta - \sin^2 \alpha$.

87. Show that $\tan(\alpha + 45^\circ) \tan(\alpha - 45^\circ) = \frac{2 \sin^2 \alpha - 1}{2 \cos^2 \alpha - 1}$.

88. Solve $\sin(x + y) \sin(x - y) = \frac{1}{2}$, $\cos(x + y) \cos(x - y) = 0$.

89. Prove $\frac{1 + \sin \alpha - \cos \alpha}{1 + \sin \alpha + \cos \alpha} = \tan \frac{\alpha}{2}$.

90. Prove $\tan 2\theta + \sec 2\theta = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$.

91. If $\tan \phi = \frac{b}{a}$, then $a \cos 2\phi + b \sin 2\phi = a$.

92. Prove $\sin^{-1} \frac{1}{\sqrt{5}} + \cot^{-1} 3 = \frac{\pi}{4}$.

93. Solve $\cos A + \cos 7A = \cos 4A$.

94. Two sides of a triangle, including an acute angle, are 5 and 7, the area is 14; find the other side.

95. Show that $\frac{3 \cos 3\theta - 2 \cos \theta - \cos 5\theta}{\sin 5\theta - 3 \sin 3\theta + 4 \sin \theta} = \tan 2\theta$.

96. A regular pyramid stands on a square base one side of which is 173.6 feet. This side makes an angle of 67° with one edge. What is the height of the pyramid?

97. From points directly opposite on the banks of a river 500 yards wide the mast of a ship lying between them is observed to be at an elevation of $10^\circ 28'.4$ and $12^\circ 14'.5$, respectively. Find the height of the mast.

98. Show that $(\sin 60^\circ - \sin 45^\circ)(\cos 30^\circ + \cos 45^\circ) = \sin^2 30^\circ$.

99. Find x if $\sin^{-1} x + \sin^{-1} \frac{x}{2} = \frac{\pi}{4}$.

100. Trace the changes in sign and value of $\sin \alpha + \cos \alpha$ as α changes from 0° to 360° .

CHAPTER VIII.

MISCELLANEOUS PROPOSITIONS.

71. The circle inscribed in a given triangle is often called the *incircle* of the triangle, its centre the *incentre*, and its radius is denoted by r . The incentre is the point of intersection of the three bisectors of the angles of the triangle (geometry).

The circle circumscribed about a triangle is called the *circumcircle*, its centre the *circumcentre*, and its radius R . The circumcentre is the point of intersection of perpendiculars erected at the middle points of the three sides of the triangle (geometry).

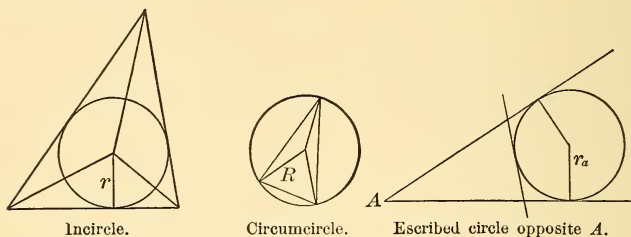


FIG. 37.

The circle which touches any side of a triangle and the other two sides produced is called the *escribed* circle; its radius is denoted by r_a , r_b , or r_c , according as the escribed circle is opposite angle A , B , or C .

Again, the altitudes from the vertices of a triangle meet in a point called the *orthocentre* of the triangle.

Finally, the medians of a triangle meet in a point called the *centroid*, which is two-thirds of the length of the median from the vertex of the angle from which that median is drawn (geometry).

Certain properties of the above will now be considered.

72. To find the radius of the incircle.

Let Δ , Δ' , Δ'' , Δ''' represent the areas of triangles ABC , COB , AOB , BOA , respectively.

Then

$$\begin{aligned}\Delta &= \Delta' + \Delta'' + \Delta''' \\ &= \frac{1}{2}(a + b + c)r = sr.\end{aligned}$$

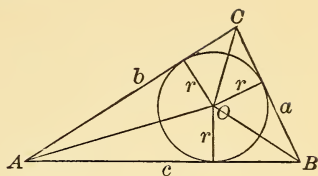


FIG. 38.

And since $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, (Art. 63)

$$\therefore r = \frac{\Delta}{s} = \frac{\sqrt{(s-a)(s-b)(s-c)}}{s}.$$

COR. To express the angles in terms of r and the sides, divide each member of the above equation by $s - a$.

$$\text{Then } \frac{r}{s-a} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan \frac{1}{2} A. \quad (\text{Art. 62})$$

$$\text{In like manner } \tan \frac{1}{2} B = \frac{r}{s-b}; \quad \tan \frac{1}{2} C = \frac{r}{s-c}.$$

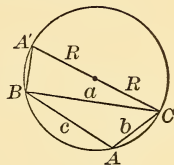
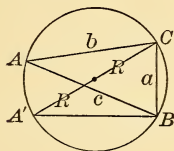
To find the radius of the circumcircle.


FIG. 39.

In the figure ABC is the given triangle, and $A'C$ a diameter of the circumcircle. Then, angle $A = A'$, or $180^\circ - A'$.

$$\therefore \sin A = \sin A'.$$

Since $A'BC$ is a right angle,

$$\sin A' = \frac{BC}{A'C} = \frac{a}{2R}.$$

$$\therefore R = \frac{\frac{1}{2}a}{\sin A}.$$

COR. 1. As above, $2R = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, which is another proof of the "law of sines."

COR. 2. From $R = \frac{a}{2 \sin A}$, we have

$$R = \frac{abc}{2bc \sin A} = \frac{abc}{4\Delta}, \text{ where } \Delta = \text{area } ABC.$$

74. To find the radii of the escribed circles.

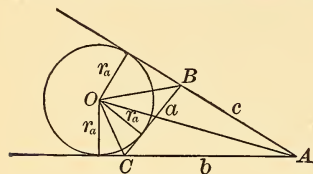


FIG. 40.

Represent areas ABC , BOA , AOC , BOC , by Δ , Δ' , Δ'' , Δ''' , respectively. Then r_a is the altitude of each of the triangles BOA , AOC , BOC .

Now

$$\begin{aligned} \Delta &= \Delta' + \Delta'' - \Delta''' \\ &= \frac{1}{2} r_a c + \frac{1}{2} r_a b - \frac{1}{2} r_a a \\ &= \frac{1}{2} r_a (c + b - a) = r_a (s - a). \end{aligned}$$

$$\therefore r_a = \frac{\Delta}{s - a}.$$

In like manner, $r_b = \frac{\Delta}{s - b}$; $r_c = \frac{\Delta}{s - c}$.

75. The orthocentre.

Denote the perpendiculars on the sides a , b , c , by AP_a , BP_b , CP_c , and let it be required to find the distances from their intersection O to the sides of the triangle, and also to the vertices.

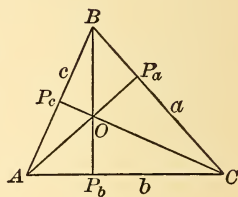


FIG. 41.

$$OP_b = AP_b \tan CAO.$$

But $AP_b = c \cos A$, and $CAO = 90^\circ - C$.

$$\therefore OP_b = c \cos A \cot C = \frac{c}{\sin C} \cos A \cos C.$$

$$= 2R \cos A \cos C. \quad (\text{Art. 73, Cor. 1})$$

In like manner, $OP_c = 2 R \cos B \cos A$,

$$OP_a = 2 R \cos C \cos B.$$

Again, the distances from the orthocentre to the vertices are,

$$\begin{aligned} OA &= \frac{AP_b}{\cos \angle CAO} = \frac{c \cos A}{\sin C} \\ &= 2 R \cos A. \end{aligned}$$

Also, $OB = 2 R \cos B$,

and $OC = 2 R \cos C$.

76. Centroid and medians.

The lengths of the medians may be computed as follows :

In the figure the medians to the sides a , b , c , are AM_a , BM_b , CM_c , meeting in the centroid O .

Now, by the law of cosines, from the triangle BM_bC ,

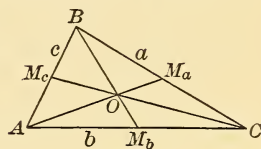


FIG. 42.

$$\begin{aligned} BM_b^2 &= a^2 + M_bC^2 - 2 a \cdot M_bC \cdot \cos C \\ &= a^2 + \frac{b^2}{4} - ab \cos C. \end{aligned}$$

$$\text{But, } \cos C = \frac{a^2 + b^2 - c^2}{2ab},$$

$$\therefore BM_b^2 = a^2 + \frac{b^2}{4} - \frac{a^2 + b^2 - c^2}{2} = \frac{2a^2 + 2c^2 - b^2}{4},$$

$$\text{whence, } BM_b = \frac{1}{2} \sqrt{2a^2 + 2c^2 - b^2} = \frac{1}{2} \sqrt{a^2 + c^2 + 2ac \cos B}$$

$$\text{since } \frac{a^2 + c^2 - b^2}{2ac} = \cos B.$$

In like manner,

$$CM_c = \frac{1}{2} \sqrt{2b^2 + 2a^2 - c^2} = \frac{1}{2} \sqrt{b^2 + a^2 + 2ba \cos C},$$

$$\text{and } AM_a = \frac{1}{2} \sqrt{2c^2 + 2b^2 - a^2} = \frac{1}{2} \sqrt{c^2 + b^2 + 2cb \cos A}.$$

EXAMPLES.

1. In the triangle, $a = 25$, $b = 35$, $c = 45$, find R , r , r_a .
2. Given $a = 0.354$, $b = 0.548$, $C = 28^\circ 34' 20''$, find the distances to C and B , from the circumcentre, the incentre, the centroid, and the orthocentre.
3. In the ambiguous triangle show that the circumcircles of the two triangles, when there are two solutions, are equal.
4. Prove that $\frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c} = \frac{1}{r}$.
5. In any triangle prove $\Delta = \sqrt{r r_a r_b r_c}$.
6. Prove that the product of the distances of the incentre from the vertices of the triangle is $4 r^2 R$.
7. Prove that the area of all triangles of given perimeter that can be circumscribed about a given circle is constant.
8. Prove that the area of the triangle ABC is $Rr(\sin A + \sin B + \sin C)$.

CHAPTER IX.

SERIES—DE MOIVRE'S THEOREM—HYPERBOLIC FUNCTIONS.

77. First consider some series by means of which logarithms of numbers and the natural functions of angles may be computed. For this purpose the following series is important :

$$e = 1 + \frac{1}{\underline{1}} + \frac{1}{\underline{2}} + \frac{1}{\underline{3}} + \frac{1}{\underline{4}} + \dots.$$

It may be derived as follows :

By the binomial theorem,

$$\begin{aligned} & \left(1 + \frac{1}{n}\right)^{nx} \\ &= 1 + nx \cdot \frac{1}{n} + \frac{nx(nx-1)}{\underline{2}} \cdot \frac{1}{n^2} + \frac{nx(nx-1)(nx-2)}{\underline{3}} \cdot \frac{1}{n^3} + \dots \\ &= 1 + x + \frac{x\left(x - \frac{1}{n}\right)}{\underline{2}} + \frac{x\left(x - \frac{1}{n}\right)\left(x - \frac{2}{n}\right)}{\underline{3}} + \dots, \end{aligned}$$

and if n increase without limit,

$$= 1 + x + \frac{x^2}{\underline{2}} + \frac{x^3}{\underline{3}} + \dots + \frac{x^r}{\underline{r}} + \dots$$

This is called the *exponential series*, and is represented by e^x , so that

$$e^x = 1 + x + \frac{x^2}{\underline{2}} + \frac{x^3}{\underline{3}} + \dots + \frac{x^r}{\underline{r}} + \dots.$$

It is shown in higher algebra that this equation holds for all values of x ; whence, if $x = 1$,

$$e = 1 + 1 + \frac{1}{\underline{2}} + \frac{1}{\underline{3}} + \dots + \frac{1}{\underline{r}} + \dots.$$

This value of e is taken as the base of the natural or Napierian system of logarithms.

This value e , however, is not the base of the system of logarithms computed by Napier, but its reciprocal instead. The natural logarithm is used in the theoretical treatment of logarithms, and, as will presently appear, it is customary to compute the common logarithm by first finding the natural, and then multiplying it by a constant multiplier called the modulus, Art. 82; *i.e.* in the Napierian system the modulus is taken as 1, and the base is computed. In the common system the base 10 is chosen and the modulus computed.

78. From the exponential series the value of e may be computed to any required degree of accuracy.

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$$

$$1 + 1 + \frac{1}{2} = 2.5$$

$$\frac{1}{3} = 0.1666666666$$

$$\frac{1}{4} = 0.0416666666$$

$$\frac{1}{5} = 0.0083333333$$

$$\frac{1}{6} = 0.0013888888$$

$$\frac{1}{7} = 0.0001984126$$

$$\frac{1}{8} = 0.0000248015$$

$$\frac{1}{9} = 0.0000027557$$

$$\frac{1}{10} = 0.0000002755$$

.

Adding, $e = 2.7182818$, correct to 7 decimal places.

79. To expand a^x in ascending powers of x .

$$e^z = 1 + z + \frac{z^2}{2} + \frac{z^3}{3} + \dots + \frac{z^r}{r} + \dots$$

Let $a^x = e^z$, then $z = \log_e a^x = x \cdot \log_e a$. (Arts. 35, 40)

Substituting

$$a^x = 1 + x \cdot \log_e a + \frac{x^2 (\log_e a)^2}{2} + \frac{x^3 (\log_e a)^3}{3} + \dots$$

Now put $1 + a$ for a , and

$$(1 + a)^x = 1 + x \cdot \log_e (1 + a) + \frac{x^2 [\log_e (1 + a)]^2}{2} + \frac{x^3 [\log_e (1 + a)]^3}{3} + \dots$$

But by the binomial theorem,

$$(1 + a)^x = 1 + xa + \frac{x(x-1)}{2} a^2 + \frac{x(x-1)(x-2)}{3} a^3 + \dots$$

Equating coefficients of x in the second members of the above equations,

$$\log_e (1 + a) = a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \dots;$$

or writing x for a ,

$$\log_e (1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

In this form the series is of little practical use, since it converges very slowly, and only when x is between $+1$ and -1 (higher algebra).

Put $-x$ for x , and

$$\log_e (1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots;$$

$$\therefore \log_e (1 + x) - \log_e (1 - x)$$

$$= \log_e \frac{1+x}{1-x} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots \right).$$

Finally, put $\frac{1}{2n+1}$ for x , and

$$\log_e \frac{n+1}{n} = \log_e (n+1) - \log_e n$$

$$= 2 \left\{ \frac{1}{2n+1} + \frac{1}{3} \left(\frac{1}{2n+1} \right)^3 + \frac{1}{5} \left(\frac{1}{2n+1} \right)^5 + \dots \right\},$$

$$\therefore \log_e (n+1) = \log_e n + 2 \left\{ \frac{1}{2n+1} + \frac{1}{3} \left(\frac{1}{2n+1} \right)^3 + \frac{1}{5} \left(\frac{1}{2n+1} \right)^5 + \dots \right\},$$

a series which is rapidly convergent.

80. From this series a table of logarithms to the base e may be computed.

To find $\log_e 2$ put $n = 1$. Then, since $\log_e 1 = 0$, the series becomes

$$\begin{aligned} \log_e 2 = \log_e 1 + 2 \left\{ \frac{1}{3} + \frac{1}{3 \cdot 3^3} + \frac{1}{5 \cdot 3^5} + \frac{1}{7 \cdot 3^7} + \frac{1}{9 \cdot 3^9} \right. \\ \left. + \frac{1}{11 \cdot 3^{11}} + \frac{1}{13 \cdot 3^{13}} + \dots \right\} = 0.693147. \end{aligned}$$

The computations may be arranged thus:

3	2.00000000	
9	.66666667	= .66666667
9	.07407407	÷ 3 = .02469136
9	.00823045	÷ 5 = .00164609
9	.00091449	÷ 7 = .00013064
9	.00010161	÷ 9 = .00001129
9	.00001129	÷ 11 = .00000103
9	.00000125	÷ 13 = .00000009
		.69314717

whence $\log_e 2 = 0.693147$, correct to 6 decimal places.

To find $\log_e 3$, put $n = 2$, and

$$\log_e 3 = \log_e 2 + 2 \left(\frac{1}{5} + \frac{1}{3 \cdot 5^3} + \frac{1}{5 \cdot 5^5} + \frac{1}{7 \cdot 5^7} + \frac{1}{9 \cdot 5^9} + \dots \right).$$

$$\begin{array}{r|l}
 5 & 2.00000000 \\
 25 & .40000000 = .40000000 \\
 25 & .01600000 \div 3 = .00533333 \\
 25 & .00640000 \div 5 = .00012800 \\
 25 & .00025600 \div 7 = .00000366 \\
 & .00000102 \div 9 = .00000011 \\
 & \hline
 & .40546510 \\
 & \log_e 2 = .69314717 \\
 & \hline
 \therefore \log_e 3 = 1.098612,
 \end{array}$$

correct to 6 decimal places.

$\log_e 4 = 2 \times \log_e 2$, $\log_e 6 = \log_e 3 + \log_e 2$, etc. (Why?)
 The logarithms of prime numbers may be computed as above by giving proper values to n .

81. Having computed the logarithms of numbers to base e , the logarithms to any other base may be computed by means of the following relation :

Let $\log_a n = x$; then $a^x = n$.

Also, $\log_b n = y$; then $b^y = n$,

$$\therefore a^x = b^y.$$

Hence, $\log_a (a^x) = \log_a (b^y)$,

and $\therefore x = y \log_a b$.

It follows that $\log_a n = \log_b n \cdot \log_a b$;

whence $\log_b n = \log_a n \cdot \frac{1}{\log_a b}$.

This factor $\frac{1}{\log_a b}$ is called the *modulus* of the system of logarithms to base b . Using it as a multiplier, logarithms of numbers to base b are computed at once from the logarithms of the same numbers to any other base a .

82. To compute the common logarithms.

Common logarithms are computed from the Napierian by use of the modulus $\frac{1}{\log_e 10}$; *i.e.*

$$\log_{10} n = \log_e n \cdot \frac{1}{\log_e 10}.$$

By Art. 80, $\log_e 10$ can be found, and

$$\frac{1}{\log_e 10} = .434294, \text{ the modulus of the common system.}$$

Ex. Compute the common logarithms of:

$$2, 3, 4, 6, 5, 10, 15, 216, 3375.$$

COMPLEX NUMBERS.

83. In algebra it is shown that the general expression for complex numbers is $a + bi$, where a represents all the real terms of the expression, b the coefficients of all the imaginary terms, and i is so defined that $i^2 = -1$; whence

$$i = \sqrt{-1}, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1, \text{ etc.}$$

The laws of operation in algebra are found to apply to complex numbers. Moreover, it is further shown that if two complex numbers are equal, the real terms are equal, and the imaginary terms are equal; *i.e.* if

$$a + bi = c + di,$$

then

$$a = b \text{ and } c = d.$$

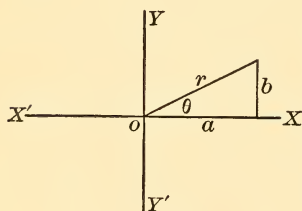


FIG. 43.

Finally, the complex number may be graphically represented as follows:

The real number is measured along OX , a units; the imaginary parallel to OY , b units. The line r is a graphic representation of $a + bi$.

Since $a = r \cos \theta$ and $b = r \sin \theta$,
 $\therefore a + bi = r(\cos \theta + i \sin \theta)$.

The properties of complex numbers are best developed by using this trigonometric form. If r be taken as unity, then $\cos \theta + i \sin \theta$ represents any complex number.

84. De Moivre's Theorem. To prove that, for any value of n ,

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta.$$

I. When n is a positive integer.

By multiplication,

$$\begin{aligned} (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta) \\ = \cos \alpha \cos \beta - \sin \alpha \sin \beta + i(\sin \alpha \cos \beta + \cos \alpha \sin \beta) \\ = \cos(\alpha + \beta) + i \sin(\alpha + \beta). \end{aligned}$$

In like manner,

$$\begin{aligned} (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta)(\cos \gamma + i \sin \gamma) \\ = \cos(\alpha + \beta + \gamma) + i \sin(\alpha + \beta + \gamma); \end{aligned}$$

and finally,

$$\begin{aligned} (\cos \alpha + i \sin \alpha)(\cos \beta + i \sin \beta)(\cos \gamma + i \sin \gamma) \cdots \text{to } n \text{ factors} \\ = \cos(\alpha + \beta + \gamma + \cdots) + i \sin(\alpha + \beta + \gamma + \cdots). \end{aligned}$$

Now let $\alpha = \beta = \gamma = \cdots$, and the above becomes

$$(\cos \alpha + i \sin \alpha)^n = \cos n\alpha + i \sin n\alpha.$$

II. When n is a negative integer.

Let $n = -m$; then

$$\begin{aligned} (\cos \alpha + i \sin \alpha)^n &= (\cos \alpha + i \sin \alpha)^{-m} \\ &= \frac{1}{(\cos \alpha + i \sin \alpha)^m} = \frac{1}{\cos m\alpha + i \sin m\alpha} \\ &= \frac{\cos m\alpha - i \sin m\alpha}{(\cos m\alpha + i \sin m\alpha)(\cos m\alpha - i \sin m\alpha)} \\ &= \frac{\cos m\alpha - i \sin m\alpha}{\cos^2 m\alpha + \sin^2 m\alpha} \\ &= \cos m\alpha - i \sin m\alpha = \cos(-m)\alpha + i \sin(-m)\alpha. \end{aligned}$$

Substituting n for $-m$, the equation becomes

$$(\cos \alpha + i \sin \alpha)^n = \cos n\alpha + i \sin n\alpha.$$

III. When n is a fraction, positive or negative.

Let $n = \frac{p}{q}$, p and q being any integers.

Now

$$\left(\cos \frac{\alpha}{q} + i \sin \frac{\alpha}{q}\right)^q = \cos q \cdot \frac{\alpha}{q} + i \sin q \cdot \frac{\alpha}{q} = \cos \alpha + i \sin \alpha \quad (\text{by I}).$$

$$\text{Then} \quad \left(\cos \frac{\alpha}{q} + i \sin \frac{\alpha}{q}\right) = (\cos \alpha + i \sin \alpha)^{\frac{1}{q}}.$$

Raising each member to the power p ,

$$(\cos \alpha + i \sin \alpha)^{\frac{p}{q}} = \left(\cos \frac{\alpha}{q} + i \sin \frac{\alpha}{q}\right)^p = \cos \frac{p}{q} \alpha + i \sin \frac{p}{q} \alpha.$$

COMPUTATIONS OF NATURAL FUNCTIONS.

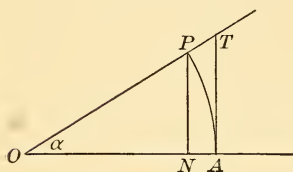


FIG. 44.

85. The radian measure of an acute angle is greater than its sine and less than its tangent, *i.e.*

$$\sin \alpha < \alpha < \tan \alpha.$$

Let α be the circular, or radian, measure of any acute angle AOP .

Then, in the figure,

area of sector $OAP <$ area of triangle OAT ,

$$\text{i.e. } \frac{1}{2} OA \cdot \text{arc } AP < \frac{1}{2} OA \cdot AT.$$

$$\therefore \text{arc } AP < AT.$$

Now, since

$$NP < \text{arc } AP,$$

$$\frac{NP}{OP} < \frac{\text{arc } AP}{OP} < \frac{AT}{OP}.$$

But $\frac{\text{arc } AP}{OP} = \text{circular measure of } AOP = \alpha;$

whence

$$\sin \alpha < \alpha < \tan \alpha.$$

86. Since $\sin \alpha < \alpha < \tan \alpha$,

$$1 < \frac{\alpha}{\sin \alpha} < \frac{1}{\cos \alpha}.$$

Hence, however small α may be, $\frac{\alpha}{\sin \alpha}$ lies between 1 and $\frac{1}{\cos \alpha}$. When α approaches 0, $\cos \alpha$ approaches unity. Therefore, by diminishing α sufficiently, we may make $\frac{\alpha}{\sin \alpha}$ differ from unity by an amount less than any assignable quantity. This we express by saying that when α approaches 0, $\frac{\alpha}{\sin \alpha}$ approaches unity as a limit, *i.e.* $\frac{\alpha}{\sin \alpha} = 1$, approximately. Multiplying by $\cos \alpha (= 1$, nearly), we have $\frac{\alpha}{\tan \alpha} = 1$, approximately. Whence, if α approaches 0, $\tan \alpha = \sin \alpha = \alpha$, approximately.

87. Sine and cosine series.

$\cos n\alpha + i \sin n\alpha = (\cos \alpha + i \sin \alpha)^n$, (De Moivre's Theorem).

Expanding the second member by the binomial formula, it becomes,

$$\begin{aligned} \cos^n \alpha + n \cos^{n-1} \alpha \cdot i \sin \alpha + \frac{n(n-1)}{2} \cos^{n-2} \alpha \cdot i^2 \sin^2 \alpha \\ + \frac{n(n-1)(n-2)}{3} \cos^{n-3} \alpha \cdot i^3 \sin^3 \alpha \\ + \frac{n(n-1)(n-2)(n-3)}{4} \cos^{n-4} \alpha \cdot i^4 \sin^4 \alpha + \dots \end{aligned}$$

Substituting the values of i^2 , i^3 , i^4 , etc., we have

$$\begin{aligned} \cos n\alpha + i \sin n\alpha = \cos^n \alpha - \frac{n(n-1)}{2} \cos^{n-2} \alpha \sin^2 \alpha \\ + \frac{n(n-1)(n-2)(n-3)}{4} \cos^{n-4} \alpha \sin^4 \alpha - \dots \\ + i \left(n \cos^{n-1} \alpha \sin \alpha - \frac{n(n-1)(n-2)}{3} \cos^{n-3} \alpha \sin^3 \alpha + \dots \right). \end{aligned}$$

Equating the real and imaginary parts in the two members,

$$\begin{aligned} \cos n\alpha &= \cos^n \alpha - \frac{n(n-1)}{2} \cos^{n-2} \alpha \sin^2 \alpha \\ &\quad + \frac{n(n-1)(n-2)(n-3)}{4} \cos^{n-4} \alpha \sin^4 \alpha - \dots, \end{aligned}$$

$$\text{and } \sin n\alpha = n \cos^{n-1} \alpha - \frac{n(n-1)(n-2)}{3} \cos^{n-3} \alpha \sin^3 \alpha + \dots.$$

Ex. 1. Find $\cos 3\alpha$; $\sin 3\alpha$.

$$\begin{aligned} \text{In the above put } n=3, \text{ and } \cos 3\alpha &= \cos^3 \alpha - 3 \cos \alpha \sin^2 \alpha \\ &= 4 \cos^3 \alpha - 3 \cos \alpha; \end{aligned}$$

$$\begin{aligned} \text{also } \sin^3 \alpha &= 3 \cos^2 \alpha \sin \alpha - \sin^3 \alpha \\ &= 3 \sin \alpha - 4 \sin^3 \alpha. \end{aligned}$$

2. Find $\sin 4\alpha$; $\cos 4\alpha$; $\sin 5\alpha$; $\cos 5\alpha$.

It will be noticed that in the series for $\cos n\alpha$ and $\sin n\alpha$ the terms are alternately positive and negative, and that the series continues till there is a zero factor in the numerator.

88. If now in the above series we let $n\alpha = \theta$, then

$$\begin{aligned} \cos \theta &= \cos^n \alpha - \frac{\frac{\theta}{\alpha} \left(\frac{\theta}{\alpha} - 1 \right)}{2} \cos^{n-2} \alpha \sin^2 \alpha \\ &\quad - \frac{\frac{\theta}{\alpha} \left(\frac{\theta}{\alpha} - 1 \right) \left(\frac{\theta}{\alpha} - 2 \right) \left(\frac{\theta}{\alpha} - 3 \right)}{4} \cos^{n-4} \alpha \sin^4 \alpha - \dots \\ &= \cos^n \alpha - \frac{\theta(\theta - \alpha)}{2} \cos^{n-2} \alpha \left(\frac{\sin \alpha}{\alpha} \right)^2 \\ &\quad + \frac{\theta(\theta - \alpha)(\theta - 2\alpha)(\theta - 3\alpha)}{4} \cos^{n-4} \alpha \left(\frac{\sin \alpha}{\alpha} \right)^4 - \dots. \end{aligned}$$

If now θ remain constant, and α decrease without limit, then will n become indefinitely great, and $\frac{\sin \alpha}{\alpha}$ and every

power thereof, and $\cos \alpha$ and every power of $\cos \alpha$ will approach unity as a limit, so that

$$\cos \theta = 1 - \frac{\theta^2}{2} + \frac{\theta^4}{24} - \frac{\theta^6}{720} + \dots$$

Similarly,
$$\sin \theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \frac{\theta^7}{5040} + \dots$$

By algebra it is shown that these series are convergent for all values of θ . By their use we can compute values of $\sin \theta$ and $\cos \theta$ to any required degree of accuracy.

Show from the above that $\tan \theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \dots$

Ex. 1. Compute the value of $\sin 1^\circ$, correct to 5 places.

In
$$\sin \theta = \theta - \frac{\theta^3}{6} + \frac{\theta^5}{120} - \frac{\theta^7}{5040} + \dots, \text{ make } \theta \text{ the radian}$$

measure of
$$1^\circ = \frac{\pi}{180} = 0.01745 +.$$

Then,
$$\theta = 0.01745 +$$

$$\frac{\theta^3}{6} = 0.0000008.$$

$$\therefore \sin \theta = 0.01745 +.$$

The terms of the series after the first do not affect the fifth place, so that the value is given by the first term, an illustration of the fact that, if α is small, $\sin \alpha = \alpha$, approximately. Compare the value of $\tan 1^\circ$.

2. Show that $\sin 10^\circ = 0.17365$; $\cos 10^\circ = 0.98481$; $\sin 15^\circ = 0.25982$; $\cos 60^\circ = 0.50000$.

3. Find the sine and cosine of $18^\circ 30'$; $22^\circ 15'$; $67^\circ 45'$.

It is unnecessary to compute the functions beyond 30° , for since

$$\sin (30^\circ + \theta) + \sin (30^\circ - \theta) = \cos \theta \text{ (why?)},$$

$$\therefore \sin (30^\circ + \theta) = \cos \theta - \sin (30^\circ - \theta).$$

So, also,
$$\cos (30^\circ + \theta) = \cos (30^\circ - \theta) - \sin \theta.$$

Giving θ proper values the functions of any angle from 30° to 45° are determined at once from the functions of angles less than 30° .

Thus,
$$\sin 31^\circ = \cos 1^\circ - \sin 29^\circ;$$

$$\cos 31^\circ = \cos 29^\circ - \sin 1^\circ.$$

4. Find sine and cosine of 40° ; of 50° .

89. The following are sometimes useful in applied mathematics :

Ex. 1. To find the sum of a series of sines of angles in A. P., such as

$$\sin \alpha + \sin (\alpha + \beta) + \sin (\alpha + 2\beta) + \dots + \sin (\alpha + [n-1]\beta).$$

$$2 \sin \alpha \sin \frac{\beta}{2} = \cos \left(\alpha - \frac{\beta}{2} \right) - \cos \left(\alpha + \frac{\beta}{2} \right),$$

$$2 \sin (\alpha + \beta) \sin \frac{\beta}{2} = \cos \left(\alpha + \frac{\beta}{2} \right) - \cos \left(\alpha + \frac{3\beta}{2} \right),$$

$$2 \sin (\alpha + 2\beta) \sin \frac{\beta}{2} = \cos \left(\alpha + \frac{3\beta}{2} \right) - \cos \left(\alpha + \frac{5\beta}{2} \right),$$

$$\dots \dots \dots$$

$$2 \sin (\alpha + [n-1]\beta) \sin \frac{\beta}{2} = \cos \left(\alpha + \frac{2n-3}{2}\beta \right) - \cos \left(\alpha + \frac{2n-1}{2}\beta \right).$$

Adding

$$\begin{aligned} 2 \{ \sin \alpha + \sin (\alpha + \beta) + \sin (\alpha + 2\beta) + \dots + \sin (\alpha + [n-1]\beta) \} \sin \frac{\beta}{2} \\ = \cos \left(\alpha - \frac{\beta}{2} \right) - \cos \left(\alpha + \frac{2n-1}{2}\beta \right) \\ = 2 \sin \left(\alpha + \frac{n-1}{2}\beta \right) \sin \frac{n}{2}\beta. \end{aligned}$$

$$\therefore \sin \alpha + \sin (\alpha + \beta) + \sin (\alpha + 2\beta) + \dots + \sin (\alpha + [n-1]\beta)$$

$$= \frac{\sin \left(\alpha + \frac{n-1}{2}\beta \right) \sin \frac{n}{2}\beta}{\sin \frac{\beta}{2}}.$$

Similarly it can be shown that

$$\cos \alpha + \cos (\alpha + \beta) + \cos (\alpha + 2\beta) + \dots + \cos (\alpha + [n-1]\beta)$$

$$= \frac{\cos \left(\alpha + \frac{n-1}{2}\beta \right) \sin \frac{n}{2}\beta}{\sin \frac{\beta}{2}}.$$

90. The series $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^r}{r} + \dots$ is proved in higher algebra to be true for all values of x , real or imaginary. Then if $x = i\theta$,

$$\begin{aligned} e^{i\theta} &= 1 + i\theta + \frac{i^2\theta^2}{2} + \frac{i^3\theta^3}{3} + \dots + \frac{i^r\theta^r}{r} + \dots \\ &= 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} - \frac{\theta^6}{6} + \dots + i\left(\theta - \frac{\theta^3}{3} + \frac{\theta^5}{5} - \frac{\theta^7}{7} + \dots\right). \end{aligned}$$

$$\therefore e^{i\theta} = \cos \theta + i \sin \theta \text{ (Art. 87).}$$

In like manner, $e^{-i\theta} = \cos \theta - i \sin \theta.$

Adding, $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2};$

subtracting, $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}.$

HYPERBOLIC FUNCTIONS.

91. Since $\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$, and $\cos \theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$ are true for all values of θ , let $\theta = i\theta$.

Then, $\sin (i\theta) = \frac{e^{-\theta} - e^{\theta}}{2i} = i \frac{e^{\theta} - e^{-\theta}}{2} = i \sinh \theta,$

and $\cos (i\theta) = \frac{e^{\theta} + e^{-\theta}}{2} = \cosh \theta,$

so that $\tan (i\theta) = \frac{\sin (i\theta)}{\cos (i\theta)} = \frac{i \sinh \theta}{\cosh \theta} = i \tanh \theta,$

where $\sinh \theta$, $\cosh \theta$, $\tanh \theta$, are called the *hyperbolic sine*, *cosine*, and *tangent* of θ . The hyperbolic cotangent, secant, and cosecant of θ are obtained from the hyperbolic sine, cosine, and tangent, just as the corresponding circular functions, cotangent, secant, and cosecant, are obtained from tangent, cosine, and sine. The hyperbolic functions have the same geometric relations to the rectangular hyper-

bola that the circular functions have to the circle, hence the name hyperbolic functions.

$$\sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2}, \quad \therefore \operatorname{csch} \theta = \frac{2}{e^{\theta} - e^{-\theta}};$$

$$\cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2}, \quad \therefore \operatorname{sech} \theta = \frac{2}{e^{\theta} + e^{-\theta}};$$

$$\tanh \theta = \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}, \quad \therefore \operatorname{coth} \theta = \frac{e^{\theta} + e^{-\theta}}{e^{\theta} - e^{-\theta}}.$$

92. From the relations of Art. 91 it appears that to any relation between the circular functions there corresponds a relation between the hyperbolic functions.

$$\text{Since} \quad \cos^2 (i\theta) + \sin^2 (i\theta) = 1,$$

$$\cosh^2 \theta + i^2 \sinh^2 \theta = 1,$$

$$\text{or} \quad \cosh^2 \theta - \sinh^2 \theta = 1.$$

This may also be derived thus:

$$\begin{aligned} \cosh^2 \theta - \sinh^2 \theta &= \left(\frac{e^{\theta} + e^{-\theta}}{2} \right)^2 - \left(\frac{e^{\theta} - e^{-\theta}}{2} \right)^2 \\ &= \frac{e^{2\theta} + 2 + e^{-2\theta} - e^{2\theta} + 2 - e^{-2\theta}}{4} = 1. \end{aligned}$$

Also since

$$\sin (i\alpha + i\beta) = \sin (i\alpha) \cos (i\beta) + \cos (i\alpha) \sin (i\beta),$$

$$\therefore i \sinh (\alpha + \beta) = i \sinh \alpha \cosh \beta + \cosh \alpha \cdot i \sinh \beta,$$

$$\text{and} \quad \sinh (\alpha + \beta) = \sinh \alpha \cosh \beta + \cosh \alpha \sinh \beta.$$

Let the student verify this relation from the exponential values of $\sinh$ and $\cosh$.

EXAMPLES.

Prove

$$1. \quad \cosh (\alpha + \beta) = \cosh \alpha \cosh \beta + \sinh \alpha \sinh \beta.$$

$$2. \quad \cosh (\alpha + \beta) - \cosh (\alpha - \beta) = 2 \sinh \alpha \sinh \beta.$$

$$3. \quad \cosh 2\theta = 1 + 2 \sinh^2 \theta = 2 \cosh^2 \theta - 1.$$

$$4. \quad \sinh 2\alpha = 2 \sinh \alpha \cosh \alpha.$$

$$5. \cosh \frac{\theta}{2} = \sqrt{\frac{1 + \cosh \theta}{2}}; \sinh \frac{\theta}{2} = \sqrt{\frac{\cosh \theta - 1}{2}}.$$

$$6. \sinh 3\theta = 3 \sinh \theta + 4 \sinh^3 \theta.$$

$$7. \sinh \theta + \sinh \phi = 2 \sinh \frac{\theta + \phi}{2} \cosh \frac{\theta - \phi}{2}.$$

$$8. \sinh \alpha + \sinh (\alpha + \beta) + \sinh (\alpha + 2\beta) + \cdots + \sinh (\alpha + [n-1]\beta)$$

$$= \frac{\sinh \left(\alpha + \frac{n-1}{2} \beta \right) \sinh \frac{n}{2} \beta}{\sinh \frac{\beta}{2}}$$

$$9. \tanh (\theta + \phi) = \frac{\tanh \theta + \tanh \phi}{1 + \tanh \theta \tanh \phi}.$$

$$10. \sinh^{-1} x = \cosh^{-1} \sqrt{1 + x^2} = \tanh^{-1} \frac{x}{\sqrt{1 + x^2}}.$$

$$11. \cosh (\alpha + \beta) \cosh (\alpha - \beta) = \cosh^2 \alpha + \sinh^2 \beta = \cosh^2 \beta + \sinh^2 \alpha.$$

$$12. 2 \cosh n\alpha \cosh \alpha = \cosh (n+1)\alpha + \cosh (n-1)\alpha.$$

$$13. \cosh \alpha = \frac{1}{2} (e^{\alpha} + e^{-\alpha}) = 1 + \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} + \cdots.$$

$$14. \sinh \alpha = \frac{1}{2} (e^{\alpha} - e^{-\alpha}) = \alpha + \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \cdots.$$

$$15. \tanh^{-1} a + \tanh^{-1} b = \tanh^{-1} \frac{a+b}{1+ab}.$$

SPHERICAL TRIGONOMETRY.

CHAPTER X.

SPHERICAL TRIANGLES.

93. Spherical trigonometry is concerned chiefly with the solution of spherical triangles. Its applications are for the most part in geodesy and astronomy.

The following definitions and theorems of geometry are for convenience of reference stated here.

A *great circle* is a plane section of a sphere passing through the centre. Other plane sections are *small circles*.

The shortest distance between two points on a sphere is measured on the arc of a great circle, less than 180° , which joins them.

A *spherical triangle* is any portion of the surface of a sphere bounded by three arcs of great circles. We shall consider only triangles whose sides are arcs not greater than 180° in length.

The *polar triangle* of any spherical triangle is the triangle whose sides are drawn with the vertices of the first triangle as poles. If ABC is the polar of $A'B'C'$, then $A'B'C'$ is the polar of ABC .

In any spherical triangle,

The sum of two sides $>$ the third side.

The greatest side is opposite the greatest angle, and conversely.

Each angle $< 180^\circ$; the sum of the angles $> 180^\circ$, and $< 540^\circ$.

Each side $< 180^\circ$; the sum of the sides $< 360^\circ$.

The sides of a spherical triangle are the supplements of the angles opposite in the polar triangle, and conversely.

If two angles are equal the sides opposite are equal, and conversely.

The sides of a spherical triangle subtend angles at the centre of the sphere which contain the same number of angle degrees as the arc does of arc degrees; *i.e.* an angle at the centre and its arc have the same measure numerically.

The arc does not measure the angle for they have not the same unit of measurement, but we say they have the same numerical measure; *i.e.* the arc contains the unit arc as many times as the angle contains the unit angle.

The angles of a spherical triangle are said to be measured by the plane angle included by tangents to the sides of the angle at their intersection. They have therefore the same numerical measure as the dihedral angle between the planes of the arcs.

In the figure the following have the same numerical measure :

arc a and angle α ;

arc b and angle β ;

arc c and angle γ ;

plane angle $A'BC'$;

spherical angle B and dihedral angle $A-BO-C$;

spherical angle C and dihedral angle $B-C'O-A$;

spherical angle A and dihedral angle $C-AO-B$.

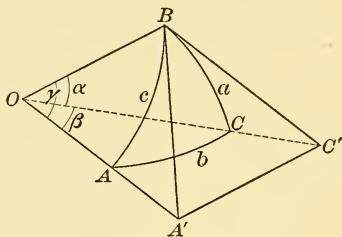


FIG. 45.

$A'C'B$ and $C'A'B$ have not the same measure as spherical angles C and A , for BA' , $A'C'$, $C'B$ are not perpendicular to OA or OC .

94. In plane trigonometry the trigonometric functions were treated as functions of the angles. But since an angle and its subtending arc vary together and have the same

numerical measure, it is clear that the trigonometric ratios are functions of the arcs, and may be so considered. All the relations between the functions are the same whether we consider them with reference to the angle or the arc, so that all the identities of plane trigonometry are true for the functions of the arcs.

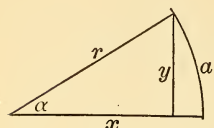


FIG. 46.

Thus in the figure we may write,

$$\sin \alpha = \frac{y}{r} \text{ or } \sin a = \frac{y}{r};$$

$$\sin^2 \alpha + \cos^2 \alpha = 1, \text{ or } \sin^2 a + \cos^2 a = 1;$$

$$\cos 2\alpha = 2\cos^2 \alpha - 1, \text{ or } \cos 2a = 2\cos^2 a - 1.$$

GENERAL FORMULÆ FOR SPHERICAL TRIANGLES.

95. The solutions of spherical triangles may be effected by formulæ now to be developed:

First it will be shown that in any spherical triangle

$$\cos a = \cos b \cos c + \sin b \sin c \cos A,$$

$$\cos b = \cos c \cos a + \sin c \sin a \cos B,$$

$$\cos c = \cos a \cos b + \sin a \sin b \cos C.$$

The following cases must be considered :

I. Both b and $c < 90^\circ$. III. Both b and $c > 90^\circ$.

II. $b > 90^\circ$, $c < 90^\circ$. IV. Either b or $c = 90^\circ$.

V. $b = c = 90^\circ$.

The figure applies to Case I.

Let ABC be a spherical triangle, a, b, c its sides, and O the centre of the sphere.

Draw AC' and AB' tangent to the sides b, c at A . (The same result would be obtained by drawing AB', AC' perpendicular to OA at any point to

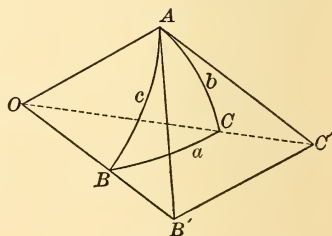


FIG. 47.

meet OB, OC .) Since these tangents lie in the planes of the circles to which they are drawn, they will meet OC and OB in C' and B' , and the angle $C'AB'$ will be the measure of the angle A of the spherical triangle ABC . Since OAB', OAC' are right angles, AOB', AOC' must be acute, and hence sides c, b are each $< 90^\circ$.

In the triangles $C'AB'$ and $C'OB'$,

$$C'B'^2 = AC'^2 + AB'^2 - 2 AC' \cdot AB' \cos C'AB',$$

and $B'C'^2 = OC'^2 + OB'^2 - 2 OC' \cdot OB' \cos C'OB'.$

Subtracting and noting that

$$\cos C'AB' = \cos A \text{ and } \cos C'OB' = \cos a,$$

we have

$$\begin{aligned} 0 &= OC'^2 - AC'^2 + OB'^2 - AB'^2 \\ &\quad + 2 AC' \cdot AB' \cos A - 2 OC' \cdot OB' \cos a. \end{aligned}$$

But $OC'^2 - AC'^2 = OA^2$ and $OB'^2 - AB'^2 = OA^2.$

Hence, $0 = OA^2 + AC' \cdot AB' \cos A - OC' \cdot OB' \cos a;$

or $\cos a = \frac{OA}{OC'} \cdot \frac{OA}{OB'} + \frac{AC'}{OC'} \cdot \frac{AB'}{OB'} \cos A.$

$$\therefore \cos a = \cos b \cos c + \sin b \sin c \cos A.$$

Similarly,

$$\cos b = \cos a \cos c + \sin a \sin c \cos B,$$

and $\cos c = \cos a \cos b + \sin a \sin b \cos C.$

These formulæ are important, and should be carefully memorized.

II. $b > 90^\circ; c < 90^\circ.$

In the triangle ABC , let $b > 90^\circ$ and $c < 90^\circ$. Complete the lune $BACA'$. Then in the triangle $A'CB$ the sides a and $A'C$ are both less than 90° , and by (I)

$$\cos A'B = \cos A'C \cos a + \sin A'C \sin a \cos A'CB.$$

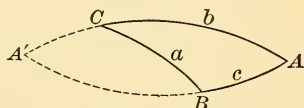


FIG. 48.

But $A'B = 180^\circ - c$, $A'C = 180^\circ - b$, and $A'CB = 180^\circ - C$.

$$\therefore \cos(180^\circ - c) = \cos(180^\circ - b) \cos a \\ + \sin(180^\circ - b) \sin a \cos(180^\circ - C);$$

or $-\cos c = (-\cos b) \cos a + \sin b \sin a (-\cos C),$

and $\cos c = \cos a \cos b + \sin a \sin b \cos C.$

A similar proof will apply in case $c > 90^\circ$, $b < 90^\circ$.

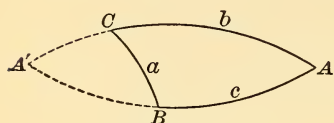


FIG. 49.

III. Both b and $c > 90^\circ$.

In the triangle ABC , let both b and $c > 90^\circ$. Complete the lune $ABA'C$. Then since $A'C$ and $A'B$ are both $< 90^\circ$,

$$\cos a = \cos A'C \cos A'B + \sin A'C \sin A'B \cos A'.$$

But $A' = A$, $A'C = 180^\circ - b$, $A'B = 180^\circ - c$.

$$\therefore \cos a = \cos(180^\circ - b) \cos(180^\circ - c) \\ + \sin(180^\circ - b) \sin(180^\circ - c) \cos A;$$

or $\cos a = \cos b \cos c + \sin b \sin c \cos A.$

Cases IV and V are left to the student as exercises.

96. Since the angles of the polar triangle are the supplements of the sides opposite in the first triangle, we have

$$a' = 180^\circ - A, \quad b' = 180^\circ - B,$$

$$c' = 180^\circ - C, \quad A' = 180^\circ - a.$$

Substituting in

$$\cos a' = \cos b' \cos c' \\ + \sin b' \sin c' \cos A',$$

we have

$$-\cos(180^\circ - A) = \cos(180^\circ - B) \cos(180^\circ - C) \\ + \sin(180^\circ - B) \sin(180^\circ - C) \cos(180^\circ - a);$$

or $-\cos A = (-\cos B)(-\cos C) + \sin B \sin C (-\cos a).$

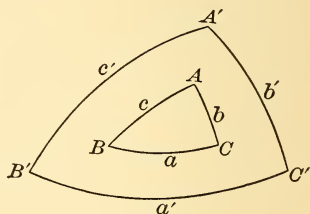


FIG. 50.

Changing signs,

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

Similarly, $\cos B = -\cos A \cos C + \sin A \sin C \cos b,$

and $\cos C = -\cos A \cos B + \sin A \sin B \cos c.$

97. In any spherical triangle to prove $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$

Since $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c};$

$$\begin{aligned} \therefore \sin^2 A &= 1 - \left(\frac{\cos a - \cos b \cos c}{\sin b \sin c} \right)^2 \\ &= \frac{\sin^2 b \sin^2 c - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} \\ &= \frac{(1 - \cos^2 b)(1 - \cos^2 c) - (\cos a - \cos b \cos c)^2}{\sin^2 b \sin^2 c} \\ &= \frac{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}{\sin^2 b \sin^2 c}. \end{aligned}$$

Hence,

$$\sin A = \frac{\sqrt{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}}{\sin b \sin c}$$

and $\frac{\sin A}{\sin a} = \frac{\sqrt{1 - \cos^2 a - \cos^2 b - \cos^2 c + 2 \cos a \cos b \cos c}}{\sin a \sin b \sin c}.$

By a similar process, $\frac{\sin B}{\sin b}$ and $\frac{\sin C}{\sin c}$ will be found equal to the same expression.

$$\therefore \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

98. Expressions for sine, cosine, and tangent of half an angle in terms of functions of the sides.

$$\begin{aligned}
 \text{We have } 2 \sin^2 \frac{A}{2} &= 1 - \cos A \\
 &= 1 - \frac{\cos a - \cos b \cos c}{\sin b \sin c} \\
 &= \frac{\cos b \cos c + \sin b \sin c - \cos a}{\sin b \sin c} \\
 &= \frac{\cos(b-c) - \cos a}{\sin b \sin c}.
 \end{aligned}$$

$$\begin{aligned}
 \text{Then } 2 \sin^2 \frac{A}{2} &= \frac{2 \sin \frac{1}{2}(a+b-c) \sin \frac{1}{2}(a-b+c)}{\sin b \sin c} \quad (\text{Art. 51}) \\
 &= \frac{2 \sin(s-b) \sin(s-c)}{\sin b \sin c},
 \end{aligned}$$

when $2s = a + b + c$.

$$\therefore \sin \frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \sin c}}.$$

$$\text{Similarly, } \sin \frac{B}{2} = \sqrt{\frac{\sin(s-c) \sin(s-a)}{\sin a \sin c}},$$

$$\text{and } \sin \frac{C}{2} = \sqrt{\frac{\sin(s-b) \sin(s-a)}{\sin a \sin b}}.$$

Also from the relation

$$\begin{aligned}
 2 \cos^2 \frac{A}{2} &= 1 + \cos A \\
 &= 1 + \frac{\cos a - \cos b \cos c}{\sin b \sin c},
 \end{aligned}$$

$$\text{we have } \cos \frac{A}{2} = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}.$$

$$\text{Also, } \cos \frac{B}{2} = \sqrt{\frac{\sin s \sin(s-b)}{\sin c \sin a}},$$

$$\text{and } \cos \frac{C}{2} = \sqrt{\frac{\sin s \sin(s-c)}{\sin a \sin b}}.$$

From the above,

$$\tan \frac{A}{2} = \frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin s \sin(s-a)}}.$$

Also,

$$\tan \frac{B}{2} = \sqrt{\frac{\sin(s-a) \sin(s-c)}{\sin s \sin(s-b)}},$$

and

$$\tan \frac{C}{2} = \sqrt{\frac{\sin(s-a) \sin(s-b)}{\sin s \sin(s-c)}}.$$

Compare the formulæ thus far derived with the corresponding formulæ for solving plane triangles. The similarity in forms will assist in memorizing the formulæ for solving spherical triangles.

99. From the formulæ of Art. 96, the student can easily prove the following relations :

$$\sin \frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\sin B \sin C}},$$

where

$$2S = A + B + C.$$

$$\sin \frac{b}{2} = ?,$$

$$\sin \frac{c}{2} = ?.$$

$$\cos \frac{a}{2} = \sqrt{\frac{\cos(S-B) \cos(S-C)}{\sin B \sin C}},$$

$$\cos \frac{b}{2} = ?,$$

$$\cos \frac{c}{2} = ?.$$

$$\tan \frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\cos(S-B) \cos(S-C)}},$$

$$\tan \frac{b}{2} = ?,$$

$$\tan \frac{c}{2} = ?.$$

100. Napier's Analogies.

Since

$$\frac{\tan \frac{A}{2}}{\tan \frac{B}{2}} = \frac{\sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin s \sin(s-a)}}}{\sqrt{\frac{\sin(s-c) \sin(s-a)}{\sin s \sin(s-b)}}}$$

$$= \sqrt{\frac{\sin^2(s-b)}{\sin^2(s-a)}} = \frac{\sin(s-b)}{\sin(s-a)};$$

by composition and division,

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2}}{\tan \frac{A}{2} - \tan \frac{B}{2}} = \frac{\sin(s-b) + \sin(s-a)}{\sin(s-b) - \sin(s-a)},$$

$$\frac{\frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} + \frac{\sin \frac{B}{2}}{\cos \frac{B}{2}}}{\frac{\sin \frac{A}{2}}{\cos \frac{A}{2}} - \frac{\sin \frac{B}{2}}{\cos \frac{B}{2}}} = \frac{\sin \frac{1}{2}(2s-a-b) \cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(2s-a-b) \sin \frac{1}{2}(a-b)},$$

(Art. 51)

$$\frac{\sin \frac{1}{2}(A+B)}{\sin \frac{1}{2}(A-B)} = \frac{\tan \frac{1}{2}(2s-a-b)}{\tan \frac{1}{2}(a-b)}$$

$$= \frac{\tan \frac{c}{2}}{\tan \frac{1}{2}(a-b)}, \text{ since } 2s-a-b=c.$$

$$\therefore \tan \frac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \frac{c}{2}.$$

To find an expression for $\tan \frac{1}{2}(A-B)$ we have only to consider the polar triangle, and by substituting $180^\circ - A$ for a , etc., $180^\circ - a$ for A , etc., we have the following relations:

$$\frac{1}{2}(a-b) = \frac{1}{2}(180^\circ - A - 180^\circ + B) = -\frac{1}{2}(A-B);$$

also, $\frac{1}{2}(A - B) = -\frac{1}{2}(a - b)$;

$$\frac{1}{2}(A + B) = \frac{1}{2}(180^\circ - a + 180^\circ - b) = 180^\circ - \frac{1}{2}(a + b);$$

and
$$\frac{c}{2} = 90^\circ - \frac{C}{2}.$$

The formula then becomes, applying Art. 29,

$$\tan \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{1}{2}(a + b)} \cot \frac{C}{2}.$$

Formulæ for $\tan \frac{1}{2}(a + b)$, $\tan \frac{1}{2}(A + B)$ are derived as follows:

Since

$$\tan \frac{A}{2} \cdot \tan \frac{B}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin s \cdot \sin(s-a)}} \cdot \sqrt{\frac{\sin(s-c) \sin(s-a)}{\sin s \cdot \sin(s-b)}},$$

$$\frac{\sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2}} = \frac{\sin(s-c)}{\sin s}.$$

By composition and division,

$$\frac{\cos \frac{A}{2} \cos \frac{B}{2} + \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{A}{2} \cos \frac{B}{2} - \sin \frac{A}{2} \sin \frac{B}{2}} = \frac{\sin s + \sin(s-c)}{\sin s - \sin(s-c)},$$

whence
$$\frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} = \frac{\tan \frac{1}{2}(a + b)}{\tan \frac{c}{2}}, \quad (\text{Art. 51})$$

since
$$2s - c = a + b,$$

or,
$$\tan \frac{1}{2}(a + b) = \frac{\cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B)} \tan \frac{c}{2}.$$

The value of $\tan \frac{1}{2}(A + B)$ is derived by substituting in terms of the corresponding elements of the polar triangle.

$$\frac{\cos \frac{1}{2}(a - b)}{-\cos \frac{1}{2}(a + b)} = \frac{-\tan \frac{1}{2}(A + B)}{\cot \frac{C}{2}},$$

$$\therefore \tan \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{1}{2}(a + b)} \cot \frac{C}{2}.$$

Similar relations among the other elements of the triangle may be derived, or they may be written from the above by proper changes of A, B, C, a, b, c in the formulæ. The student should write them out as exercises.

101. Delambre's Analogies.

$$\text{Since} \quad \sin \frac{1}{2}(A + B) = \sin \frac{A}{2} \cos \frac{B}{2} + \cos \frac{A}{2} \sin \frac{B}{2},$$

then

$$\sin \frac{1}{2}(A + B) = \frac{\sin(s - b) + \sin(s - a)}{\sin c} \cdot \sqrt{\frac{\sin s \cdot \sin(s - c)}{\sin a \cdot \sin b}}.$$

(Art. 98)

$$\begin{aligned} \text{Hence,} \quad \frac{\sin \frac{1}{2}(A + B)}{\cos \frac{C}{2}} &= \frac{\sin(s - b) + \sin(s - a)}{\sin c} \\ &= \frac{2 \sin \frac{c}{2} \cos \frac{1}{2}(a - b)}{2 \sin \frac{c}{2} \cos \frac{c}{2}}, \quad (\text{Art. 51}) \end{aligned}$$

$$\text{and} \quad \sin \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a - b)}{\cos \frac{c}{2}} \cos \frac{C}{2};$$

In like manner derive

$$\sin \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a - b)}{\sin \frac{c}{2}} \cos \frac{C}{2};$$

$$\cos \frac{1}{2}(A + B) = \frac{\cos \frac{1}{2}(a + b)}{\cos \frac{c}{2}} \sin \frac{C}{2};$$

$$\cos \frac{1}{2}(A - B) = \frac{\sin \frac{1}{2}(a + b)}{\sin \frac{c}{2}} \sin \frac{C}{2}.$$

These formulæ are often called Gauss's Formulæ, but they were first discovered by Delambre in 1807. Afterwards Gauss, independently, discovered them, and published them in his *Theoria Motus*.

102. Formulæ for solving *right spherical triangles* are derived from the foregoing by putting $C = 90^\circ$, whence $\sin C = 1$, $\cos C = 0$.

$$\cos c = \cos a \cos b + \sin a \sin b \cos C \quad (\text{Art. 95})$$

becomes $\cos c = \cos a \cos b.$ (1)

Substituting the value of $\cos a$ from (1), and simplifying,

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} \quad (\text{Art. 95})$$

becomes $\cos A = \frac{\tan b}{\tan c}.$ (2)

Again, $\frac{\sin A}{\sin a} = \frac{\sin C}{\sin c}$ (Art. 97)

in the right triangle is

$$\sin A = \frac{\sin a}{\sin c}. \quad (3)$$

Dividing (3) by (2),

$$\tan A = \frac{\sin a \cos b}{\cos c \sin b} = \frac{\sin a \cos a \cos b}{\cos c \cos a \sin b} = \frac{\sin a}{\cos a \sin b},$$

since $\cos a \cos b = \cos c.$

$$\therefore \tan A \sin b = \tan a. \quad (4)$$

$$\text{From (4)} \quad \tan a = \tan A \sin b,$$

$$\text{also,} \quad \tan b = \tan B \sin a.$$

$$\text{Multiplying, } \tan a \tan b = \tan A \tan B \sin a \sin b,$$

$$\text{or,} \quad \cot A \cot B = \cos a \cos b = \cos c. \quad (5)$$

From (2) and (3), by division,

$$\frac{\cos A}{\sin B} = \frac{\frac{\tan b}{\tan c}}{\frac{\sin b}{\sin c}} = \frac{\cos c}{\cos b} = \cos a.$$

$$\therefore \cos A = \cos a \sin B. \quad (6)$$

Let the student write formulæ (2), (3), (4), (6) for B . It will be noticed that (1) and (5) give values for c only, while (2), (3), (4), (6) apply only to A and B .

103. Formulæ (1)–(6) are sufficient for the solution of right spherical triangles if any two parts besides the right angle are given. They are easily remembered by comparison with corresponding formulæ in plane trigonometry. Two rules, invented by Napier, and called *Napier's Rules of Circular Parts*, include all the formulæ of Art. 102.

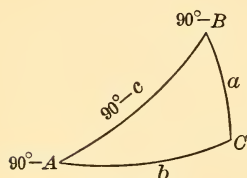


FIG. 51.

Omitting C , and taking the complements of A , c , and B , the parts of the triangle taken in order are a , b , $90^\circ - A$, $90^\circ - c$, $90^\circ - B$. These are called the *circular parts* of the triangle.

Any one of the five parts may be selected as the *middle part*, the two parts next to it are called the *adjacent parts*, and the remaining two the *opposite parts*. Thus, if a be taken as the middle part, $90^\circ - B$ and b are the adjacent parts, and $90^\circ - c$, $90^\circ - A$ the opposite parts.

Napier's Two Rules are as follows :

The sine of the middle part equals the product of the tangents of the adjacent parts.

The sine of the middle part equals the product of the cosines of the opposite parts.

It will aid the memory somewhat to notice that **i** occurs in sine and middle, **a** in tangent and adjacent, and **o** in cosine and opposite, these words being associated in the rules.

The value of the above rules is frequently questioned, most computers preferring to associate the formulæ with the corresponding formulæ of plane trigonometry.

These rules may be proved by taking each of the parts as the middle part, and showing that the formulæ derived from the rules reduce to one of the six formulæ of Art. 102.

Then, if b is the middle part, by the rules,

$$\sin b = \tan a \tan (90^\circ - A) = \tan a \cot A, \text{ or } \tan A = \frac{\tan a}{\sin b},$$

$$\sin b = \cos (90^\circ - c) \cos (90^\circ - B) = \sin c \sin B,$$

$$\text{or} \quad \sin B = \frac{\sin b}{\sin c},$$

results which agree with (4). and (3), Art. 102. If any other part be taken as the middle part, the rules will be found to hold.

104. Area of the spherical triangle.

If r = radius of the sphere,

$$E = \text{spherical excess of the triangle} = A + B + C - 180^\circ,$$

Δ = area of spherical triangle, then by geometry

$$\Delta = Er^2 \times \frac{\pi}{180}.$$

If the three angles are not known, E may be computed by one of the following methods, and Δ found as above.

Cagnoli's Method.

$$\begin{aligned}
\sin \frac{E}{2} &= \sin \frac{1}{2}(A + B + C - 180^\circ) \\
&= \sin \frac{1}{2}(A + B) \sin \frac{C}{2} - \cos \frac{1}{2}(A + B) \cos \frac{C}{2} \\
&= [\cos \frac{1}{2}(a - b) - \cos \frac{1}{2}(a + b)] \frac{\sin \frac{C}{2} \cos \frac{C}{2}}{\cos \frac{c}{2}} \quad (\text{Art. 101}) \\
&= \frac{2 \sin \frac{a}{2} \sin \frac{b}{2}}{\cos \frac{c}{2}} \cdot \frac{\sqrt{\sin s \sin(s - a) \sin(s - b) \sin(s - c)}}{\sin a \sin b} \\
&\quad (\text{Arts. 51, 98}) \\
\sin \frac{E}{2} &= \frac{\sqrt{\sin s \sin(s - a) \sin(s - b) \sin(s - c)}}{2 \cos \frac{a}{2} \cos \frac{b}{2} \cos \frac{c}{2}}.
\end{aligned}$$

Lhuillier's Method.

$$\tan \frac{E}{4} = \frac{\sin \frac{1}{4}(A + B + C - 180^\circ)}{\cos \frac{1}{4}(A + B + C - 180^\circ)}.$$

Now, multiply each term of the fraction by

$$2 \cos \frac{1}{4}(A + B - C + 180^\circ),$$

and by Art. 51, (1) and (3), the equation becomes

$$\begin{aligned}
\tan \frac{E}{4} &= \frac{\sin \frac{1}{2}(A + B) - \cos \frac{C}{2}}{\cos \frac{1}{2}(A + B) + \sin \frac{C}{2}} \\
&= \frac{\left[\cos \frac{1}{2}(a - b) - \cos \frac{c}{2} \right] \cos \frac{C}{2}}{\left[\cos \frac{1}{2}(a + b) + \cos \frac{c}{2} \right] \sin \frac{C}{2}} \quad (\text{Art. 101}) \\
&= \frac{\sin \frac{1}{2}(s - b) \sin \frac{1}{2}(s - a)}{\cos \frac{s}{2} \cos \frac{1}{2}(s - c)} \cdot \sqrt{\frac{\sin s \sin(s - c)}{\sin(s - a) \sin(s - b)}} \\
&\quad (\text{Art. 51})
\end{aligned}$$

By Art. 52, introducing the coefficient under the radical,

$$\tan \frac{E}{4} = \sqrt{\tan \frac{s}{2} \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)}.$$

If two sides and the included angle are given, E may be determined as follows:

$$\begin{aligned} \cos \frac{E}{2} &= \cos \frac{1}{2}(A+B+C-180^\circ) \\ &= \cos \frac{1}{2}(A+B) \sin \frac{C}{2} + \sin \frac{1}{2}(A+B) \cos \frac{C}{2} \\ &= \cos \frac{1}{2}(a+b) \sin^2 \frac{C}{2} + \cos \frac{1}{2}(a-b) \cos^2 \frac{C}{2} \quad (\text{Art. 101}) \\ &= \frac{\cos \frac{a}{2} \cos \frac{b}{2} + \sin \frac{a}{2} \sin \frac{b}{2} \cos C}{\cos \frac{c}{2}}. \end{aligned}$$

$$\text{But } \sin \frac{E}{2} = \frac{\sin \frac{a}{2} \sin \frac{b}{2} \cdot 2 \sin \frac{C}{2} \cos \frac{C}{2}}{\cos \frac{c}{2}}. \quad (\text{Cagnoli's Method})$$

Dividing this equation by the above,

$$\tan \frac{E}{2} = \frac{\sin \frac{a}{2} \sin \frac{b}{2} \sin C}{\cos \frac{a}{2} \cos \frac{b}{2} + \sin \frac{a}{2} \sin \frac{b}{2} \cos C}.$$

This formula is not suitable for logarithmic computations. Usually it is better to compute the angles by Napier's Analogies, and solve by $\Delta = Er^2 \times \frac{\pi}{180}$.

EXAMPLES.

1. Show that $\cos a = \cos b \cos c + \sin b \sin c \cos A$ becomes $\sec A = 1 + \sec a$, when $a = b = c$.

2. If $a + b + c = \pi$, prove

$$(a) \cos a = \tan \frac{B}{2} \tan \frac{C}{2}.$$

$$(b) \cos^2 \frac{A}{2} = \frac{\cos a}{\sin b \sin c}.$$

$$(c) \sin^2 \frac{A}{2} = \cot b \cot c.$$

$$(d) \cos A + \cos B + \cos C = 1.$$

$$(e) \sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} + \sin^2 \frac{C}{2} = 1.$$

$$3. \text{ Prove } \frac{\sin \frac{E}{2} \cos \frac{1}{2}(A - E)}{\sin \frac{A}{2}} = \frac{\sin \frac{S}{2} \sin \frac{1}{2}(s - a)}{\cos \frac{a}{2}}. \quad (\text{Art. 104})$$

4. Show that $\cos a \sin b = \sin a \cos b \cos C + \sin c \cos A$.

CHAPTER XI.

SOLUTION OF SPHERICAL TRIANGLES.

105. According to the principles of spherical geometry any three parts are sufficient to *determine* a spherical triangle; the other parts are *computed*, if any three are given, by the formulæ of trigonometry. The known parts may be :

I. Three sides, or three angles.

II. Two sides and the included angle, or two angles and the included side.

III. Two sides and an angle opposite one, or two angles and a side opposite one.

It will appear that, as in plane geometry, III may be ambiguous.

The *signs* of the functions in the formulæ are important since the cosines and tangents of arcs and angles greater than 90° are negative; whether the part sought is greater or less than 90° is therefore determined by the sign of the function in terms of which it is found unless this function be sine. In this case the result is ambiguous, since $\sin \alpha$ and $\sin(180^\circ - \alpha)$ have the same sign and value. Thus if the solution gives $\log \sin \alpha = 9.56504$, we may have either $\alpha = 21^\circ 33'$, or $158^\circ 27'$. The conditions of the problem must determine which values apply to the triangle in question.

The negative signs, when they occur, will be indicated thus:

$$\log \cos 115^\circ 20' = 9.63135\bar{7},$$

indicating, not that the logarithm is negative, but that in the final result account must be made of the fact that $\cos 115^\circ 20'$ is negative.

106. *Formulæ for the solution of triangles.*

$$\text{I.} \quad \frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

$$\text{II.} \quad \tan \frac{A}{2} = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin s \sin(s-a)}}.$$

$$\text{III.} \quad \tan \frac{a}{2} = \sqrt{\frac{-\cos S \cos(S-A)}{\cos(S-B) \cos(S-C)}}.$$

$$\text{IV.} \quad \tan \frac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \frac{c}{2}.$$

$$\text{V.} \quad \tan \frac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \frac{c}{2}.$$

$$\text{VI.} \quad \tan \frac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \frac{C}{2}.$$

$$\text{VII.} \quad \tan \frac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \frac{C}{2}.$$

$$\text{VIII.} \quad \Delta = Er^2 \frac{\pi}{180},$$

where E is determined by

$$\tan \frac{E}{4} = \sqrt{\tan \frac{s}{2} \tan \frac{1}{2}(s-a) \tan \frac{1}{2}(s-b) \tan \frac{1}{2}(s-c)}.$$

Right triangles may be solved as special cases of oblique triangles, or by the following :

$$(1) \quad \cos c = \cos a \cos b. \quad (4) \quad \tan A \sin b = \tan a.$$

$$(2) \quad \cos A = \frac{\tan b}{\tan c}. \quad (5) \quad \cot A \cot B = \cos c.$$

$$(3) \quad \sin A = \frac{\sin a}{\sin c}. \quad (6) \quad \cos A = \cos a \sin B.$$

The formula to be used in any case may be determined by applying Napier's Rule of Circular Parts.

107. In solving a triangle the student should select formulæ

in which all parts save one are known, and solve for that one (see page 77). Referring to Arts. 105 and 106, it will appear that solutions are effected as follows :

Case I by formulæ II, or III, check by I.

Case II by formulæ VI, VII, I, or IV, V, I, check by IV or VI.

Case III by formulæ I, IV, or I, VI, check by VI or IV.

MODEL SOLUTIONS.

108. 1. Given $a = 46^\circ 24'$, $b = 67^\circ 14'$, $c = 81^\circ 12'$. Solve.

$$\tan \frac{A}{2} = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin s \sin(s-a)}}, \quad \tan \frac{B}{2} = \sqrt{\frac{\sin(s-a)\sin(s-c)}{\sin s \sin(s-b)}},$$

$$\tan \frac{C}{2} = \sqrt{\frac{\sin(s-a)\sin(s-b)}{\sin s \sin(s-c)}}. \quad \text{Check: } \frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}.$$

Arrange and solve as in Example 1, page 80.

$$\text{Ans. } A = 46^\circ 13'.5, B = \quad, C = \quad.$$

Solve: (1) $A = 96^\circ 45'$, $B = 108^\circ 30'$, $C = 116^\circ 15'$.

(Use formulæ III in the same manner as in Example 1.)

$$(2) \quad a = 108^\circ 14', \quad b = 75^\circ 29', \quad c = 56^\circ 37'.$$

$$(3) \quad A = 57^\circ 50', \quad B = 98^\circ 20', \quad C = 63^\circ 40'.$$

2. Given $b = 113^\circ 3'$, $c = 82^\circ 39'$, $A = 138^\circ 50'$. Solve.

$$\tan \frac{1}{2}(B+C) = \frac{\cos \frac{1}{2}(b-c)}{\cos \frac{1}{2}(b+c)} \cot \frac{A}{2}, \quad \tan \frac{1}{2}(B-C) = \frac{\sin \frac{1}{2}(b-c)}{\sin \frac{1}{2}(b+c)} \cot \frac{A}{2},$$

$$\frac{1}{2}(B+C) \pm \frac{1}{2}(B-C) = B, \text{ or } C, \quad \sin a = \frac{\sin A \sin b}{\sin B}.$$

$$\text{Check: } \tan \frac{a}{2} = \frac{\tan \frac{1}{2}(b-c) \sin \frac{1}{2}(B+C)}{\sin \frac{1}{2}(B-C)}.$$

$$\begin{aligned} b &= 113^\circ 3' & \log \cos \frac{1}{2}(b-c) &= 9.98453 & \log \sin \frac{1}{2}(b-c) &= 9.41861 \\ c &= 82^\circ 39' & \text{colog } \cos \frac{1}{2}(b+c) &= 0.86461 & \text{colog } \sin \frac{1}{2}(b+c) &= 0.00409 \\ \frac{1}{2}(b+c) &= 97^\circ 51' & \log \cot \frac{A}{2} &= 9.57466 & \log \cot \frac{A}{2} &= 9.57466 \\ \frac{1}{2}(b-c) &= 15^\circ 12' & & & & \\ \frac{1}{2}A &= 69^\circ 25' & \log \tan \frac{1}{2}(B+C) &= 0.42380 & \log \tan \frac{1}{2}(B-C) &= 8.99736 \\ & & \frac{1}{2}(B+C) &= 110^\circ 39' & \frac{1}{2}(B-C) &= 5^\circ 40'.6 \\ & & \frac{1}{2}(B-C) &= 5^\circ 40'.6 & & \\ & & \therefore B &= 116^\circ 19'.6 & & \\ & & \text{and } C &= 104^\circ 58'.4 & & \end{aligned}$$

Check :

$$\log \sin A = 9.81839$$

$$\log \sin b = 9.96387$$

$$\text{cologs in } B = 0.04756$$

$$\log \sin a = 9.82982$$

$$a = 137^\circ 29'$$

$$\log \tan \frac{1}{2}(b - c) = 9.43408$$

$$\log \sin \frac{1}{2}(B + C) = 9.97116$$

$$\text{colog} \sin \frac{1}{2}(B - C) = 1.00474$$

$$\log \tan \frac{a}{2} = 0.40998$$

$$a = 137^\circ 29'$$

Notice that $\tan \frac{1}{2}(B + C)$ is —. Hence, $\frac{1}{2}(B + C)$ is greater than 90° , i.e. $110^\circ 39'$.

Solve: (1) $A = 68^\circ 40'$, $B = 56^\circ 20'$, $c = 84^\circ 30'$.

(Use formulæ IV, V, I. Compare Example 2.)

(2) $a = 102^\circ 22'$, $b = 78^\circ 17'$, $C = 125^\circ 28'$.

(3) $A = 130^\circ 5'$, $B = 32^\circ 26'$, $c = 51^\circ 6'$.

109. Ambiguous cases. By the principles of geometry the spherical triangle is not necessarily determined by two sides and an angle opposite, nor by two angles and a side opposite. The triangle may be ambiguous. By geometrical principles it is shown that the marks of the ambiguous spherical triangle are:

1. The parts given are two angles and the side opposite one, or two sides and the angle opposite one.

2. The side, or angle, opposite differs from 90° more than the other given side, or angle.

3. Both sides, or angles, given are either greater than 90° , or less than 90° .

In the right triangle ABC_2 ,

$$\sin a = \sin A \sin c. \text{ (formula (3))}$$

Therefore there will be no solution, one solution, or two solutions, according as $\sin a \leq \sin A \sin c$, i.e. according as $a \leq$ the perpendicular p . (See Art. 65.)

But the most expeditious means of determining the ambiguity is found in the solution of the triangle. The use of formula I gives the solution in terms of sine, so that it is to be expected that two values of the part sought may be possible; and whether the triangle be ambiguous or not, there must be some means of determining which of the two

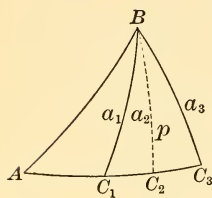


FIG. 52.

angles, α and $180^\circ - \alpha$, that have the same sine is to be used. If there are two solutions, both values are used.

This is determined in the further solution of the triangle by formula VI, which may be written

$$\tan \frac{b}{2} = \frac{\cos \frac{1}{2}(A + C) \tan \frac{1}{2}(a + c)}{\cos \frac{1}{2}(A - C)}.$$

Now $\frac{b}{2} < 90^\circ$, whence $\tan \frac{b}{2}$ is +. Then if for both values of C , found by the sine formula, the second member is +, there are two solutions; if the second member is - for either value of C , there is but one solution; while if both values of C make the second member -, there is no solution. The various cases will be illustrated by problems.

3. Given $a = 62^\circ 15'.4$, $b = 103^\circ 18'.8$, $A = 53^\circ 42'.6$. Solve.

$$\sin B = \frac{\sin b \sin A}{\sin a}, \quad \tan \frac{c}{2} = \frac{\cos \frac{1}{2}(A + B) \tan \frac{1}{2}(a + b)}{\cos \frac{1}{2}(A - B)},$$

$$\sin C = \frac{\sin c \sin A}{\sin a}. \quad \text{Check: } \cot \frac{C}{2} = \frac{\tan \frac{1}{2}(A - B) \sin \frac{1}{2}(a + b)}{\sin \frac{1}{2}(a - b)}.$$

Solving the first formula gives

$$\log \sin B = 9.94756,$$

whence

$$B_1 = 62^\circ 24'.4,$$

$$B_2 = 117^\circ 35'.6.$$

For each of the values B_1 and B_2 ,

$$\frac{\cos \frac{1}{2}(A + B) \tan \frac{1}{2}(a + b)}{\cos \frac{1}{2}(A - B)}$$

is + and therefore equal to $\tan \frac{c}{2}$. Hence there are two solutions. Find

$$c = 153^\circ 9'.6, \text{ or } 70^\circ 25'.4$$

and

$$C = 155^\circ 43'.2, \text{ or } 59^\circ 6'.2$$

4. Given $a = 46^\circ 45'.5$, $A = 73^\circ 11'.3$, $B = 61^\circ 18'.2$. Solve.

$$\sin b = \frac{\sin a \sin B}{\sin A}, \quad \cot \frac{C}{2} = \frac{\tan \frac{1}{2}(A - B) \cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(a - b)},$$

$$\sin c = \frac{\sin a \sin C}{\sin A}. \quad \text{Check: } \tan \frac{c}{2} = \frac{\tan \frac{1}{2}(a - b) \sin \frac{1}{2}(A + B)}{\sin \frac{1}{2}(A - B)}.$$

Solving for b gives $\log \sin b = 9.82446$,

whence $b_1 = 41^\circ 52'.5$,

and $b_2 = 138^\circ 7'.5$.

For the value b_1 the fraction

$$\frac{\tan \frac{1}{2}(A - B) \cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(a - b)}$$

is +, but for $b_2 \cos \frac{1}{2}(a + b)$ is -, making the fraction -, and hence it can not equal $\cot \frac{C}{2}$, which is +. There is then but one solution. Find

$$C = 60^\circ 42'.7, \quad c = 41^\circ 35'.1.$$

5. Given $a = 162^\circ 30'$, $A = 49^\circ 50'$, $B = 57^\circ 52'$. Solve.

Solving gives $\log \sin b = 9.52274$,

whence $b_1 = 19^\circ 27'.9$,

$$b_2 = 160^\circ 32'.1.$$

For both values, b_1 and b_2 , $\cos \frac{1}{2}(a + b)$ is -. Therefore,

$$\frac{\tan \frac{1}{2}(A - B) \cos \frac{1}{2}(a + b)}{\cos \frac{1}{2}(a - b)}$$

is - and not equal to $\cot \frac{C}{2}$. Hence the triangle is impossible.

Solve, testing for the number of solutions:

$$(1) \quad b = 106^\circ 24'.5, \quad c = 40^\circ 20', \quad C = 38^\circ 45'.6.$$

$$(2) \quad a = 80^\circ 50', \quad A = 131^\circ 40', \quad B = 65^\circ 25'.$$

$$(3) \quad a = 60^\circ 31'.4, \quad b = 147^\circ 32'.1, \quad B = 143^\circ 50'.$$

$$(4) \quad a = 55^\circ 30', \quad c = 139^\circ 5', \quad A = 43^\circ 25'.$$

RIGHT TRIANGLES.

110. Right triangles are a special case of oblique triangles, but are usually solved by formulæ (1) to (6), Art. 106. Students should have no difficulty in applying these.

Computers generally question the utility of Napier's Rules of Circular Parts. For those who prefer the rules a problem will be solved by their use.

6. Given $c = 86^\circ 51'$, $B = 18^\circ 3'5$, $C = 90^\circ$.

The parts sought are a , b , A , and it is immaterial which is computed first. a and A are adjacent to c and B , while b is the middle part of c and B . Then by Napier's first rule

$$\sin (90^\circ - B) = \tan (90^\circ - c) \tan a;$$

or
$$\tan a = \frac{\cos B}{\cot c} = \cos B \tan c,$$

which is formula (2).

By the same rule

$$\sin (90^\circ - c) = \tan (90^\circ - A) \tan (90^\circ - B),$$

or
$$\cot A = \frac{\cos c}{\cot B} = \cos c \tan B, \quad \text{formula (5).}$$

Finally by the second rule

$$\sin b = \cos (90^\circ - c) \cos (90^\circ - B) = \sin c \sin B, \quad \text{formula (3).}$$

The solutions give $a = 86^\circ 41'2$, $b = 18^\circ 1'8$, $A = 88^\circ 58'4$. Verify.

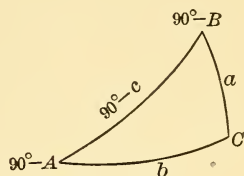


FIG. 53.

111. Species. Two angles or sides of a spherical triangle are said to be of the *same species* if they are both less, or both greater, than 90° . They are of *opposite species* when one is greater and the other less than 90° . Since the sides and angles of a spherical triangle may, any or all, be less or greater than 90° , it is necessary in solutions to determine whether each part is more or less than 90° . The directions already given are sufficient in oblique triangles. In right triangles the sign of the function will determine if the solution gives the result in terms of cosine or tangent, but not if the result is found in terms of sine. Thus in Example 6, above, we have $\log \sin b = 9.49068$, whence $b = 18^\circ 1'8$, or $161^\circ 58'2$. By formula (4) $\sin b = \frac{\tan a}{\tan A}$. Now $\sin b$ is always +, therefore, $\tan a$ and $\tan A$ must be of the same sign, whence *in any right spherical triangle an oblique angle and its opposite side must be of the same species*.

Again by formula (1) $\cos c = \cos a \cos b$. Now $\cos c$ is + or - according as c is less or greater than 90° . If then $c < 90^\circ$, $\cos a$ and $\cos b$ are of the same sign, but if $c > 90^\circ$, $\cos a$ and $\cos b$ are of opposite sign. Therefore, *if the*

hypotenuse of a right spherical triangle is less than 90° , the other sides, and hence the angles opposite, are of the same species; but if the hypotenuse be greater than 90° , the other sides, and the angles opposite, are of opposite species.

112. Ambiguous right triangles.

When the parts given are a side adjacent to the right angle, and the angle opposite this side, the triangle is ambiguous, for solving for the hypotenuse by formula (3) gives

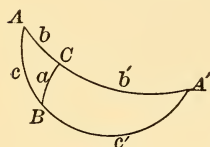


FIG. 54.

$$\sin c = \frac{\sin a}{\sin A},$$

from which there result two values of c .

By the last rule of species it follows that to the values of c , one $< 90^\circ$, the other $> 90^\circ$, there will correspond two values for b , one of the same species as a , the other of opposite species.

Clearly $\sin c \leq 1$, according as $\sin a \leq \sin A$, and hence there will be no solution, one solution, or two solutions, according as $\sin a \geq \sin A$.

Solve the spherical triangles, right angled at C , given :

- (1) $b = 73^\circ 21'.4$, $c = 84^\circ 48'.7$.
- (2) $c = 54^\circ 28'$, $B = 128^\circ 12'.6$.
- (3) $b = 45^\circ 42'$, $B = 135^\circ 42'$.
- (4) $a = 108^\circ 22'.3$, $b = 120^\circ 14'.5$.
- (5) $a = 70^\circ 50'$, $A = 170^\circ 40'$.
- (6) $b = 32^\circ 8'.4$, $B = 46^\circ 2'.8$.
- (7) $b = 34^\circ 28'$, $c = 62^\circ 50'$.
- (8) $c = 102^\circ 35'$, $B = 17^\circ 45'$.
- (9) $a = 92^\circ 16'$, $c = 57^\circ 35'$.

Solve, given :

EXAMPLES.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>A</i>	<i>B</i>	<i>C</i>
1.	97° 35'	27° 8'.4	119° 8'.4			
2.		67° 33'.4	94° 5'	99° 57'.6		
3.	40° 20'	70° 40'		40°		
4.		82° 39'.5		116° 20'		70° 7'
5.	155° 47'.1		110° 46'.4			90°
6.				49° 44'.3	121° 10'.4	26° 6'.3
7.	144° 10'		41° 44'.2		130°	
8.		127° 30'		132° 16'	139° 44'	
9.		155° 5'.3		110° 10'		70° 20'.8
10.	62° 42'			50° 12'	58° 8'	
11.	120° 30'	70° 20'.3	69° 35'			
12.	50° 15'			75° 30'		90°
13.				116° 20'	104° 59'.1	138° 50'.2
14.	84° 14'.5				32° 26'.1	36° 45'.4
15.	100°	50°	60°			
16.			87° 12'	88° 12'		90°
17.	63° 50'	80° 19'		50° 30'		
18.				34° 15'	42° 15'.2	121° 36'.2
19.	50°			63° 15'		90°
20.	159° 50'			159° 43'	123° 40'	
21.	124° 12'.5	54° 18'	97° 12'.5			
22.				48° 31'.3	62° 55'.7	125° 18'.9
23.	76° 36'		40° 20'		42° 15'.2	
24.			28° 45'.1	44° 22'.2	122° 25'.1	
25.		44° 53'	53° 52'			90°
26.	98° 21'.7	109° 50'.4	115° 13'.5			
27.	99° 40'.8	64° 23'.2		95° 38'.1		

APPLICATIONS TO GEODESY AND ASTRONOMY.

113. Geodesy is concerned in measuring portions of the earth's surface, considering the earth as a sphere.

To find the distance on the earth's surface between two points whose latitudes and longitudes are known.

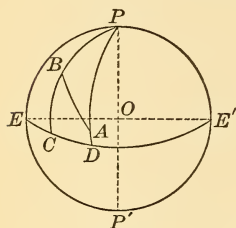


FIG. 55.

If A and B are two places on the earth, P the north pole, $ECDE'$ the equator, and PEP' the principal meridian, *e.g.* the meridian of Greenwich, and if the latitude and longitude of A and B are known, then AB can be computed.

For

$$AP = 90^\circ - \text{latitude } A,$$

$$BP = 90^\circ - \text{latitude } B,$$

$$\text{angle } APB = \text{longitude } A - \text{longitude } B.$$

$\therefore$ two sides and the included angle of the triangle APB are known, and AB can be computed.

Ex. 1. Find the distance between Ann Arbor, $42^\circ 19' \text{ N.}$, $83^\circ 43'.8 \text{ W.}$, and San Juan, $18^\circ 29' \text{ N.}$, $66^\circ 7' \text{ W.}$

2. How far is Manila, $14^\circ 36' \text{ N.}$, $120^\circ 58' \text{ E.}$, from Honolulu, $21^\circ 18' \text{ N.}$, $157^\circ 55' \text{ W.}$? Honolulu from San Francisco, $37^\circ 47'.9 \text{ N.}$, $122^\circ 24'.5 \text{ W.}$? San Francisco from Manila?

114. *The celestial sphere.* The heavenly bodies appear to be situated on a sphere of indefinitely great radius with the centre at the point of observation. This is called the *celestial sphere*.

A tangent plane to the earth at the point of observation cuts the celestial sphere in a great circle called the *horizon*.

The points of the horizon directly south, west, north, east are called the *south, west, north, east points*.

A vertical line through the point of observation cuts the celestial sphere above in the *zenith*, and below in the *nadir*, the zenith and nadir being *poles* of the horizon.

The earth's axis produced is the *axis of the celestial sphere*, cutting it in the north and south poles of the equator.

The *altitude* of a star is its distance from the horizon measured on an arc of a great circle drawn through the star and the zenith.

The *azimuth*, or *bearing*, of a star, is the arc of the horizon measured from some fixed point to the foot of the great circle through the star and the zenith. The fixed point is usually the south point.

The *declination* of a star is its distance from the celestial equator. The circle drawn through the pole and the star is the *hour circle*, and the angle at the pole between the prime meridian and the hour circle is the *hour angle* of the star.

Let an observer be at O on the surface of the earth, and let P be the position of a star.

Then Z is the zenith, Z' the nadir, EQE' the celestial equator, N its north pole, S its south pole, HRH' the horizon, NPS the meridian, or hour circle, of P , and ZNP the hour angle. The declination of the star is PQ , its altitude PR , and its azimuth, or bearing, NZP . The *astronomical triangle* NZP can be solved if any three of its parts are known.

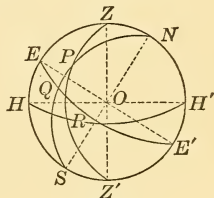


FIG. 56.

EXAMPLES.

1. What will be the altitude of the sun at 9 A.M. in Detroit, lat. $42^{\circ} 20' N.$, its declination being $27^{\circ} 30'.5$?
2. At what time will the sun rise at San Francisco, lat. $37^{\circ} 47'.9$, if its declination is $32^{\circ} 46'.2$?
3. Find the azimuth and altitude of a star to an observer in lat. $42^{\circ} 20' N.$, when the hour angle of the star is 3 h. 42.3 m. E., and the declination is $42^{\circ} 31' N.$
4. The latitude of Sayre Observatory is $40^{\circ} 36'.4 N.$; the sun's altitude is $47^{\circ} 15'.3$, its azimuth $80^{\circ} 23'.1$. Find its declination and hour angle.
5. At Ann Arbor, March 13, 1891, the altitude of Regulus is $32^{\circ} 10'.3$, and the azimuth is $283^{\circ} 5'.1$. Find the declination and hour angle.

FIVE-PLACE
LOGARITHMIC AND TRIGONOMETRIC
TABLES

ADAPTED FROM GAUSS'S TABLES

BY

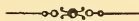
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Norwood Press
J. S. Cushing & Co. — Berwick & Smith
Norwood Mass. U.S.A.

Reprin + 20 Oct 05
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TABLE I.
THE COMMON LOGARITHMS OF NUMBERS
FROM 1 TO 10009.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
100	00	000	043	087	130	173	217	260	303	346	389	
101		432	475	518	561	604	647	689	732	775	817	44 43 42
102		860	903	945	988	*030	*072	*115	*157	*199	*242	1 4,4 4,3 4,2
103	01	284	326	368	410	452	494	536	578	620	662	2 8,8 8,6 8,4
104		703	745	787	828	870	912	953	995	*036	*078	3 13,2 12,9 12,6
												4 17,6 17,2 16,8
105	02	119	160	202	243	284	325	366	407	449	490	5 22,0 21,5 21,0
106		531	572	612	653	694	735	776	816	857	898	6 26,4 25,8 25,2
107		938	979	*019	*060	*100	*141	*181	*222	*262	*302	7 30,8 30,1 29,4
108	03	342	383	423	463	503	543	583	623	663	703	8 35,2 34,4 33,6
109		743	782	822	862	902	941	981	*021	*060	*100	9 39,6 38,7 37,8
110	04	139	179	218	258	297	336	376	415	454	493	41 40 39
111		532	571	610	650	689	727	766	805	844	883	1 4,1 4,0 3,9
112		922	961	999	*038	*077	*115	*154	*192	*231	*269	2 8,2 8,0 7,8
113	05	308	346	385	423	461	500	538	576	614	652	3 12,3 12,0 11,7
114		690	729	767	805	843	881	918	956	994	*032	4 16,4 16,0 15,6
												5 20,5 20,0 19,5
115	06	070	108	145	183	221	258	296	333	371	408	6 24,6 24,0 23,4
116		446	483	521	558	595	633	670	707	744	781	7 28,7 28,0 27,3
117		819	856	893	930	967	*004	*041	*078	*115	*151	8 32,8 32,0 31,2
118	07	188	225	262	298	335	372	408	445	482	518	9 36,9 36,0 35,1
119		555	591	628	664	700	737	773	809	846	882	
120		918	954	990	*027	*063	*099	*135	*171	*207	*243	38 37 36
121	08	279	314	350	386	422	458	493	529	565	600	1 3,8 3,7 3,6
122		636	672	707	743	778	814	849	884	920	955	2 7,6 7,4 7,2
123		991	*026	*061	*096	*132	*167	*202	*237	*272	*307	3 11,4 11,1 10,8
124	09	342	377	412	447	482	517	552	587	621	656	4 15,2 14,8 14,4
												5 19,0 18,5 18,0
125		691	726	760	795	830	864	899	934	968	*003	6 22,6 22,2 21,6
126	10	037	072	106	140	175	209	243	278	312	346	7 26,6 25,9 25,2
127		380	415	449	483	517	551	585	619	653	687	8 30,4 29,6 28,8
128		721	755	789	823	857	890	924	958	992	*025	9 34,2 33,3 32,4
129	11	059	093	126	160	193	227	261	294	327	361	
130		394	428	461	494	528	561	594	628	661	694	35 34 33
131		727	760	793	826	860	893	926	959	992	*024	1 3,5 3,4 3,3
132	12	057	090	123	156	189	222	254	287	320	352	2 7,0 6,8 6,6
133		385	418	450	483	516	548	581	613	646	678	3 10,5 10,2 9,9
134		710	743	775	808	840	872	905	937	969	*001	4 14,0 13,6 13,2
												5 17,5 17,0 16,5
135	13	033	066	098	130	162	194	226	258	290	322	6 21,0 20,4 19,8
136		354	386	418	450	481	513	545	577	609	640	7 24,5 23,8 23,1
137		672	704	735	767	799	830	862	893	925	956	8 28,0 27,2 26,4
138		988	*019	*051	*082	*114	*145	*176	*208	*239	*270	9 31,5 30,6 29,7
139	14	301	333	364	395	426	457	489	520	551	582	
140		613	644	675	706	737	768	799	829	860	891	32 31 30
141		922	953	983	*014	*045	*076	*106	*137	*168	*198	1 3,2 3,1 3,0
142	15	229	259	290	320	351	381	412	442	473	503	2 6,4 6,2 6,0
143		534	564	594	625	655	685	715	746	776	806	3 9,6 9,3 9,0
144		836	866	897	927	957	987	*017	*047	*077	*107	4 12,8 12,4 12,0
												5 16,0 15,5 15,0
145	16	137	167	197	227	256	286	316	346	376	406	6 19,2 18,6 18,0
146		435	465	495	524	554	584	613	643	673	702	7 22,4 21,7 21,0
147		732	761	791	820	850	879	909	938	967	997	8 25,6 24,8 24,0
148	17	026	056	085	114	143	173	202	231	260	289	9 28,8 27,9 27,0
149		319	348	377	406	435	464	493	522	551	580	
150		609	638	667	696	725	754	782	811	840	869	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
150	17	609	638	667	696	725	754	782	811	840	869	
151		898	926	955	984	*013	*041	*070	*099	*127	*156	29
152	18	184	213	241	270	298	327	355	384	412	441	28
153		469	498	526	554	583	611	639	667	696	724	I 2,9 2,8
154		752	780	808	837	865	893	921	949	977	*005	2 5,8 5,6
												3 8,7 8,4
155	19	033	061	089	117	145	173	201	229	257	285	4 11,6 11,2
156		312	340	368	396	424	451	479	507	535	562	5 14,5 14,0
157		590	618	645	673	700	728	756	783	811	838	6 17,4 16,8
158		866	893	921	948	976	*003	*030	*058	*085	*112	7 20,3 19,6
159	20	140	167	194	222	249	276	303	330	358	385	8 23,2 22,4
												9 26,1 25,2
160		412	439	466	493	520	548	575	602	629	656	
161		683	710	737	763	790	817	844	871	898	925	27
162		952	978	*005	*032	*059	*085	*112	*139	*165	*192	26
163	21	219	245	272	299	325	352	378	405	431	458	I 2,7 2,6
164		484	511	537	564	590	617	643	669	696	722	2 5,4 5,2
												3 8,1 7,8
165		748	775	801	827	854	880	906	932	958	985	4 10,8 10,4
166	22	011	037	063	089	115	141	167	194	220	246	5 13,5 13,0
167		272	298	324	350	376	401	427	453	479	505	6 16,2 15,6
168		531	557	583	608	634	660	686	712	737	763	7 18,9 18,2
169		789	814	840	866	891	917	943	968	994	*019	8 21,6 20,8
												9 24,3 23,4
170	23	045	070	096	121	147	172	198	223	249	274	
171		300	325	350	376	401	426	452	477	502	528	25
172		553	578	603	629	654	679	704	729	754	779	I 2,5 2,0
173		805	830	855	880	905	930	955	980	*005	*030	2 5,0 4,6
174	24	055	080	105	130	155	180	204	229	254	279	3 7,5 7,0
												4 10,0 9,6
175		304	329	353	378	403	428	452	477	502	527	5 12,5 12,0
176		551	576	601	625	650	674	699	724	748	773	6 15,0 14,5
177		797	822	846	871	895	920	944	969	993	*018	7 17,5 17,0
178	25	042	066	091	115	139	164	188	212	237	261	8 20,0 19,5
179		285	310	334	358	382	406	431	455	479	503	9 22,5 22,0
180		527	551	574	600	624	648	672	696	720	744	
181		768	792	816	840	864	888	912	935	959	983	24
182	26	007	031	055	079	102	126	150	174	198	221	23
183		245	269	293	316	340	364	387	411	435	458	I 2,4 2,3
184		482	505	529	553	576	600	623	647	670	694	2 4,8 4,6
												3 7,2 6,9
185		717	741	764	788	811	834	858	881	905	928	4 9,6 9,2
186		951	975	998	*021	*045	*068	*091	*114	*138	*161	5 12,0 11,5
187	27	184	207	231	254	277	300	323	346	370	393	6 14,4 13,8
188		416	439	462	485	508	531	554	577	600	623	7 16,8 16,1
189		646	669	692	715	738	761	784	807	830	852	8 19,2 18,4
												9 21,6 20,7
190		875	898	921	944	967	989	*012	*035	*058	*081	
191	28	103	126	149	171	194	217	240	262	285	307	22
192		330	353	375	398	421	443	466	488	511	533	21
193		556	578	601	623	646	668	691	713	735	758	I 2,2 2,1
194		780	803	825	847	870	892	914	937	959	981	2 4,4 4,2
												3 6,6 6,3
195	29	003	026	048	070	092	115	137	159	181	203	4 8,8 8,4
196		226	248	270	292	314	336	358	380	403	425	5 11,0 10,5
197		447	469	491	513	535	557	579	601	623	645	6 13,2 12,6
198		667	688	710	732	754	776	798	820	842	863	7 15,4 14,7
199		885	907	929	951	973	994	*016	*038	*060	*081	8 17,6 16,8
												9 19,8 18,9
200	30	103	125	146	168	190	211	233	255	276	298	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
200	30 103	125	146	168	190	211	233	255	276	298	22 21 1 2,2 2,1 2 4,4 4,2 3 6,6 6,3 4 8,8 8,4 5 11,0 10,5 6 13,2 12,6 7 15,4 14,7 8 17,6 16,8 9 19,8 18,9
201	320	341	363	384	406	428	449	471	492	514	
202	535	557	578	600	621	643	664	685	707	728	
203	750	771	792	814	835	856	878	899	920	942	
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	
205	31 175	197	218	239	260	281	302	323	345	366	20 1 2,0 2 4,0 3 6,0 4 8,0 5 10,0 6 12,0 7 14,0 8 16,0 9 18,0
206	387	408	429	450	471	492	513	534	555	576	
207	597	618	639	660	681	702	723	744	765	785	
208	806	827	848	869	890	911	931	952	973	994	
209	32 015	035	056	077	098	118	139	160	181	201	
210	222	243	263	284	305	325	346	366	387	408	19 1 1,9 2 3,8 3 5,7 4 7,6 5 9,5 6 11,4 7 13,3 8 15,2 9 17,1
211	428	449	469	490	510	531	552	572	593	613	
212	634	654	675	695	715	736	756	777	797	818	
213	838	858	879	899	919	940	960	980	*001	*021	
214	33 041	062	082	102	122	143	163	183	203	224	
215	244	264	284	304	325	345	365	385	405	425	18 1 1,8 2 3,6 3 5,4 4 7,2 5 9,0 6 10,8 7 12,6 8 14,4 9 16,2
216	445	465	486	506	526	546	566	586	606	626	
217	646	666	686	706	726	746	766	786	806	826	
218	846	866	885	905	925	945	965	985	*005	*025	
219	34 044	064	084	104	124	143	163	183	203	223	
220	242	262	282	301	321	341	361	380	400	420	17 1 1,7 2 3,4 3 5,1 4 6,8 5 8,5 6 10,2 7 11,9 8 13,6 9 15,3
221	439	459	479	498	518	537	557	577	596	616	
222	635	655	674	694	713	733	753	772	792	811	
223	830	850	869	889	908	928	947	967	986	*005	
224	35 025	044	064	083	102	122	141	160	180	199	
225	218	238	257	276	295	315	334	353	372	392	16 1 1,6 2 3,2 3 4,8 4 6,4 5 8,0 6 9,6 7 11,2 8 12,8 9 14,4
226	411	430	449	468	488	507	526	545	564	583	
227	603	622	641	660	679	698	717	736	755	774	
228	793	813	832	851	870	889	908	927	946	965	
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	
230	36 173	192	211	229	248	267	286	305	324	342	15 1 1,5 2 3,0 3 4,5 4 6,0 5 7,5 6 9,0 7 10,5 8 12,0 9 13,5

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
250	39	794	811	829	846	863	881	898	915	933	950	18 1 1,8 2 3,6 3 5,4 4 7,2 5 9,0 6 10,8 7 12,6 8 14,4 9 16,2
251		967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40	140	157	175	192	209	226	243	261	278	295	
253		312	329	346	364	381	398	415	432	449	466	
254		483	500	518	535	552	569	586	603	620	637	
255		654	671	688	705	722	739	756	773	790	807	17 1 1,7 2 3,4 3 5,1 4 6,8 5 8,5 6 10,2 7 11,9 8 13,6 9 15,3
256		824	841	858	875	892	909	926	943	960	976	
257		993	*010	*027	*044	*061	*078	*095	*111	*128	*145	
258	41	162	179	196	212	229	246	263	280	296	313	
259		330	347	363	380	397	414	430	447	464	481	
260		497	514	531	547	564	581	597	614	631	647	16 1 1,6 2 3,2 3 4,8 4 6,4 5 8,0 6 9,6 7 11,2 8 12,8 9 14,4
261		664	681	697	714	731	747	764	780	797	814	
262		830	847	863	880	896	913	929	946	963	979	
263		996	*012	*029	*045	*062	*078	*095	*111	*127	*144	
264	42	160	177	193	210	226	243	259	275	292	308	
265		325	341	357	374	390	406	423	439	455	472	15 1 1,5 2 3,0 3 4,5 4 6,0 5 7,5 6 9,0 7 10,5 8 12,0 9 13,5
266		488	504	521	537	553	570	586	602	619	635	
267		651	667	684	700	716	732	749	765	781	797	
268		813	830	846	862	878	894	911	927	943	959	
269		975	991	*008	*024	*040	*056	*072	*088	*104	*120	
270	43	136	152	169	185	201	217	233	249	265	281	14 1 1,4 2 2,8 3 4,2 4 5,6 5 7,0 6 8,4 7 9,8 8 11,2 9 12,6
271		297	313	329	345	361	377	393	409	425	441	
272		457	473	489	505	521	537	553	569	584	600	
273		616	632	648	664	680	696	712	727	743	759	
274		775	791	807	823	838	854	870	886	902	917	
275		933	949	965	981	996	*012	*028	*044	*059	*075	13 1 1,3 2 2,6 3 3,9 4 5,2 5 6,5 6 7,8 7 9,1 8 10,4 9 11,7
276	44	091	107	122	138	154	170	185	201	217	232	
277		248	264	279	295	311	326	342	358	373	389	
278		404	420	436	451	467	483	498	514	529	545	
279		560	576	592	607	623	638	654	669	685	700	
280		716	731	747	762	778	793	809	824	840	855	12 1 1,2 2 2,4 3 3,6 4 4,8 5 6,0 6 7,2 7 8,4 8 9,6 9 10,8
281		871	886	902	917	932	948	963	979	994	*010	
282	45	025	040	056	071	086	102	117	133	148	163	
283		179	194	209	225	240	255	271	286	301	317	
284		332	347	362	378	393	408	423	439	454	469	
285		484	500	515	530	545	561	576	591	606	621	11 1 1,1 2 2,2 3 3,3 4 4,4 5 5,5 6 6,6 7 7,7 8 8,8 9 9,9
286		637	652	667	682	697	712	728	743	758	773	
287		788	803	818	834	849	864	879	894	909	924	
288		939	954	969	984	*000	*015	*030	*045	*060	*075	
289	46	090	105	120	135	150	165	180	195	210	225	
290		240	255	270	285	300	315	330	345	359	374	10 1 1,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
291		389	404	419	434	449	464	479	494	509	523	
292		538	553	568	583	598	613	627	642	657	672	
293		687	702	716	731	746	761	776	790	805	820	
294		835	850	864	879	894	909	923	938	953	967	
295		982	997	*012	*026	*041	*056	*070	*085	*100	*114	9 1 1,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
296	47	129	144	159	173	188	202	217	232	246	261	
297		276	290	305	319	334	349	363	378	392	407	
298		422	436	451	465	480	494	509	524	538	553	
299		567	582	596	611	625	640	654	669	683	698	
300		712	727	741	756	770	784	799	813	828	842	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
300	47	712	727	741	756	770	784	799	813	828	842	15
301		857	871	885	900	914	929	943	958	972	986	
302	48	001	015	029	044	058	073	087	101	116	130	
303		144	159	173	187	202	216	230	244	259	273	
304		287	302	316	330	344	359	373	387	401	416	
305		430	444	458	473	487	501	515	530	544	558	
306		572	586	601	615	629	643	657	671	686	700	
307		714	728	742	756	770	785	799	813	827	841	
308		855	869	883	897	911	926	940	954	968	982	
309		996	*010	*024	*038	*052	*066	*080	*094	*108	*122	
310	49	136	150	164	178	192	206	220	234	248	262	14
311		276	290	304	318	332	346	360	374	388	402	
312		415	429	443	457	471	485	499	513	527	541	
313		554	568	582	596	610	624	638	651	665	679	
314		693	707	721	734	748	762	776	790	803	817	
315		831	845	859	872	886	900	914	927	941	955	
316		969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50	106	120	133	147	161	174	188	202	215	229	
318		243	256	270	284	297	311	325	338	352	365	
319		379	393	406	420	433	447	461	474	488	501	
320		515	529	542	556	569	583	596	610	623	637	13
321		651	664	678	691	705	718	732	745	759	772	
322		786	799	813	826	840	853	866	880	893	907	
323		920	934	947	961	974	987	*001	*014	*028	*041	
324	51	055	068	081	095	108	121	135	148	162	175	
325		188	202	215	228	242	255	268	282	295	308	
326		322	335	348	362	375	388	402	415	428	441	
327		455	468	481	495	508	521	534	548	561	574	
328		587	601	614	627	640	654	667	680	693	706	
329		720	733	746	759	772	786	799	812	825	838	
330		851	865	878	891	904	917	930	943	957	970	12
331		983	996	*009	*022	*035	*048	*061	*075	*088	*101	
332	52	114	127	140	153	166	179	192	205	218	231	
333		244	257	270	284	297	310	323	336	349	362	
334		375	388	401	414	427	440	453	466	479	492	
335		504	517	530	543	556	569	582	595	608	621	
336		634	647	660	673	686	699	711	724	737	750	
337		763	776	789	802	815	827	840	853	866	879	
338		892	905	917	930	943	956	969	982	994	*007	
339	53	020	033	046	058	071	084	097	110	122	135	
340		148	161	173	186	199	212	224	237	250	263	11,7
341		275	288	301	314	326	339	352	364	377	390	
342		403	415	428	441	453	466	479	491	504	517	
343		529	542	555	567	580	593	605	618	631	643	
344		656	668	681	694	706	719	732	744	757	769	
345		782	794	807	820	832	845	857	870	882	895	
346		908	920	933	945	958	970	983	*008	*020	936	
347	54	033	045	058	070	083	095	108	120	133	145	
348		158	170	183	195	208	220	233	245	258	270	
349		283	295	307	320	332	345	357	370	382	394	
350		407	419	432	444	456	469	481	494	506	518	
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
350	54	407	419	432	444	456	469	481	494	506	518	13 1 1,3 2 2,6 3 3,9 4 5,2 5 6,5 6 7,8 7 9,1 8 10,4 9 11,7
351		531	543	555	568	580	593	605	617	630	642	
352		654	667	679	691	704	716	728	741	753	765	
353		777	790	802	814	827	839	851	864	876	888	
354		900	913	925	937	949	962	974	986	998	*011	
355	55	023	035	047	060	072	084	096	108	121	133	
356		145	157	169	182	194	206	218	230	242	255	
357		267	279	291	303	315	328	340	352	364	376	
358		388	400	413	425	437	449	461	473	485	497	
359		509	522	534	546	558	570	582	594	606	618	
360		630	642	654	666	678	691	703	715	727	739	12 1 1,2 2 2,4 3 3,6 4 4,8 5 6,0 6 7,2 7 8,4 8 9,6 9 10,8
361		751	763	775	787	799	811	823	835	847	859	
362		871	883	895	907	919	931	943	955	967	979	
363		991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56	110	122	134	146	158	170	182	194	205	217	
365		229	241	253	265	277	289	301	312	324	336	
366		348	360	372	384	396	407	419	431	443	455	
367		467	478	490	502	514	526	538	549	561	573	
368		585	597	608	620	632	644	656	667	679	691	
369		703	714	726	738	750	761	773	785	797	808	
370		820	832	844	855	867	879	891	902	914	926	11 1 1,1 2 2,2 3 3,3 4 4,4 5 5,5 6 6,6 7 7,7 8 8,8 9 9,9
371		937	949	961	972	984	996	*008	*019	*031	*043	
372	57	054	066	078	089	101	113	124	136	148	159	
373		171	183	194	206	217	229	241	252	264	276	
374		287	299	310	322	334	345	357	368	380	392	
375		403	415	426	438	449	461	473	484	496	507	
376		519	530	542	553	565	576	588	600	611	623	
377		634	646	657	669	680	692	703	715	726	738	
378		749	761	772	784	795	807	818	830	841	852	
379		864	875	887	898	910	921	933	944	955	967	
380		978	990	*001	*013	*024	*035	*047	*058	*070	*081	10 1 1,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
381	58	092	104	115	127	138	149	161	172	184	195	
382		206	218	229	240	252	263	274	286	297	309	
383		320	331	343	354	365	377	388	399	410	422	
384		433	444	456	467	478	490	501	512	524	535	
385		546	557	569	580	591	602	614	625	636	647	
386		659	670	681	692	704	715	726	737	749	760	
387		771	782	794	805	816	827	838	850	861	872	
388		883	894	906	917	928	939	950	961	973	984	
389		995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59	106	118	129	140	151	162	173	184	195	207	9 1 1,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
391		218	229	240	251	262	273	284	295	306	318	
392		329	340	351	362	373	384	395	406	417	428	
393		439	450	461	472	483	494	506	517	528	539	
394		550	561	572	583	594	605	616	627	638	649	
395		660	671	682	693	704	715	726	737	748	759	
396		770	780	791	802	813	824	835	846	857	868	
397		879	890	901	912	923	934	945	956	966	977	
398		988	*999	*010	*021	*032	*043	*054	*065	*076	*086	
399	60	097	108	119	130	141	152	163	173	184	195	
400		206	217	228	239	249	260	271	282	293	304	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
400	60	206	217	228	239	249	260	271	282	293	304	11 1 1,1 2 2,2 3 3,3 4 4,4 5 5,5 6 6,6 7 7,7 8 8,8 9 9,9
401		314	325	336	347	358	369	379	390	401	412	
402		423	433	444	455	466	477	487	498	509	520	
403		531	541	552	563	574	584	595	606	617	627	
404		638	649	660	670	681	692	703	713	724	735	
405		746	756	767	778	788	799	810	821	831	842	
406		853	863	874	885	895	906	917	927	938	949	
407		959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61	066	077	087	098	109	119	130	140	151	162	
409		172	183	194	204	215	225	236	247	257	268	
410		278	289	300	310	321	331	342	352	363	374	10 1 1,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
411		384	395	405	416	426	437	448	458	469	479	
412		490	500	511	521	532	542	553	563	574	584	
413		595	606	616	627	637	648	658	669	679	690	
414		700	711	721	731	742	752	763	773	784	794	
415		805	815	826	836	847	857	868	878	888	899	
416		909	920	930	941	951	962	972	982	993	*003	
417	62	014	024	034	045	055	066	076	086	097	107	
418		118	128	138	149	159	170	180	190	201	211	
419		221	232	242	252	263	273	284	294	304	315	
420		325	335	346	356	366	377	387	397	408	418	9 1 0,9 2 1,8 3 2,7 4 3,6 5 4,5 6 5,4 7 6,3 8 7,2 9 8,1
421		428	439	449	459	469	480	490	500	511	521	
422		531	542	552	562	572	583	593	603	613	624	
423		634	644	655	665	675	685	696	706	716	726	
424		737	747	757	767	778	788	798	808	818	829	
425		839	849	859	870	880	890	900	910	921	931	
426		941	951	961	972	982	992	*002	*012	*022	*033	
427	63	043	053	063	073	083	094	104	114	124	134	
428		144	155	165	175	185	195	205	215	225	236	
429		246	256	266	276	286	296	306	317	327	337	
430		347	357	367	377	387	397	407	417	428	438	8 1 0,8 2 1,7 3 2,6 4 3,5 5 4,4 6 5,3 7 6,2 8 7,1
431		448	458	468	478	488	498	508	518	528	538	
432		548	558	568	579	589	599	609	619	629	639	
433		649	659	669	679	689	699	709	719	729	739	
434		749	759	769	779	789	799	809	819	829	839	
435		849	859	869	879	889	899	909	919	929	939	
436		949	959	969	979	988	998	*008	*018	*028	*038	
437	64	048	058	068	078	088	098	108	118	128	137	
438		147	157	167	177	187	197	207	217	227	237	
439		246	256	266	276	286	296	306	316	326	335	
440		345	355	365	375	385	395	404	414	424	434	7 1 0,7 2 1,6 3 2,5 4 3,4 5 4,3 6 5,2 7 6,1
441		444	454	464	473	483	493	503	513	523	532	
442		542	552	562	572	582	591	601	611	621	631	
443		640	650	660	670	680	689	699	709	719	729	
444		738	748	758	768	777	787	797	807	816	826	
445		836	846	856	865	875	885	895	904	914	924	
446		933	943	953	963	972	982	992	*002	*011	*021	
447	65	031	040	050	060	070	079	089	099	108	118	
448		128	137	147	157	167	176	186	196	205	215	
449		225	234	244	254	263	273	283	292	302	312	
450		321	331	341	350	360	369	379	389	398	408	6 1 0,6 2 1,5 3 2,4 4 3,3 5 4,2 6 5,1
N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
450	65	32I	33I	34I	350	360	369	379	389	398	408	10 1 I,0 2 2,0 3 3,0 4 4,0 5 5,0 6 6,0 7 7,0 8 8,0 9 9,0
451		4I8	427	437	447	456	466	475	485	495	504	
452		5I4	523	533	543	552	562	57I	58I	59I	600	
453		6I0	6I9	629	639	648	658	667	677	686	696	
454		706	7I5	725	734	744	753	763	772	782	792	
455		80I	8II	820	830	839	849	858	868	877	887	
456		896	906	9I6	925	935	944	954	963	973	982	
457		992	*00I	*0II	*020	*030	*039	*049	*058	*068	*077	
458	66	087	096	IO6	II5	124	134	143	153	162	172	
459		18I	19I	200	2IO	2I9	229	238	247	257	266	
460		276	285	295	304	3I4	323	332	342	35I	36I	9 1 0,9 2 I,8 3 2,7 4 3,6 5 4,5 6 5,4 7 6,3 8 7,2 9 8,I
461		370	380	389	398	408	417	427	436	445	455	
462		464	474	483	492	502	51I	52I	530	539	549	
463		558	567	577	586	596	605	6I4	624	633	642	
464		652	66I	67I	680	689	699	708	7I7	727	736	
465		745	755	764	773	783	792	80I	8II	820	829	
466		839	848	857	867	876	885	894	904	913	922	
467		932	94I	950	960	969	978	987	997	*006	*0I5	
468	67	025	034	043	052	062	07I	080	089	099	IO8	
469		II7	127	136	145	154	164	173	182	19I	20I	
470		2IO	2I9	228	237	247	256	265	274	284	293	8 1 0,8 2 I,6 3 2,4 4 3,2 5 4,0 6 4,8 7 5,6 8 6,4 9 7,2
471		302	3II	32I	330	339	348	357	367	376	385	
472		394	403	413	422	43I	440	449	459	468	477	
473		486	495	504	514	523	532	54I	550	560	569	
474		578	587	596	605	6I4	624	633	642	65I	660	
475		669	679	688	697	706	715	724	733	742	752	
476		76I	770	779	788	797	806	815	825	834	843	
477		852	86I	870	879	888	897	906	916	925	934	
478		943	952	96I	970	979	988	997	*006	*015	*024	
479	68	034	043	052	06I	070	079	088	097	IO6	II5	
480		124	133	142	15I	160	169	178	187	196	205	7 1 0,7 2 I,5 3 2,3 4 3,1 5 3,9 6 4,7 7 5,5 8 6,3 9 7,1
481		215	224	233	242	25I	260	269	278	287	296	
482		305	314	323	332	34I	350	359	368	377	386	
483		395	404	413	422	43I	440	449	458	467	476	
484		485	494	502	51I	520	529	538	547	556	565	
485		574	583	592	60I	610	619	628	637	646	655	
486		664	673	68I	690	699	708	717	726	735	744	
487		753	762	77I	780	789	797	806	815	824	833	
488		842	85I	860	869	878	886	895	904	913	922	
489		93I	940	949	958	966	975	984	993	*002	*01I	
490	69	020	028	037	046	055	064	073	082	090	099	6 1 0,6 2 I,4 3 2,2 4 3,0 5 3,8 6 4,6 7 5,4 8 6,2 9 7,0
491		108	117	126	135	144	152	161	170	179	188	
492		197	205	214	223	232	24I	249	258	267	276	
493		285	294	302	31I	320	329	338	346	355	364	
494		373	38I	390	399	408	417	425	434	443	452	
495		46I	469	478	487	496	504	513	522	53I	539	
496		548	557	566	574	583	592	60I	609	618	627	
497		636	644	653	662	67I	679	688	697	705	714	
498		723	732	740	749	758	767	775	784	793	80I	
499		810	819	827	836	845	854	862	87I	880	888	
500		897	906	914	923	932	940	949	958	966	975	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
500	69	897	906	914	923	932	940	949	958	966	975	9 1 0,9 2 1,8 3 2,7 4 3,6 5 4,5 6 5,4 7 6,3 8 7,2 9 8,1
501		984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70	070	079	088	096	105	114	122	131	140	148	
503		157	165	174	183	191	200	209	217	226	234	
504		243	252	260	269	278	286	295	303	312	321	
505		329	338	346	355	364	372	381	389	398	406	
506		415	424	432	441	449	458	467	475	484	492	
507		501	509	518	526	535	544	552	561	569	578	
508		586	595	603	612	621	629	638	646	655	663	
509		672	680	689	697	706	714	723	731	740	749	
510		757	766	774	783	791	800	808	817	825	834	8 1 0,8 2 1,6 3 2,4 4 3,2 5 4,0 6 4,8 7 5,6 8 6,4 9 7,2
511		842	851	859	868	876	885	893	902	910	919	
512		927	935	944	952	961	969	978	986	995	*003	
513	71	012	020	029	037	046	054	063	071	079	088	
514		096	105	113	122	130	139	147	155	164	172	
515		181	189	198	206	214	223	231	240	248	257	
516		265	273	282	290	299	307	315	324	332	341	
517		349	357	366	374	383	391	399	408	416	425	
518		433	441	450	458	466	475	483	492	500	508	
519		517	525	533	542	550	559	567	575	584	592	
520		600	609	617	625	634	642	650	659	667	675	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
521		684	692	700	709	717	725	734	742	750	759	
522		767	775	784	792	800	809	817	825	834	842	
523		850	858	867	875	883	892	900	908	917	925	
524		933	941	950	958	966	975	983	991	999	*008	
525	72	016	024	032	041	049	057	066	074	082	090	
526		099	107	115	123	132	140	148	156	165	173	
527		181	189	198	206	214	222	230	239	247	255	
528		263	272	280	288	296	304	313	321	329	337	
529		346	354	362	370	378	387	395	403	411	419	
530		428	436	444	452	460	469	477	485	493	501	6 1 0,6 2 1,3 3 2,0 4 2,7 5 3,4 6 4,1 7 4,8 8 5,5 9 6,2
531		509	518	526	534	542	550	558	567	575	583	
532		591	599	607	616	624	632	640	648	656	665	
533		673	681	689	697	705	713	722	730	738	746	
534		754	762	770	779	787	795	803	811	819	827	
535		835	843	852	860	868	876	884	892	900	908	
536		916	925	933	941	949	957	965	973	981	989	
537		997	*006	*014	*022	*030	*038	*046	*054	*062	*070	
538	73	078	086	094	102	111	119	127	135	143	151	
539		159	167	175	183	191	199	207	215	223	231	
540		239	247	255	263	272	280	288	296	304	312	5 1 0,5 2 1,1 3 1,7 4 2,3 5 2,9 6 3,5 7 4,1 8 4,7 9 5,3
541		320	328	336	344	352	360	368	376	384	392	
542		400	408	416	424	432	440	448	456	464	472	
543		480	488	496	504	512	520	528	536	544	552	
544		560	568	576	584	592	600	608	616	624	632	
545		640	648	656	664	672	679	687	695	703	711	
546		719	727	735	743	751	759	767	775	783	791	
547		799	807	815	823	830	838	846	854	862	870	
548		878	886	894	902	910	918	926	933	941	949	
549		957	965	973	981	989	997	*005	*013	*020	*028	
550	74	036	044	052	060	068	076	084	092	099	107	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
550	74	036	044	052	060	068	076	084	092	099	107	8 1 0,8 2 1,6 3 2,4 4 3,2 5 4,0 6 4,8 7 5,6 8 6,4 9 7,2
551		115	123	131	139	147	155	162	170	178	186	
552		194	202	210	218	225	233	241	249	257	265	
553		273	280	288	296	304	312	320	327	335	343	
554		351	359	367	374	382	390	398	406	414	421	
555		429	437	445	453	461	468	476	484	492	500	
556		507	515	523	531	539	547	554	562	570	578	
557		586	593	601	609	617	624	632	640	648	656	
558		663	671	679	687	695	702	710	718	726	733	
559		741	749	757	764	772	780	788	796	803	811	
560		819	827	834	842	850	858	865	873	881	889	8 1 0,8 2 1,6 3 2,4 4 3,2 5 4,0 6 4,8 7 5,6 8 6,4 9 7,2
561		896	904	912	920	927	935	943	950	958	966	
562		974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75	051	059	066	074	082	089	097	105	113	120	
564		128	136	143	151	159	166	174	182	189	197	
565		205	213	220	228	236	243	251	259	266	274	
566		282	289	297	305	312	320	328	335	343	351	
567		358	366	374	381	389	397	404	412	420	427	
568		435	442	450	458	465	473	481	488	496	504	
569		511	519	526	534	542	549	557	565	572	580	
570		587	595	603	610	618	626	633	641	648	656	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
571		664	671	679	686	694	702	709	717	724	732	
572		740	747	755	762	770	778	785	793	800	808	
573		815	823	831	838	846	853	861	868	876	884	
574		891	899	906	914	921	929	937	944	952	959	
575		967	974	982	989	997	*005	*012	*020	*027	*035	
576	76	042	050	057	065	072	080	087	095	103	110	
577		118	125	133	140	148	155	163	170	178	185	
578		193	200	208	215	223	230	238	245	253	260	
579		268	275	283	290	298	305	313	320	328	335	
580		343	350	358	365	373	380	388	395	403	410	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
581		418	425	433	440	448	455	462	470	477	485	
582		492	500	507	515	522	530	537	545	552	559	
583		567	574	582	589	597	604	612	619	626	634	
584		641	649	656	664	671	678	686	693	701	708	
585		716	723	730	738	745	753	760	768	775	782	
586		790	797	805	812	819	827	834	842	849	856	
587		864	871	879	886	893	901	908	916	923	930	
588		938	945	953	960	967	975	982	989	997	*004	
589	77	012	019	026	034	041	048	056	063	070	078	
590		085	093	100	107	115	122	129	137	144	151	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
591		159	166	173	181	188	195	203	210	217	225	
592		232	240	247	254	262	269	276	283	291	298	
593		305	313	320	327	335	342	349	357	364	371	
594		379	386	393	401	408	415	422	430	437	444	
595		452	459	466	474	481	488	495	503	510	517	
596		525	532	539	546	554	561	568	576	583	590	
597		597	605	612	619	627	634	641	648	656	663	
598		670	677	685	692	699	706	714	721	728	735	
599		743	750	757	764	772	779	786	793	801	808	
600		815	822	830	837	844	851	859	866	873	880	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
600	77	815	822	830	837	844	851	859	866	873	880	8 1 0,8 2 1,6 3 2,4 4 3,2 5 4,0 6 4,8 7 5,6 8 6,4 9 7,2
601		887	895	902	909	916	924	931	938	945	952	
602		960	967	974	981	988	996	*003	*010	*017	*025	
603	78	032	039	046	053	061	068	075	082	089	097	
604		104	111	118	125	132	140	147	154	161	168	
605		176	183	190	197	204	211	219	226	233	240	
606		247	254	262	269	276	283	290	297	305	312	
607		319	326	333	340	347	355	362	369	376	383	
608		390	398	405	412	419	426	433	440	447	455	
609		462	469	476	483	490	497	504	512	519	526	
610		533	540	547	554	561	569	576	583	590	597	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
611		604	611	618	625	633	640	647	654	661	668	
612		675	682	689	696	704	711	718	725	732	739	
613		746	753	760	767	774	781	789	796	803	810	
614		817	824	831	838	845	852	859	866	873	880	
615		888	895	902	909	916	923	930	937	944	951	
616		958	965	972	979	986	993	*000	*007	*014	*021	
617	79	029	036	043	050	057	064	071	078	085	092	
618		099	106	113	120	127	134	141	148	155	162	
619		169	176	183	190	197	204	211	218	225	232	
620		239	246	253	260	267	274	281	288	295	302	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
621		309	316	323	330	337	344	351	358	365	372	
622		379	386	393	400	407	414	421	428	435	442	
623		449	456	463	470	477	484	491	498	505	511	
624		518	525	532	539	546	553	560	567	574	581	
625		588	595	602	609	616	623	630	637	644	650	
626		657	664	671	678	685	692	699	706	713	720	
627		727	734	741	748	754	761	768	775	782	789	
628		796	803	810	817	824	831	837	844	851	858	
629		865	872	879	886	893	900	906	913	920	927	
630		934	941	948	955	962	969	975	982	989	996	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
631	80	003	010	017	024	030	037	044	051	058	065	
632		072	079	085	092	099	106	113	120	127	134	
633		140	147	154	161	168	175	182	188	195	202	
634		209	216	223	229	236	243	250	257	264	271	
635		277	284	291	298	305	312	318	325	332	339	
636		346	353	359	366	373	380	387	393	400	407	
637		414	421	428	434	441	448	455	462	468	475	
638		482	489	496	502	509	516	523	530	536	543	
639		550	557	564	570	577	584	591	598	604	611	
640		618	625	632	638	645	652	659	665	672	679	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
641		686	693	699	706	713	720	726	733	740	747	
642		754	760	767	774	781	787	794	801	808	814	
643		821	828	835	841	848	855	862	868	875	882	
644		889	895	902	909	916	922	929	936	943	949	
645		956	963	969	976	983	990	996	*003	*010	*017	
646	81	023	030	037	043	050	057	064	070	077	084	
647		090	097	104	111	117	124	131	137	144	151	
648		158	164	171	178	184	191	198	204	211	218	
649		224	231	238	245	251	258	265	271	278	285	
650		291	298	305	311	318	325	331	338	345	351	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
650	81	291	298	305	311	318	325	331	338	345	351	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
651		358	365	371	378	385	391	398	405	411	418	
652		425	431	438	445	451	458	465	471	478	485	
653		491	498	505	511	518	525	531	538	544	551	
654		558	564	571	578	584	591	598	604	611	617	
655		624	631	637	644	651	657	664	671	677	684	
656		690	697	704	710	717	723	730	737	743	750	
657		757	763	770	776	783	790	796	803	809	816	
658		823	829	836	842	849	856	862	869	875	882	
659		889	895	902	908	915	921	928	935	941	948	
660		954	961	968	974	981	987	994	*000	*007	*014	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
661	82	020	027	033	040	046	053	060	066	073	079	
662		086	092	099	105	112	119	125	132	138	145	
663		151	158	164	171	178	184	191	197	204	210	
664		217	223	230	236	243	249	256	263	269	276	
665		282	289	295	302	308	315	321	328	334	341	
666		347	354	360	367	373	380	387	393	400	406	
667		413	419	426	432	439	445	452	458	465	471	
668		478	484	491	497	504	510	517	523	530	536	
669		543	549	556	562	569	575	582	588	595	601	
670		607	614	620	627	633	640	646	653	659	666	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
671		672	679	685	692	698	705	711	718	724	730	
672		737	743	750	756	763	769	776	782	789	795	
673		802	808	814	821	827	834	840	847	853	860	
674		866	872	879	885	892	898	905	911	918	924	
675		930	937	943	950	956	963	969	975	982	988	
676		995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83	059	065	072	078	085	091	097	104	110	117	
678		123	129	136	142	149	155	161	168	174	181	
679		187	193	200	206	213	219	225	232	238	245	
680		251	257	264	270	276	283	289	296	302	308	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
681		315	321	327	334	340	347	353	359	366	372	
682		378	385	391	398	404	410	417	423	429	436	
683		442	448	455	461	467	474	480	487	493	499	
684		506	512	518	525	531	537	544	550	556	563	
685		569	575	582	588	594	601	607	613	620	626	
686		632	639	645	651	658	664	670	677	683	689	
687		696	702	708	715	721	727	734	740	746	753	
688		759	765	771	778	784	790	797	803	809	816	
689		822	828	835	841	847	853	860	866	872	879	
690		885	891	897	904	910	916	923	929	935	942	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
691		948	954	960	967	973	979	985	992	998	*004	
692	84	011	017	023	029	036	042	048	055	061	067	
693		073	080	086	092	098	105	111	117	123	130	
694		136	142	148	155	161	167	173	180	186	192	
695		198	205	211	217	223	230	236	242	248	255	
696		261	267	273	280	286	292	298	305	311	317	
697		323	330	336	342	348	354	361	367	373	379	
698		386	392	398	404	410	417	423	429	435	442	
699		448	454	460	466	473	479	485	491	497	504	
700		510	516	522	528	535	541	547	553	559	566	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
700	84	510	516	522	528	535	541	547	553	559	566	7 1 0,7 2 1,4 3 2,1 4 2,8 5 3,5 6 4,2 7 4,9 8 5,6 9 6,3
701		572	578	584	590	597	603	609	615	621	628	
702		634	640	646	652	658	665	671	677	683	689	
703		696	702	708	714	720	726	733	739	745	751	
704		757	763	770	776	782	788	794	800	807	813	
705		819	825	831	837	844	850	856	862	868	874	
706		880	887	893	899	905	911	917	924	930	936	
707		942	948	954	960	967	973	979	985	991	997	
708	85	003	009	016	022	028	034	040	046	052	058	
709		065	071	077	083	089	095	101	107	114	120	
710		126	132	138	144	150	156	163	169	175	181	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
711		187	193	199	205	211	217	224	230	236	242	
712		248	254	260	266	272	278	285	291	297	303	
713		309	315	321	327	333	339	345	352	358	364	
714		370	376	382	388	394	400	406	412	418	425	
715		431	437	443	449	455	461	467	473	479	485	
716		491	497	503	509	516	522	528	534	540	546	
717		552	558	564	570	576	582	588	594	600	606	
718		612	618	625	631	637	643	649	655	661	667	
719		673	679	685	691	697	703	709	715	721	727	
720		733	739	745	751	757	763	769	775	781	788	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
721		794	800	806	812	818	824	830	836	842	848	
722		854	860	866	872	878	884	890	896	902	908	
723		914	920	926	932	938	944	950	956	962	968	
724		974	980	986	992	998	*004	*010	*016	*022	*028	
725	86	034	040	046	052	058	064	070	076	082	088	
726		094	100	106	112	118	124	130	136	141	147	
727		153	159	165	171	177	183	189	195	201	207	
728		213	219	225	231	237	243	249	255	261	267	
729		273	279	285	291	297	303	308	314	320	326	
730		332	338	344	350	356	362	368	374	380	386	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
731		392	398	404	410	415	421	427	433	439	445	
732		451	457	463	469	475	481	487	493	499	504	
733		510	516	522	528	534	540	546	552	558	564	
734		570	576	581	587	593	599	605	611	617	623	
735		629	635	641	646	652	658	664	670	676	682	
736		688	694	700	705	711	717	723	729	735	741	
737		747	753	759	764	770	776	782	788	794	800	
738		806	812	817	823	829	835	841	847	853	859	
739		864	870	876	882	888	894	900	906	911	917	
740		923	929	935	941	947	953	958	964	970	976	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
741		982	988	994	999	*005	*011	*017	*023	*029	*035	
742	87	040	046	052	058	064	070	075	081	087	093	
743		099	105	111	116	122	128	134	140	146	151	
744		157	163	169	175	181	186	192	198	204	210	
745		216	221	227	233	239	245	251	256	262	268	
746		274	280	286	291	297	303	309	315	320	326	
747		332	338	344	349	355	361	367	373	379	384	
748		390	396	402	408	413	419	425	431	437	442	
749		448	454	460	466	471	477	483	489	495	500	
750		506	512	518	523	529	535	541	547	552	558	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
750	87	506	512	518	523	529	535	541	547	552	558	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
751		564	570	576	581	587	593	599	604	610	616	
752		622	628	633	639	645	651	656	662	668	674	
753		679	685	691	697	703	708	714	720	726	731	
754		737	743	749	754	760	766	772	777	783	789	
755		795	800	806	812	818	823	829	835	841	846	
756		852	858	864	869	875	881	887	892	898	904	
757		910	915	921	927	933	938	944	950	955	961	
758		967	973	978	984	990	996	*001	*007	*013	*018	
759	88	024	030	036	041	047	053	058	064	070	076	
760		081	087	093	098	104	110	116	121	127	133	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
761		138	144	150	156	161	167	173	178	184	190	
762		195	201	207	213	218	224	230	235	241	247	
763		252	258	264	270	275	281	287	292	298	304	
764		309	315	321	326	332	338	343	349	355	360	
765		366	372	377	383	389	395	400	406	412	417	
766		423	429	434	440	446	451	457	463	468	474	
767		480	485	491	497	502	508	513	519	525	530	
768		536	542	547	553	559	564	570	576	581	587	
769		593	598	604	610	615	621	627	632	638	643	
770		649	655	660	666	672	677	683	689	694	700	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
771		705	711	717	722	728	734	739	745	750	756	
772		762	767	773	779	784	790	795	801	807	812	
773		818	824	829	835	840	846	852	857	863	868	
774		874	880	885	891	897	902	908	913	919	925	
775		930	936	941	947	953	958	964	969	975	981	
776		986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89	042	048	053	059	064	070	076	081	087	092	
778		098	104	109	115	120	126	131	137	143	148	
779		154	159	165	170	176	182	187	193	198	204	
780		209	215	221	226	232	237	243	248	254	260	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
781		265	271	276	282	287	293	298	304	310	315	
782		321	326	332	337	343	348	354	360	365	371	
783		376	382	387	393	398	404	409	415	421	426	
784		432	437	443	448	454	459	465	470	476	481	
785		487	492	498	504	509	515	520	526	531	537	
786		542	548	553	559	564	570	575	581	586	592	
787		597	603	609	614	620	625	631	636	642	647	
788		653	658	664	669	675	680	686	691	697	702	
789		708	713	719	724	730	735	741	746	752	757	
790		763	768	774	779	785	790	796	801	807	812	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
791		818	823	829	834	840	845	851	856	862	867	
792		873	878	883	889	894	900	905	911	916	922	
793		927	933	938	944	949	955	960	966	971	977	
794		982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90	037	042	048	053	059	064	069	075	080	086	
796		091	097	102	108	113	119	124	129	135	140	
797		146	151	157	162	168	173	179	184	189	195	
798		200	206	211	217	222	227	233	238	244	249	
799		255	260	266	271	276	282	287	293	298	304	
800		309	314	320	325	331	336	342	347	352	358	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
800	90	309	314	320	325	331	336	342	347	352	358	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
801		363	369	374	380	385	390	396	401	407	412	
802		417	423	428	434	439	445	450	455	461	466	
803		472	477	482	488	493	499	504	509	515	520	
804		526	531	536	542	547	553	558	563	569	574	
805		580	585	590	596	601	607	612	617	623	628	
806		634	639	644	650	655	660	666	671	677	682	
807		687	693	698	703	709	714	720	725	730	736	
808		741	747	752	757	763	768	773	779	784	789	
809		795	800	806	811	816	822	827	832	838	843	
810		849	854	859	865	870	875	881	886	891	897	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
811		902	907	913	918	924	929	934	940	945	950	
812		956	961	966	972	977	982	988	993	998	*004	
813	91	009	014	020	025	030	036	041	046	052	057	
814		062	068	073	078	084	089	094	100	105	110	
815		116	121	126	132	137	142	148	153	158	164	
816		169	174	180	185	190	196	201	206	212	217	
817		222	228	233	238	243	249	254	259	265	270	
818		275	281	286	291	297	302	307	312	318	323	
819		328	334	339	344	350	355	360	365	371	376	
820		381	387	392	397	403	408	413	418	424	429	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
821		434	440	445	450	455	461	466	471	477	482	
822		487	492	498	503	508	514	519	524	529	535	
823		540	545	551	556	561	566	572	577	582	587	
824		593	598	603	609	614	619	624	630	635	640	
825		645	651	656	661	666	672	677	682	687	693	
826		698	703	709	714	719	724	730	735	740	745	
827		751	756	761	766	772	777	782	787	793	798	
828		803	808	814	819	824	829	834	840	845	850	
829		855	861	866	871	876	882	887	892	897	903	
830		908	913	918	924	929	934	939	944	950	955	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
831		960	965	971	976	981	986	991	997	*002	*007	
832	92	012	018	023	028	033	038	044	049	054	059	
833		065	070	075	080	085	091	096	101	106	111	
834		117	122	127	132	137	143	148	153	158	163	
835		169	174	179	184	189	195	200	205	210	215	
836		221	226	231	236	241	247	252	257	262	267	
837		273	278	283	288	293	298	304	309	314	319	
838		324	330	335	340	345	350	355	361	366	371	
839		376	381	387	392	397	402	407	412	418	423	
840		428	433	438	443	449	454	459	464	469	474	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
841		480	485	490	495	500	505	511	516	521	526	
842		531	536	542	547	552	557	562	567	572	578	
843		583	588	593	598	603	609	614	619	624	629	
844		634	639	645	650	655	660	665	670	675	681	
845		686	691	696	701	706	711	716	722	727	732	
846		737	742	747	752	758	763	768	773	778	783	
847		788	793	799	804	809	814	819	824	829	834	
848		840	845	850	855	860	865	870	875	881	886	
849		891	896	901	906	911	916	921	927	932	937	
850		942	947	952	957	962	967	973	978	983	988	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.	
850		92	942	947	952	957	962	967	973	978	983	988	6 1 0,6 2 1,2 3 1,8 4 2,4 5 3,0 6 3,6 7 4,2 8 4,8 9 5,4
851			993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93	044	049	054	059	064	069	075	080	085	090		
853			095	100	105	110	115	120	125	131	136	141	
854			146	151	156	161	166	171	176	181	186	192	
855			197	202	207	212	217	222	227	232	237	242	
856			247	252	258	263	268	273	278	283	288	293	
857			298	303	308	313	318	323	328	333	339	344	
858			349	354	359	364	369	374	379	384	389	394	
859			399	404	409	414	420	425	430	435	440	445	
860			450	455	460	465	470	475	480	485	490	495	
861			500	505	510	515	520	526	531	536	541	546	
862			551	556	561	566	571	576	581	586	591	596	
863			601	606	611	616	621	626	631	636	641	646	
864			651	656	661	666	671	676	682	687	692	697	
865			702	707	712	717	722	727	732	737	742	747	
866			752	757	762	767	772	777	782	787	792	797	
867			802	807	812	817	822	827	832	837	842	847	
868			852	857	862	867	872	877	882	887	892	897	
869			902	907	912	917	922	927	932	937	942	947	
870			952	957	962	967	972	977	982	987	992	997	
871	94	002	007	012	017	022	027	032	037	042	047	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5	
872		052	057	062	067	072	077	082	086	091	096		
873		101	106	111	116	121	126	131	136	141	146		
874		151	156	161	166	171	176	181	186	191	196		
875		201	206	211	216	221	226	231	236	240	245		
876		250	255	260	265	270	275	280	285	290	295		
877		300	305	310	315	320	325	330	335	340	345		
878		349	354	359	364	369	374	379	384	389	394		
879		399	404	409	414	419	424	429	433	438	443		
880			448	453	458	463	468	473	478	483	488		493
881			498	503	507	512	517	522	527	532	537	542	
882			547	552	557	562	567	571	576	581	586	591	
883			596	601	606	611	616	621	626	630	635	640	
884			645	650	655	660	665	670	675	680	685	689	
885			694	699	704	709	714	719	724	729	734	738	
886			743	748	753	758	763	768	773	778	783	787	
887			792	797	802	807	812	817	822	827	832	836	
888			841	846	851	856	861	866	871	876	880	885	
889			890	895	900	905	910	915	919	924	929	934	
890			939	944	949	954	959	963	968	973	978	983	
891			988	993	998	*002	*007	*012	*017	*022	*027	*032	
892	95	036	041	046	051	056	061	066	071	075	080	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6	
893		085	090	095	100	105	109	114	119	124	129		
894		134	139	143	148	153	158	163	168	173	177		
895			182	187	192	197	202	207	211	216	221		226
896			231	236	240	245	250	255	260	265	270		274
897			279	284	289	294	299	303	308	313	318		323
898			328	332	337	342	347	352	357	361	366		371
899			376	381	386	390	395	400	405	410	415		419
900			424	429	434	439	444	448	453	458	463		468
N.	L.	0	1	2	3	4	5	6	7	8	9		P. P.

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
900	95	424	429	434	439	444	448	453	458	463	468	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
901		472	477	482	487	492	497	501	506	511	516	
902		521	525	530	535	540	545	550	554	559	564	
903		569	574	578	583	588	593	598	602	607	612	
904		617	622	626	631	636	641	646	650	655	660	
905		665	670	674	679	684	689	694	698	703	708	
906		713	718	722	727	732	737	742	746	751	756	
907		761	766	770	775	780	785	789	794	799	804	
908		809	813	818	823	828	832	837	842	847	852	
909		856	861	866	871	875	880	885	890	895	899	
910		904	909	914	918	923	928	933	938	942	947	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
911		952	957	961	966	971	976	980	985	990	995	
912		999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96	047	052	057	061	066	071	076	080	085	090	
914		095	099	104	109	114	118	123	128	133	137	
915		142	147	152	156	161	166	171	175	180	185	
916		190	194	199	204	209	213	218	223	227	232	
917		237	242	246	251	256	261	265	270	275	280	
918		284	289	294	298	303	308	313	317	322	327	
919		332	336	341	346	350	355	360	365	369	374	
920		379	384	388	393	398	402	407	412	417	421	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
921		426	431	435	440	445	450	454	459	464	468	
922		473	478	483	487	492	497	501	506	511	515	
923		520	525	530	534	539	544	548	553	558	562	
924		567	572	577	581	586	591	595	600	605	609	
925		614	619	624	628	633	638	642	647	652	656	
926		661	666	670	675	680	685	689	694	699	703	
927		708	713	717	722	727	731	736	741	745	750	
928		755	759	764	769	774	778	783	788	792	797	
929		802	806	811	816	820	825	830	834	839	844	
930		848	853	858	862	867	872	876	881	886	890	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
931		895	900	904	909	914	918	923	928	932	937	
932		942	946	951	956	960	965	970	974	979	984	
933		988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97	035	039	044	049	053	058	063	067	072	077	
935		081	086	090	095	100	104	109	114	118	123	
936		128	132	137	142	146	151	155	160	165	169	
937		174	179	183	188	192	197	202	206	211	216	
938		220	225	230	234	239	243	248	253	257	262	
939		267	271	276	280	285	290	294	299	304	308	
940		313	317	322	327	331	336	340	345	350	354	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
941		359	364	368	373	377	382	387	391	396	400	
942		405	410	414	419	424	428	433	437	442	447	
943		451	456	460	465	470	474	479	483	488	493	
944		497	502	506	511	516	520	525	529	534	539	
945		543	548	552	557	562	566	571	575	580	585	
946		589	594	598	603	607	612	617	621	626	630	
947		635	640	644	649	653	658	663	667	672	676	
948		681	685	690	695	699	704	708	713	717	722	
949		727	731	736	740	745	749	754	759	763	768	
950		772	777	782	786	791	795	800	804	809	813	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

N.	L.	0	1	2	3	4	5	6	7	8	9	P. P.
950	97	772	777	782	786	791	795	800	804	809	813	5 1 0,5 2 1,0 3 1,5 4 2,0 5 2,5 6 3,0 7 3,5 8 4,0 9 4,5
951		818	823	827	832	836	841	845	850	855	859	
952		864	868	873	877	882	886	891	896	900	905	
953		909	914	918	923	928	932	937	941	946	950	
954		955	959	964	968	973	978	982	987	991	996	
955	98	000	005	009	014	019	023	028	032	037	041	
956		046	050	055	059	064	068	073	078	082	087	
957		091	096	100	105	109	114	118	123	127	132	
958		137	141	146	150	155	159	164	168	173	177	
959		182	186	191	195	200	204	209	214	218	223	
960		227	232	236	241	245	250	254	259	263	268	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
961		272	277	281	286	290	295	299	304	308	313	
962		318	322	327	331	336	340	345	349	354	358	
963		363	367	372	376	381	385	390	394	399	403	
964		408	412	417	421	426	430	435	439	444	448	
965		453	457	462	466	471	475	480	484	489	493	
966		498	502	507	511	516	520	525	529	534	538	
967		543	547	552	556	561	565	570	574	579	583	
968		588	592	597	601	605	610	614	619	623	628	
969		632	637	641	646	650	655	659	664	668	673	
970		677	682	686	691	695	700	704	709	713	717	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
971		722	726	731	735	740	744	749	753	758	762	
972		767	771	776	780	784	789	793	798	802	807	
973		811	816	820	825	829	834	838	843	847	851	
974		856	860	865	869	874	878	883	887	892	896	
975		900	905	909	914	918	923	927	932	936	941	
976		945	949	954	958	963	967	972	976	981	985	
977		989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99	034	038	043	047	052	056	061	065	069	074	
979		078	083	087	092	096	100	105	109	114	118	
980		123	127	131	136	140	145	149	154	158	162	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
981		167	171	176	180	185	189	193	198	202	207	
982		211	216	220	224	229	233	238	242	247	251	
983		255	260	264	269	273	277	282	286	291	295	
984		300	304	308	313	317	322	326	330	335	339	
985		344	348	352	357	361	366	370	374	379	383	
986		388	392	396	401	405	410	414	419	423	427	
987		432	436	441	445	449	454	458	463	467	471	
988		476	480	484	489	493	498	502	506	511	515	
989		520	524	528	533	537	542	546	550	555	559	
990		564	568	572	577	581	585	590	594	599	603	4 1 0,4 2 0,8 3 1,2 4 1,6 5 2,0 6 2,4 7 2,8 8 3,2 9 3,6
991		607	612	616	621	625	629	634	638	642	647	
992		651	656	660	664	669	673	677	682	686	691	
993		695	699	704	708	712	717	721	726	730	734	
994		739	743	747	752	756	760	765	769	774	778	
995		782	787	791	795	800	804	808	813	817	822	
996		826	830	835	839	843	848	852	856	861	865	
997		870	874	878	883	887	891	896	900	904	909	
998		913	917	922	926	930	935	939	944	948	952	
999		957	961	965	970	974	978	983	987	991	996	
1000		00 000	004	009	013	017	022	026	030	035	039	P. P.
N.	L.	0	1	2	3	4	5	6	7	8	9	

NOTES ON TABLES I AND II.

The logarithms of numbers are in general incommensurable. In these tables they are given correct to five places of decimals. If the sixth place is 5 or more, the next larger number is used in the fifth place. Thus $\log 8102 = 3.908549+$; in five-place tables this is written $3.9085\bar{5}$, the dash above the 5 showing that the logarithm is less than given.

So $\log 8133 = 3.910251-$; in five-place tables this is written $3.9102\dot{5}$, the dot above the 5 showing that the logarithm is more than given.

In the natural functions of the angles (Table II) all numbers are decimals for sine and cosine (why?), and for tangent and cotangent, except where the decimal point is used to indicate that part of the number is integral. When no decimal point is printed in the tables it is to be understood. When the natural function is a pure decimal the characteristic of the logarithm is negative. Accordingly, in the tables 10 is added, and in the result this must be allowed for. Thus

$$\text{nat. sin } 44^\circ 20' = 0.69883, \quad \log \sin 44^\circ 20' = \bar{1}.84437,$$

or, as printed in the tables, 9.84437, which means $9.84437 - 10$.

TABLE II.

THE LOGARITHMIC AND NATURAL SINES, COSINES,
TANGENTS, AND COTANGENTS OF ANGLES
FROM 0° TO 90° .

	Nat. Sin Log.	d.	Nat. Cos Log.	Nat. Tan Log.	c. d.	Log. Cot Nat.	
0	00000 —		10000 0.00000	00000 —		∞	60
1	029 6.46373	30103	000 0.00000	029 6.46373	30103	3.53627	59
2	058 6.76476	17609	000 0.00000	058 6.76476	17609	3.23524	58
3	087 6.94085	12494	000 0.00000	087 6.94085	12494	3.05915	57
4	116 7.06579	9691	000 0.00000	116 7.06579	9691	2.93421	56
5	00145 7.16270	7918	10000 0.00000	00145 7.16270	7918	2.83730	55
6	175 7.24188	6694	000 0.00000	175 7.24188	6694	2.75812	54
7	204 7.30882	5800	000 0.00000	204 7.30882	5800	2.69118	53
8	233 7.36682	5115	000 0.00000	233 7.36682	5115	2.63318	52
9	262 7.41797	4576	000 0.00000	262 7.41797	4576	2.58203	51
10	00291 7.46373	4139	10000 0.00000	00291 7.46373	4139	2.53627	50
11	320 7.50512	3779	99999 0.00000	320 7.50512	3779	2.49488	49
12	349 7.54291	3476	999 0.00000	349 7.54291	3476	2.45709	48
13	378 7.57767	3218	999 0.00000	378 7.57767	3219	2.42233	47
14	407 7.60985	2997	999 0.00000	407 7.60985	2996	2.39014	46
15	00436 7.63982	2802	99999 0.00000	00436 7.63982	2803	2.36018	45
16	465 7.66784	2633	999 0.00000	465 7.66785	2633	2.33215	44
17	495 7.69417	2483	999 9.99999	495 7.69418	2482	2.30582	43
18	524 7.71900	2348	999 9.99999	524 7.71900	2348	2.28100	42
19	553 7.74248	2227	998 9.99999	553 7.74248	2228	2.25752	41
20	00582 7.76475	2119	99998 9.99999	00582 7.76476	2119	2.23524	40
21	611 7.78595	2021	998 9.99999	611 7.78595	2020	2.21405	39
22	640 7.80615	1930	998 9.99999	640 7.80615	1931	2.19385	38
23	669 7.82545	1848	998 9.99999	669 7.82546	1848	2.17454	37
24	698 7.84393	1773	998 9.99999	698 7.84394	1773	2.15606	36
25	00727 7.86160	1704	99997 9.99999	00727 7.86167	1704	2.13833	35
26	756 7.87870	1639	997 9.99999	756 7.87871	1639	2.12129	34
27	785 7.89509	1579	997 9.99999	785 7.89510	1579	2.10490	33
28	814 7.91088	1524	997 9.99999	815 7.91089	1524	2.08911	32
29	844 7.92612	1472	996 9.99998	844 7.92613	1473	2.07387	31
30	00873 7.94084	1424	99996 9.99998	00873 7.94086	1424	2.05914	30
31	902 7.95508	1379	996 9.99998	902 7.95510	1379	2.04490	29
32	931 7.96887	1336	996 9.99998	931 7.96889	1336	2.03111	28
33	960 7.98223	1297	995 9.99998	960 7.98225	1297	2.01775	27
34	989 7.99520	1259	995 9.99998	989 7.99522	1259	2.00478	26
35	01018 8.00779	1223	99995 9.99998	01018 8.00781	1223	1.99219	25
36	047 8.02002	1190	995 9.99998	047 8.02004	1190	1.97996	24
37	076 8.03192	1158	994 9.99997	076 8.03194	1159	1.96806	23
38	105 8.04350	1128	994 9.99997	105 8.04353	1128	1.95647	22
39	134 8.05478	1100	994 9.99997	135 8.05481	1100	1.94519	21
40	01164 8.06573	1072	99993 9.99997	01164 8.06581	1072	1.93419	20
41	193 8.07650	1046	993 9.99997	193 8.07653	1047	1.92347	19
42	222 8.08696	1022	993 9.99997	222 8.08700	1022	1.91300	18
43	251 8.09718	999	992 9.99997	251 8.09722	998	1.90278	17
44	280 8.10717	976	992 9.99996	280 8.10720	976	1.89280	16
45	01309 8.11693	954	99991 9.99996	01309 8.11696	955	1.88304	15
46	338 8.12647	934	991 9.99996	338 8.12651	934	1.87349	14
47	367 8.13581	914	991 9.99996	367 8.13585	915	1.86415	13
48	396 8.14495	896	990 9.99996	396 8.14500	895	1.85500	12
49	425 8.15391	877	990 9.99996	425 8.15395	878	1.84605	11
50	01454 8.16268	860	99989 9.99995	01455 8.16273	860	1.83727	10
51	483 8.17128	843	989 9.99995	484 8.17133	843	1.82867	9
52	513 8.17971	827	989 9.99995	513 8.17976	828	1.82024	8
53	542 8.18798	812	988 9.99995	542 8.18804	812	1.81196	7
54	571 8.19610	797	988 9.99995	571 8.19616	797	1.80384	6
55	01600 8.20407	782	99987 9.99994	01600 8.20413	782	1.79587	5
56	629 8.21189	769	987 9.99994	629 8.21195	769	1.78805	4
57	658 8.21958	755	986 9.99994	658 8.21964	756	1.78036	3
58	687 8.22713	743	986 9.99994	687 8.22720	742	1.77280	2
59	716 8.23456	730	985 9.99994	716 8.23462	730	1.76538	1
60	745 8.24186		985 9.99993	746 8.24192		1.75808	0
	Nat. Cos Log.	d.	Nat. Sin Log.	Nat. Cot Log.	c. d.	Log. Tan Nat.	

'	Nat. Sin Log.	d.	Nat. Cos Log.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	01745	8.24186	717	99985	0.99993	01746	8.24192	718	1.75808	57.290	60
1	774	8.24903	706	984	0.99993	775	8.24910	706	1.75090	56.351	59
2	803	8.25600	695	984	0.99993	804	8.25616	696	1.74384	55.442	58
3	832	8.26304	684	983	0.99993	833	8.26312	684	1.73688	54.561	57
4	862	8.26988	673	983	0.99992	862	8.26996	673	1.73004	53.709	56
5	01891	8.27661	663	99982	0.99992	01891	8.27669	663	1.72331	52.882	55
6	920	8.28324	653	982	0.99992	920	8.28332	654	1.71668	.081	54
7	949	8.28977	644	981	0.99992	949	8.28986	643	1.71014	51.303	53
8	978	8.29621	634	980	0.99992	978	8.29629	634	1.70371	50.549	52
9	02007	8.30255	624	980	0.99991	02007	8.30263	625	1.69737	49.816	51
10	02036	8.30879	616	99979	0.99991	02036	8.30888	617	1.69112	49.104	50
11	065	8.31495	608	979	0.99991	066	8.31505	607	1.68495	48.412	49
12	094	8.32103	599	978	0.99990	095	8.32112	599	1.67888	47.740	48
13	123	8.32702	590	977	0.99990	124	8.32711	591	1.67289	.085	47
14	152	8.33292	583	977	0.99990	153	8.33302	584	1.66698	46.449	46
15	02181	8.33875	575	99976	0.99990	02182	8.33886	575	1.66114	45.829	45
16	211	8.34450	568	976	0.99989	211	8.34461	570	1.65539	.226	44
17	240	8.35018	560	975	0.99989	240	8.35029	561	1.64971	44.639	43
18	269	8.35578	553	974	0.99989	269	8.35590	553	1.64410	.066	42
19	298	8.36131	547	974	0.99989	298	8.36143	546	1.63857	43.508	41
20	02327	8.36678	539	99973	0.99988	02328	8.36689	540	1.63311	42.964	40
21	356	8.37217	533	972	0.99988	357	8.37229	533	1.62771	.433	39
22	385	8.37750	526	972	0.99988	386	8.37762	527	1.62238	41.916	38
23	414	8.38276	520	971	0.99987	415	8.38289	520	1.61711	.411	37
24	443	8.38796	514	970	0.99987	444	8.38809	514	1.61191	40.917	36
25	02472	8.39310	508	99969	0.99987	02473	8.39323	509	1.60677	40.436	35
26	501	8.39818	502	969	0.99986	502	8.39832	502	1.60168	39.965	34
27	530	8.40320	496	968	0.99986	531	8.40334	496	1.59666	.506	33
28	560	8.40816	491	967	0.99986	560	8.40830	491	1.59170	.057	32
29	589	8.41307	485	966	0.99985	589	8.41321	486	1.58679	38.618	31
30	02618	8.41792	480	99966	0.99985	02619	8.41807	480	1.58193	38.188	30
31	647	8.42272	474	965	0.99985	648	8.42287	475	1.57713	37.769	29
32	676	8.42746	470	964	0.99984	677	8.42762	475	1.57238	.358	28
33	705	8.43216	464	963	0.99984	706	8.43232	464	1.56768	36.956	27
34	734	8.43680	459	963	0.99984	735	8.43696	460	1.56304	.563	26
35	02763	8.44139	455	99962	0.99983	02764	8.44156	455	1.55844	36.178	25
36	792	8.44594	450	961	0.99983	793	8.44611	450	1.55389	35.801	24
37	821	8.45044	445	960	0.99983	822	8.45061	446	1.54939	.431	23
38	850	8.45489	441	959	0.99982	851	8.45507	441	1.54493	.070	22
39	879	8.45930	436	959	0.99982	881	8.45948	437	1.54052	34.715	21
40	02908	8.46366	433	99958	0.99982	02910	8.46385	432	1.53615	34.368	20
41	938	8.46799	427	957	0.99981	939	8.46817	432	1.53183	.027	19
42	967	8.47226	424	956	0.99981	968	8.47245	424	1.52755	33.964	18
43	996	8.47650	419	955	0.99981	997	8.47669	420	1.52331	.366	17
44	03025	8.48069	416	954	0.99980	03026	8.48089	416	1.51911	.045	16
45	03054	8.48485	411	99953	0.99980	03055	8.48505	412	1.51495	32.730	15
46	083	8.48890	408	952	0.99979	084	8.48917	408	1.51083	.421	14
47	112	8.49304	404	952	0.99979	114	8.49325	404	1.50675	.118	13
48	141	8.49708	400	951	0.99979	143	8.49729	401	1.50271	31.821	12
49	170	8.50108	396	950	0.99978	172	8.50130	397	1.49870	.528	11
50	03199	8.50504	393	99949	0.99978	03201	8.50527	393	1.49473	31.242	10
51	228	8.50897	390	948	0.99977	230	8.50920	390	1.49080	30.960	9
52	257	8.51287	386	947	0.99977	259	8.51310	386	1.48690	.683	8
53	286	8.51673	382	946	0.99977	288	8.51696	383	1.48304	.412	7
54	316	8.52055	379	945	0.99976	317	8.52079	380	1.47921	.145	6
55	03345	8.52434	376	99944	0.99976	03346	8.52459	376	1.47541	29.882	5
56	374	8.52810	373	943	0.99975	376	8.52835	373	1.47165	.624	4
57	403	8.53183	369	942	0.99975	405	8.53208	370	1.46792	.371	3
58	432	8.53552	367	941	0.99974	434	8.53578	367	1.46422	.122	2
59	461	8.53910	363	940	0.99974	463	8.53945	363	1.46055	28.877	1
60	490	8.54282		939	0.99974	492	8.54308		1.45692	.636	0
	Nat. Cos Log.	d.	Nat. Sin Log.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'				

'	Nat. Sin Log.			Nat. Cos Log.			Nat. Tan Log.			c.d.	Log. Cot Nat.		
0	03490	8.54282	360	99939	0.99974	03492	8.54308	361	1.45692	28.636	60		
1	519	8.54642	357	938	0.99973	521	8.54669	358	1.45331	.399	59		
2	548	8.54999	355	937	0.99973	550	8.55027	355	1.44973	.166	58		
3	577	8.55354	351	936	0.99972	579	8.55382	352	1.44618	27.937	57		
4	606	8.55705	349	935	0.99972	609	8.55734	349	1.44266	.712	56		
5	03635	8.56054	346	99934	0.99971	03638	8.56083	346	1.43917	27.490	55		
6	664	8.56400	343	933	0.99971	667	8.56429	346	1.43571	.271	54		
7	693	8.56743	341	932	0.99970	696	8.56773	344	1.43227	.057	53		
8	723	8.57084	337	931	0.99970	725	8.57114	341	1.42886	26.845	52		
9	752	8.57421	336	930	0.99969	754	8.57452	338	1.42548	.637	51		
10	03781	8.57757	332	99929	0.99969	03783	8.57788	336	1.42212	26.432	50		
11	810	8.58089	330	927	0.99968	812	8.58121	333	1.41879	.230	49		
12	839	8.58419	328	926	0.99968	842	8.58451	330	1.41549	.031	48		
13	868	8.58747	325	925	0.99967	871	8.58779	326	1.41221	25.835	47		
14	897	8.59072	323	924	0.99967	900	8.59105	323	1.40895	.642	46		
15	03926	8.59395	320	99923	0.99967	03929	8.59428	321	1.40572	25.452	45		
16	955	8.59715	318	922	0.99966	958	8.59749	319	1.40251	.264	44		
17	984	8.60033	316	921	0.99966	987	8.60068	316	1.39932	.080	43		
18	04013	8.60349	313	919	0.99965	04016	8.60384	314	1.39616	24.898	42		
19	042	8.60662	311	918	0.99964	046	8.60698	311	1.39302	.719	41		
20	04071	8.60973	309	99917	0.99964	04075	8.61009	310	1.38991	24.542	40		
21	100	8.61282	307	916	0.99963	104	8.61319	307	1.38681	.368	39		
22	129	8.61589	305	915	0.99963	133	8.61626	305	1.38374	.196	38		
23	159	8.61894	302	913	0.99962	162	8.61931	303	1.38060	.026	37		
24	188	8.62196	301	912	0.99962	191	8.62234	301	1.37766	23.859	36		
25	04217	8.62497	298	99911	0.99961	04220	8.62535	299	1.37465	23.695	35		
26	246	8.62795	296	910	0.99961	250	8.62834	297	1.37166	.532	34		
27	275	8.63091	294	909	0.99960	279	8.63131	295	1.36860	.372	33		
28	304	8.63385	293	907	0.99960	308	8.63426	292	1.36574	.214	32		
29	333	8.63678	290	906	0.99959	337	8.63718	291	1.36282	.058	31		
30	04362	8.63968	288	99905	0.99959	04366	8.64009	289	1.35991	22.994	30		
31	391	8.64256	287	904	0.99958	395	8.64298	287	1.35702	.752	29		
32	420	8.64543	284	902	0.99958	424	8.64585	285	1.35415	.602	28		
33	449	8.64827	283	901	0.99957	454	8.64870	284	1.35130	.454	27		
34	478	8.65110	281	900	0.99956	483	8.65154	281	1.34846	.308	26		
35	04507	8.65391	279	99898	0.99956	04512	8.65435	280	1.34565	22.164	25		
36	536	8.65670	277	897	0.99955	541	8.65715	278	1.34285	.022	24		
37	565	8.65947	276	896	0.99955	570	8.65993	276	1.34007	21.881	23		
38	594	8.66223	274	894	0.99954	599	8.66269	274	1.33731	.743	22		
39	623	8.66497	272	893	0.99954	628	8.66543	271	1.33457	.606	21		
40	04653	8.66769	270	99892	0.99953	04658	8.66816	273	1.33184	21.470	20		
41	682	8.67039	269	890	0.99952	687	8.67087	269	1.32913	.337	19		
42	711	8.67308	267	889	0.99952	716	8.67356	268	1.32644	.205	18		
43	740	8.67575	266	888	0.99951	745	8.67624	266	1.32376	.075	17		
44	769	8.67841	263	886	0.99951	774	8.67890	264	1.32110	20.946	16		
45	04798	8.68104	263	99885	0.99950	04803	8.68154	263	1.31846	20.819	15		
46	827	8.68367	260	883	0.99949	833	8.68417	261	1.31583	.693	14		
47	856	8.68627	259	882	0.99949	862	8.68678	260	1.31322	.569	13		
48	885	8.68886	258	881	0.99948	891	8.68938	258	1.31062	.446	12		
49	914	8.69144	256	879	0.99948	920	8.69196	257	1.30804	.325	11		
50	04943	8.69400	254	99878	0.99947	04949	8.69453	255	1.30547	20.206	10		
51	972	8.69654	253	876	0.99946	978	8.69708	252	1.30292	.087	9		
52	05001	8.69907	252	875	0.99946	05007	8.69962	254	1.30038	19.970	8		
53	030	8.70159	250	873	0.99945	037	8.70214	251	1.29786	.855	7		
54	059	8.70409	249	872	0.99944	066	8.70465	249	1.29535	.740	6		
55	05088	8.70658	247	99870	0.99944	05095	8.70714	248	1.29286	19.627	5		
56	117	8.70905	246	869	0.99943	124	8.70962	246	1.29038	.516	4		
57	146	8.71151	244	867	0.99942	153	8.71208	245	1.28792	.405	3		
58	175	8.71395	243	866	0.99942	182	8.71453	244	1.28547	.296	2		
59	205	8.71638	242	864	0.99941	212	8.71697	243	1.28303	.188	1		
60	234	8.71880		863	0.99940	241	8.71940		1.28060	.081	0		
	Nat. Cos Log. d.			Nat. Sin Log.			Nat. Cot Log.			c.d.	Log. Tan Nat.		

'	Nat. Sin Log.		d.	Nat. Cos Log.		Nat. Tan Log.		c.d.	Log. Cot Nat.			
0	05234	8.71880			99863	9.99040	05341	8.71940		1.28060	19.081	60
1	263	8.72120	239		861	9.99040	270	8.72181	241	1.27819	18.976	59
2	292	8.72359	238		860	9.99039	299	8.72420	239	1.27580	.871	58
3	321	8.72597	237		858	9.99038	328	8.72659	237	1.27341	.768	57
4	350	8.72834	235		857	9.99038	357	8.72896	236	1.27104	.666	56
5	05379	8.73069	234		99855	9.99037	05387	8.73132	234	1.26868	18.564	55
6	408	8.73303	232		854	9.99036	416	8.73366	234	1.26634	.464	54
7	437	8.73535	232		852	9.99036	445	8.73600	232	1.26400	.366	53
8	466	8.73797	230		851	9.99035	474	8.73832	231	1.26168	.268	52
9	495	8.73997	229		849	9.99034	503	8.74063	229	1.25937	.171	51
10	05524	8.74226	228		99847	9.99034	05533	8.74292	229	1.25708	18.075	50
11	553	8.74454	226		846	9.99033	562	8.74521	227	1.25479	17.980	49
12	582	8.74680	226		844	9.99032	591	8.74748	226	1.25252	.886	48
13	611	8.74906	224		842	9.99032	620	8.74974	225	1.25026	.793	47
14	640	8.75130	223		841	9.99031	649	8.75199	224	1.24801	.702	46
15	05669	8.75353	222		99839	9.99030	05678	8.75423	222	1.24577	17.611	45
16	698	8.75575	220		838	9.99029	708	8.75645	222	1.24355	.521	44
17	727	8.75795	220		836	9.99029	737	8.75867	220	1.24133	.431	43
18	756	8.76015	219		834	9.99028	766	8.76087	219	1.23913	.343	42
19	785	8.76234	217		833	9.99027	795	8.76306	219	1.23694	.256	41
20	05814	8.76451	216		99831	9.99026	05824	8.76525	217	1.23475	17.169	40
21	844	8.76667	216		829	9.99026	854	8.76742	216	1.23258	.084	39
22	873	8.76883	214		827	9.99025	883	8.76958	215	1.23042	16.999	38
23	902	8.77097	213		826	9.99024	912	8.77173	214	1.22827	.915	37
24	931	8.77310	212		824	9.99023	941	8.77387	213	1.22613	.832	36
25	05960	8.77522	211		99822	9.99023	05970	8.77600	211	1.22400	16.750	35
26	989	8.77733	210		821	9.99022	999	8.77811	211	1.22189	.668	34
27	06018	8.77943	209		819	9.99021	06029	8.78022	210	1.21978	.587	33
28	047	8.78152	208		817	9.99020	058	8.78232	209	1.21768	.507	32
29	076	8.78360	208		815	9.99020	087	8.78441	208	1.21559	.428	31
30	06105	8.78568	206		99813	9.99019	06116	8.78649	206	1.21351	16.350	30
31	134	8.78774	205		812	9.99018	145	8.78855	206	1.21145	.272	29
32	163	8.78979	204		810	9.99017	175	8.79061	205	1.20939	.195	28
33	192	8.79183	203		808	9.99017	204	8.79266	205	1.20734	.119	27
34	221	8.79386	202		806	9.99016	233	8.79470	204	1.20530	.043	26
35	06250	8.79588	201		99804	9.99015	06262	8.79673	203	1.20327	15.969	25
36	279	8.79780	201		803	9.99014	291	8.79875	201	1.20125	.895	24
37	308	8.79990	199		801	9.99013	321	8.80070	201	1.19924	.821	23
38	337	8.80180	199		799	9.99013	350	8.80277	199	1.19723	.748	22
39	366	8.80388	197		797	9.99012	379	8.80476	198	1.19524	.676	21
40	06395	8.80585	197		99795	9.99011	06408	8.80674	198	1.19326	15.605	20
41	424	8.80782	196		793	9.99010	438	8.80872	196	1.19128	.534	19
42	453	8.80978	195		792	9.99009	467	8.81068	196	1.18932	.464	18
43	482	8.81173	194		790	9.99009	496	8.81264	195	1.18736	.394	17
44	511	8.81367	193		788	9.99008	525	8.81459	194	1.18541	.325	16
45	06540	8.81560	192		99786	9.99007	06554	8.81653	193	1.18347	15.257	15
46	569	8.81752	192		784	9.99006	584	8.81846	192	1.18154	.189	14
47	598	8.81944	190		782	9.99005	613	8.82038	192	1.17962	.122	13
48	627	8.82134	190		780	9.99004	642	8.82230	190	1.17770	.056	12
49	656	8.82324	189		778	9.99004	671	8.82420	190	1.17580	14.990	11
50	06685	8.82513	188		99776	9.99003	06700	8.82610	189	1.17390	14.924	10
51	714	8.82701	187		774	9.99002	730	8.82799	188	1.17201	.860	9
52	743	8.82888	187		772	9.99001	759	8.82987	188	1.17013	.795	8
53	773	8.83075	186		770	9.99000	788	8.83175	186	1.16825	.732	7
54	802	8.83261	185		768	9.99000	817	8.83361	186	1.16639	.669	6
55	06831	8.83446	184		99766	9.99000	06847	8.83547	185	1.16453	14.606	5
56	860	8.83630	183		764	9.99000	876	8.83732	184	1.16268	.544	4
57	889	8.83813	183		762	9.99000	905	8.83916	184	1.16084	.482	3
58	918	8.83996	181		760	9.99000	934	8.84100	182	1.15900	.421	2
59	947	8.84177	181		758	9.99000	963	8.84282	182	1.15718	.361	1
60	976	8.84358			756	9.99000	993	8.84464		1.15536	.301	0
	Nat. Cos Log.		d.	Nat. Sin Log.		Nat. Cot Log.		c.d.	Log. Tan Nat.		'	

'	Nat. Sin Log. d.			Nat. Cos Log.		Nat. Tan Log.		c.d.	Log. Cot Nat.		
0	06976	8.84358	181	99756	9.99894	06993	8.84464	182	1.15536	14.301	60
1	07005	8.84539	179	754	9.99893	07022	8.84646	180	1.15354	.241	59
2	034	8.84718	179	752	9.99892	051	8.84826	180	1.15174	.182	58
3	063	8.84897	178	750	9.99891	080	8.85006	179	1.14994	.124	57
4	092	8.85075	177	748	9.99890	110	8.85185	178	1.14815	.065	56
5	07121	8.85252	177	99746	9.99890	07139	8.85363	177	1.14637	14.008	55
6	150	8.85429	176	744	9.99889	168	8.85540	177	1.14460	13.951	54
7	179	8.85605	175	742	9.99888	197	8.85717	176	1.14283	.894	53
8	208	8.85780	175	740	9.99887	227	8.85893	176	1.14107	.838	52
9	237	8.85955	173	738	9.99886	256	8.86069	174	1.13931	.782	51
10	07266	8.86128	173	99736	9.99885	07285	8.86243	174	1.13757	13.727	50
11	295	8.86301	173	734	9.99884	314	8.86417	174	1.13583	.672	49
12	324	8.86474	173	731	9.99883	344	8.86591	174	1.13409	.617	48
13	353	8.86645	171	729	9.99882	373	8.86763	173	1.13237	.563	47
14	382	8.86816	171	727	9.99881	402	8.86935	172	1.13065	.510	46
15	07411	8.86987	171	99725	9.99880	07431	8.87106	171	1.12894	13.457	45
16	440	8.87156	169	723	9.99879	461	8.87277	171	1.12723	.404	44
17	469	8.87325	169	721	9.99879	490	8.87447	170	1.12553	.352	43
18	498	8.87494	169	719	9.99878	519	8.87616	169	1.12384	.300	42
19	527	8.87661	167	716	9.99877	548	8.87785	169	1.12215	.248	41
20	07556	8.87829	168	99714	9.99876	07578	8.87953	168	1.12047	13.197	40
21	585	8.87995	166	712	9.99875	607	8.88120	167	1.11880	.146	39
22	614	8.88161	166	710	9.99874	636	8.88287	167	1.11713	.096	38
23	643	8.88326	165	708	9.99873	665	8.88453	166	1.11547	.046	37
24	672	8.88490	164	705	9.99872	695	8.88618	165	1.11382	12.996	36
25	07701	8.88654	164	99703	9.99871	07724	8.88783	165	1.11217	12.947	35
26	730	8.88817	163	701	9.99870	753	8.88948	165	1.11052	.898	34
27	759	8.88980	163	699	9.99869	782	8.89111	163	1.10880	.850	33
28	788	8.89142	162	696	9.99868	812	8.89274	163	1.10726	.801	32
29	817	8.89304	162	694	9.99867	841	8.89437	161	1.10563	.754	31
30	07846	8.89464	160	99692	9.99866	07870	8.89598	162	1.10402	12.706	30
31	875	8.89625	161	689	9.99865	899	8.89760	162	1.10240	.659	29
32	904	8.89784	159	687	9.99864	929	8.89920	160	1.10080	.612	28
33	933	8.89943	159	685	9.99863	958	8.90080	160	1.09920	.566	27
34	962	8.90102	159	683	9.99862	987	8.90240	160	1.09760	.520	26
35	07991	8.90260	158	99680	9.99861	08017	8.90399	159	1.09601	12.474	25
36	08020	8.90417	157	678	9.99860	046	8.90557	158	1.09443	.429	24
37	049	8.90574	157	676	9.99859	075	8.90715	158	1.09285	.384	23
38	078	8.90730	156	673	9.99858	104	8.90872	157	1.09128	.339	22
39	107	8.90885	155	671	9.99857	134	8.91029	157	1.08971	.295	21
40	08136	8.91040	155	99668	9.99856	08163	8.91185	156	1.08815	12.251	20
41	165	8.91195	155	666	9.99855	192	8.91340	155	1.08660	.207	19
42	194	8.91349	154	664	9.99854	221	8.91495	155	1.08505	.163	18
43	223	8.91502	153	661	9.99853	251	8.91650	155	1.08350	.120	17
44	252	8.91655	153	659	9.99852	280	8.91803	153	1.08197	.077	16
45	08281	8.91807	152	99657	9.99851	08309	8.91957	154	1.08043	12.035	15
46	310	8.91959	152	654	9.99850	339	8.92110	153	1.07890	11.992	14
47	339	8.92110	151	652	9.99848	368	8.92262	152	1.07738	.950	13
48	368	8.92261	151	649	9.99847	397	8.92414	152	1.07586	.909	12
49	397	8.92411	150	647	9.99846	427	8.92565	151	1.07435	.867	11
50	08426	8.92561	150	99644	9.99845	08456	8.92716	151	1.07284	11.826	10
51	455	8.92710	149	642	9.99844	485	8.92866	150	1.07134	.785	9
52	484	8.92859	149	639	9.99843	514	8.93016	150	1.06984	.745	8
53	513	8.93007	148	637	9.99842	544	8.93165	149	1.06835	.705	7
54	542	8.93154	147	635	9.99841	573	8.93313	148	1.06687	.664	6
55	08571	8.93301	147	99632	9.99840	08602	8.93462	149	1.06538	11.625	5
56	600	8.93448	147	630	9.99839	632	8.93609	147	1.06391	.585	4
57	629	8.93594	146	627	9.99838	661	8.93756	147	1.06244	.546	3
58	658	8.93740	146	625	9.99837	690	8.93903	147	1.06097	.507	2
59	687	8.93885	145	622	9.99836	720	8.94049	146	1.05951	.468	1
60	716	8.94030	145	619	9.99834	749	8.94195	146	1.05805	.430	0
	Nat. Cos Log. d.			Nat. Sin Log.		Nat. Cot Log.		c.d. Log. Tan Nat.		'	

'	Nat. Sin Log.		d.	Nat. Cos Log.		Nat. Tan Log.		c.d.	Log. Cot Nat.		
0	08716	8.94030		99619	9.99834	08749	8.94195		1.05805	11.430	60
1	745	8.94174	144	617	9.99833	778	8.94340	145	1.05660	.392	59
2	774	8.94317	143	614	9.99832	807	8.94485	145	1.05515	.354	58
3	803	8.94461	144	612	9.99831	837	8.94630	145	1.05370	.316	57
4	831	8.94603	142	609	9.99830	866	8.94773	143	1.05227	.279	56
5	08860	8.94746	143	99607	9.99829	08895	8.94917	144	1.05083	11.242	55
6	889	8.94887	141	604	9.99828	925	8.95060	143	1.04940	.205	54
7	918	8.95029	142	602	9.99827	954	8.95202	142	1.04798	.168	53
8	947	8.95170	141	599	9.99825	983	8.95344	142	1.04656	.132	52
9	976	8.95310	140	596	9.99824	09013	8.95486	142	1.04514	.095	51
10	09005	8.95450	140	99594	9.99823	09042	8.95627	141	1.04373	11.059	50
11	034	8.95589	139	591	9.99822	071	8.95707	140	1.04233	.024	49
12	063	8.95728	139	588	9.99821	101	8.95908	141	1.04092	10.888	48
13	092	8.95867	139	586	9.99820	130	8.96047	139	1.03953	.953	47
14	121	8.96005	138	583	9.99819	159	8.96187	140	1.03813	.918	46
15	09150	8.96143	138	99580	9.99817	09189	8.96325	138	1.03675	10.883	45
16	179	8.96280	137	578	9.99816	218	8.96404	139	1.03536	.848	44
17	208	8.96417	137	575	9.99815	247	8.96602	138	1.03398	.814	43
18	237	8.96553	136	572	9.99814	277	8.96739	137	1.03261	.780	42
19	266	8.96689	136	570	9.99813	306	8.96877	138	1.03123	.746	41
20	09295	8.96825	136	99567	9.99812	09335	8.97013	136	1.02987	10.712	40
21	324	8.96960	135	564	9.99810	305	8.97150	137	1.02850	.678	39
22	353	8.97095	135	562	9.99809	394	8.97285	135	1.02715	.645	38
23	382	8.97229	134	559	9.99808	423	8.97421	136	1.02579	.612	37
24	411	8.97363	134	556	9.99807	453	8.97556	135	1.02444	.579	36
25	09440	8.97499	133	99553	9.99806	09482	8.97691	135	1.02309	10.546	35
26	469	8.97629	133	551	9.99804	511	8.97825	134	1.02175	.514	34
27	498	8.97762	133	548	9.99803	541	8.97959	134	1.02041	.481	33
28	527	8.97894	132	545	9.99802	570	8.98092	133	1.01908	.449	32
29	556	8.98026	132	542	9.99801	600	8.98225	133	1.01775	.417	31
30	09585	8.98157	131	99540	9.99800	09629	8.98358	133	1.01642	10.385	30
31	614	8.98288	131	537	9.99798	658	8.98490	132	1.01510	.354	29
32	642	8.98419	131	534	9.99797	688	8.98622	132	1.01378	.322	28
33	671	8.98549	130	531	9.99796	717	8.98753	131	1.01247	.291	27
34	700	8.98679	130	528	9.99795	746	8.98884	131	1.01116	.260	26
35	09729	8.98808	129	99526	9.99793	09776	8.99015	131	1.00985	10.229	25
36	758	8.98937	129	523	9.99792	805	8.99145	130	1.00855	.199	24
37	787	8.99066	129	520	9.99791	834	8.99275	130	1.00725	.168	23
38	816	8.99194	128	517	9.99790	864	8.99405	129	1.00595	.138	22
39	845	8.99322	128	514	9.99788	893	8.99534	128	1.00466	.108	21
40	09874	8.99450	128	99511	9.99787	09923	8.99662	128	1.00338	10.078	20
41	903	8.99577	127	508	9.99786	952	8.99791	129	1.00209	.048	19
42	932	8.99704	127	506	9.99785	981	8.99919	128	1.00081	.019	18
43	961	8.99830	126	503	9.99783	10011	9.00046	127	0.99954	9.9893	17
44	990	8.99956	126	500	9.99782	040	9.00174	128	0.99826	.601	16
45	10019	9.00082	126	99497	9.99781	10069	9.00301	127	0.99699	9.9310	15
46	048	9.00207	125	494	9.99780	099	9.00427	126	0.99573	.021	14
47	077	9.00332	125	491	9.99778	128	9.00553	126	0.99447	9.8734	13
48	106	9.00456	124	488	9.99777	158	9.00679	126	0.99321	.448	12
49	135	9.00581	125	485	9.99776	187	9.00805	126	0.99195	.164	11
50	10164	9.00704	123	99482	9.99775	10216	9.00930	125	0.99070	9.7882	10
51	192	9.00828	124	479	9.99773	246	9.01055	125	0.98945	.601	9
52	221	9.00951	123	476	9.99772	275	9.01179	124	0.98821	.322	8
53	250	9.01074	123	473	9.99771	305	9.01303	124	0.98697	.044	7
54	279	9.01196	122	470	9.99769	334	9.01427	124	0.98573	9.6768	6
55	10308	9.01318	122	99467	9.99768	10363	9.01550	123	0.98450	9.6493	5
56	337	9.01440	121	464	9.99767	393	9.01673	123	0.98327	.220	4
57	366	9.01561	121	461	9.99765	422	9.01796	123	0.98204	9.5949	3
58	395	9.01682	121	458	9.99764	452	9.01918	122	0.98082	.679	2
59	424	9.01803	121	455	9.99763	481	9.02040	122	0.97960	.411	1
60	453	9.01923	120	452	9.99761	510	9.02162	122	0.97838	.144	0
	Nat. Cos Log.		d.	Nat. Sin Log.		Nat. Cot Log.		c.d.	Log. Tan Nat.		'

'	Nat. Sin Log.	d.	Nat. Cos Log.	Nat. Tan Log.	c.d.	Log. Cot Nat.			
0	10453	9.01923	99452	9.99761	10510	9.02162	0.97838	9.5144	60
1	482	9.02043	449	9.99760	540	9.02283	0.97717	9.4878	59
2	511	9.02163	446	9.99759	569	9.02404	0.97596	614	58
3	540	9.02283	443	9.99757	599	9.02525	0.97475	352	57
4	569	9.02402	440	9.99756	628	9.02645	0.97355	090	56
5	10597	9.02520	99437	9.99755	10657	9.02766	0.97234	9.3831	55
6	626	9.02639	434	9.99753	687	9.02885	0.97115	572	54
7	655	9.02757	431	9.99752	716	9.03005	0.96995	315	53
8	684	9.02874	428	9.99751	746	9.03124	0.96876	060	52
9	713	9.02992	424	9.99749	775	9.03242	0.96758	9.2806	51
10	10742	9.03109	99421	9.99748	10805	9.03361	0.96639	9.2553	50
11	771	9.03226	418	9.99747	834	9.03479	0.96521	302	49
12	800	9.03342	415	9.99745	863	9.03597	0.96403	052	48
13	829	9.03458	412	9.99744	893	9.03714	0.96286	9.1803	47
14	858	9.03574	409	9.99742	922	9.03832	0.96168	555	46
15	10887	9.03690	99406	9.99741	10952	9.03948	0.96052	9.1309	45
16	916	9.03805	402	9.99740	981	9.04065	0.95935	065	44
17	945	9.03920	399	9.99738	11011	9.04181	0.95819	9.0821	43
18	973	9.04034	396	9.99737	040	9.04297	0.95703	579	42
19	11002	9.04149	393	9.99736	070	9.04413	0.95587	338	41
20	11031	9.04262	99390	9.99734	11099	9.04528	0.95472	9.0098	40
21	060	9.04376	386	9.99733	128	9.04643	0.95357	8.9860	39
22	089	9.04490	383	9.99731	158	9.04758	0.95242	623	38
23	118	9.04603	380	9.99730	187	9.04873	0.95127	387	37
24	147	9.04715	377	9.99728	217	9.04987	0.95013	152	36
25	11176	9.04828	99374	9.99727	11246	9.05101	0.94899	8.8919	35
26	205	9.04940	370	9.99726	276	9.05214	0.94786	686	34
27	234	9.05052	367	9.99724	305	9.05328	0.94672	455	33
28	263	9.05164	364	9.99723	335	9.05441	0.94559	225	32
29	291	9.05275	360	9.99721	364	9.05553	0.94447	8.7996	31
30	11320	9.05386	99357	9.99720	11394	9.05666	0.94334	8.7769	30
31	349	9.05497	354	9.99718	423	9.05778	0.94222	542	29
32	378	9.05607	351	9.99717	452	9.05890	0.94110	317	28
33	407	9.05717	347	9.99716	482	9.06002	0.93998	093	27
34	436	9.05827	344	9.99714	511	9.06113	0.93887	8.6870	26
35	11465	9.05937	99341	9.99713	11541	9.06224	0.93776	8.6648	25
36	494	9.06046	337	9.99711	570	9.06335	0.93665	427	24
37	523	9.06155	334	9.99710	600	9.06445	0.93555	208	23
38	552	9.06264	331	9.99708	629	9.06556	0.93444	8.5989	22
39	580	9.06372	327	9.99707	659	9.06666	0.93334	772	21
40	11609	9.06481	99324	9.99705	11688	9.06775	0.93225	8.5555	20
41	638	9.06589	320	9.99704	718	9.06885	0.93115	340	19
42	667	9.06696	317	9.99702	747	9.06994	0.93006	126	18
43	696	9.06804	314	9.99701	777	9.07103	0.92897	8.4913	17
44	725	9.06911	310	9.99699	806	9.07211	0.92789	701	16
45	11754	9.07018	99307	9.99698	11836	9.07320	0.92680	8.4490	15
46	783	9.07124	303	9.99696	865	9.07428	0.92572	280	14
47	812	9.07231	300	9.99695	895	9.07536	0.92464	071	13
48	840	9.07337	297	9.99693	924	9.07643	0.92357	8.3863	12
49	869	9.07442	293	9.99692	954	9.07751	0.92249	656	11
50	11898	9.07548	99290	9.99690	11983	9.07858	0.92142	8.3450	10
51	927	9.07653	286	9.99689	12013	9.07964	0.92036	245	9
52	956	9.07758	283	9.99687	042	9.08071	0.91929	041	8
53	985	9.07863	279	9.99686	072	9.08177	0.91823	8.2838	7
54	12014	9.07968	276	9.99684	101	9.08283	0.91717	636	6
55	12043	9.08072	99272	9.99683	12131	9.08389	0.91611	8.2434	5
56	071	9.08176	269	9.99681	160	9.08495	0.91505	234	4
57	100	9.08280	265	9.99680	190	9.08600	0.91400	035	3
58	129	9.08383	262	9.99678	219	9.08705	0.91295	8.1837	2
59	158	9.08486	258	9.99677	249	9.08810	0.91190	640	1
60	187	9.08589	255	9.99675	278	9.08914	0.91086	443	0
	Nat. Cos Log.	d.	Nat. Sin Log.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'		

'	Nat. Sin Log.	d.	Nat. Cos Log.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	12187	0.08589	103	99255	9.99675	12278	0.08914	105	0.91086	8.1443	60
1	216	0.08692	103	251	9.99674	308	0.09019	104	0.90981	248	59
2	245	0.08795	102	248	9.99672	338	0.09123	104	0.90877	054	58
3	274	0.08897	102	244	9.99670	367	0.09227	103	0.90773	8.0860	57
4	302	0.08999	102	240	9.99669	397	0.09330	104	0.90670	667	56
5	12331	0.09101	101	99237	9.99667	12426	0.09434	103	0.90566	8.0476	55
6	360	0.09202	102	233	9.99666	456	0.09537	103	0.90463	285	54
7	389	0.09304	101	230	9.99664	485	0.09640	102	0.90360	095	53
8	418	0.09405	101	226	9.99663	515	0.09742	103	0.90258	7.9906	52
9	447	0.09506	100	222	9.99661	544	0.09845	102	0.90155	718	51
10	12476	0.09606	101	99219	9.99659	12574	0.09947	102	0.90053	7.9530	50
11	504	0.09707	100	215	9.99658	603	0.10049	101	0.89951	344	49
12	533	0.09807	100	211	9.99656	633	0.10150	102	0.89850	158	48
13	562	0.09907	99	208	9.99655	662	0.10252	101	0.89748	7.8973	47
14	591	0.10006	100	204	9.99653	692	0.10353	101	0.89647	789	46
15	12620	0.10106	99	99200	9.99651	12722	0.10454	101	0.89546	7.8606	45
16	649	0.10205	99	197	9.99650	751	0.10555	101	0.89445	424	44
17	678	0.10304	98	193	9.99648	781	0.10656	100	0.89344	243	43
18	706	0.10402	99	189	9.99647	810	0.10756	100	0.89244	062	42
19	735	0.10501	98	186	9.99645	840	0.10856	100	0.89144	7.7882	41
20	12764	0.10599	98	99182	9.99643	12869	0.10956	100	0.89044	7.7704	40
21	793	0.10697	98	178	9.99642	899	0.11056	99	0.88944	525	39
22	822	0.10795	98	175	9.99640	929	0.11155	99	0.88845	348	38
23	851	0.10893	97	171	9.99638	958	0.11254	99	0.88746	171	37
24	880	0.10990	97	167	9.99637	988	0.11353	99	0.88647	7.6996	36
25	12908	0.11087	97	99163	9.99635	13017	0.11452	99	0.88548	7.6821	35
26	937	0.11184	97	160	9.99633	047	0.11551	98	0.88449	647	34
27	966	0.11281	96	156	9.99632	076	0.11649	98	0.88351	473	33
28	995	0.11377	97	152	9.99630	106	0.11747	98	0.88253	301	32
29	13024	0.11474	96	148	9.99629	136	0.11845	98	0.88155	129	31
30	13053	0.11570	96	99144	9.99627	13165	0.11943	97	0.88057	7.5958	30
31	081	0.11666	95	141	9.99625	195	0.12040	97	0.87960	787	29
32	110	0.11761	96	137	9.99624	224	0.12138	98	0.87862	618	28
33	139	0.11857	95	133	9.99622	254	0.12235	97	0.87765	449	27
34	168	0.11952	95	129	9.99620	284	0.12332	96	0.87668	281	26
35	13197	0.12047	95	99125	9.99618	13313	0.12428	97	0.87572	7.5113	25
36	226	0.12142	94	122	9.99617	343	0.12525	96	0.87475	7.4947	24
37	254	0.12236	95	118	9.99615	372	0.12621	96	0.87379	781	23
38	283	0.12331	95	114	9.99613	402	0.12717	96	0.87283	615	22
39	312	0.12425	94	110	9.99612	432	0.12813	96	0.87187	451	21
40	13341	0.12519	93	99106	9.99610	13461	0.12909	95	0.87091	7.4287	20
41	370	0.12612	94	102	9.99608	491	0.13004	95	0.86996	124	19
42	399	0.12706	94	098	9.99607	521	0.13099	95	0.86901	7.3962	18
43	427	0.12799	93	094	9.99605	550	0.13194	95	0.86806	800	17
44	456	0.12892	93	091	9.99603	580	0.13289	95	0.86711	639	16
45	13485	0.12985	93	99087	9.99601	13609	0.13384	94	0.86616	7.3479	15
46	514	0.13078	93	083	9.99600	639	0.13478	94	0.86522	319	14
47	543	0.13171	92	079	9.99598	669	0.13573	94	0.86427	160	13
48	572	0.13263	92	075	9.99596	698	0.13667	94	0.86333	002	12
49	600	0.13355	92	071	9.99595	728	0.13761	93	0.86239	7.2844	11
50	13629	0.13447	92	99067	9.99593	13758	0.13854	94	0.86146	7.2687	10
51	658	0.13539	91	063	9.99591	787	0.13948	93	0.86052	531	9
52	687	0.13630	92	059	9.99589	817	0.14041	93	0.85959	375	8
53	716	0.13722	91	055	9.99588	846	0.14134	93	0.85866	220	7
54	744	0.13813	91	051	9.99586	876	0.14227	93	0.85773	066	6
55	13773	0.13904	90	99047	9.99584	13906	0.14320	92	0.85680	7.1912	5
56	802	0.13994	91	043	9.99582	935	0.14412	92	0.85588	759	4
57	831	0.14085	90	039	9.99581	965	0.14504	92	0.85496	607	3
58	860	0.14175	91	035	9.99579	995	0.14597	91	0.85403	455	2
59	889	0.14266	90	031	9.99577	14024	0.14688	92	0.85312	304	1
60	917	0.14356	90	027	9.99575	054	0.14780	92	0.85220	154	0
	Nat. Cos Log.	d.	Nat. Sin Log.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'				

	Nat. Sin Log.	d.	Nat. Cos Log.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	13917	9.14356	89	99027	9.99575	14054	9.14780	92	0.85220	7.1154	60
1	946	9.14445	89	023	9.99574	084	9.14872	91	0.85128	004	59
2	975	9.14535	89	019	9.99572	113	9.14963	91	0.85037	7.0855	58
3	14004	9.14624	90	015	9.99570	143	9.15054	91	0.84946	7.0657	57
4	033	9.14714	90	011	9.99568	173	9.15145	91	0.84855	558	56
5	14061	9.14803	88	99006	9.99566	14202	9.15236	91	0.84764	7.0410	55
6	090	9.14891	88	002	9.99565	232	9.15327	91	0.84673	264	54
7	119	9.14980	89	98998	9.99563	262	9.15417	90	0.84583	117	53
8	148	9.15069	89	994	9.99561	291	9.15508	91	0.84492	6.9972	52
9	177	9.15157	88	990	9.99559	321	9.15598	90	0.84402	827	51
10	14205	9.15245	88	98986	9.99557	14351	9.15688	89	0.84312	6.9682	50
11	234	9.15333	88	982	9.99556	381	9.15777	89	0.84223	538	49
12	263	9.15421	87	978	9.99554	410	9.15867	90	0.84133	395	48
13	292	9.15508	88	973	9.99552	440	9.15956	90	0.84044	252	47
14	320	9.15596	87	969	9.99550	470	9.16046	90	0.83954	110	46
15	14349	9.15683	87	98965	9.99548	14499	9.16135	89	0.83865	6.8969	45
16	378	9.15770	87	961	9.99546	529	9.16224	88	0.83776	828	44
17	407	9.15857	87	957	9.99545	559	9.16312	88	0.83688	687	43
18	436	9.15944	86	953	9.99543	588	9.16401	89	0.83599	548	42
19	464	9.16030	86	948	9.99541	618	9.16489	88	0.83511	408	41
20	14493	9.16116	87	98944	9.99539	14648	9.16577	88	0.83423	6.8269	40
21	522	9.16203	86	940	9.99537	678	9.16665	88	0.83335	131	39
22	551	9.16289	85	936	9.99535	707	9.16753	88	0.83247	6.7994	38
23	580	9.16374	86	931	9.99533	737	9.16841	87	0.83159	856	37
24	608	9.16460	85	927	9.99532	767	9.16928	88	0.83072	720	36
25	14637	9.16545	86	98923	9.99530	14796	9.17016	87	0.82984	6.7584	35
26	666	9.16631	85	919	9.99528	826	9.17103	87	0.82897	448	34
27	695	9.16716	85	914	9.99526	856	9.17190	87	0.82810	313	33
28	723	9.16801	85	910	9.99524	886	9.17277	87	0.82723	179	32
29	752	9.16886	84	906	9.99522	915	9.17363	87	0.82637	045	31
30	14781	9.16970	85	98902	9.99520	14945	9.17450	86	0.82550	6.6912	30
31	810	9.17055	84	897	9.99518	975	9.17536	86	0.82464	779	29
32	838	9.17139	84	893	9.99517	15005	9.17622	86	0.82378	646	28
33	867	9.17223	84	889	9.99515	034	9.17708	86	0.82292	514	27
34	896	9.17307	84	884	9.99513	064	9.17794	86	0.82206	383	26
35	14925	9.17391	83	98880	9.99511	15094	9.17880	85	0.82120	6.6252	25
36	954	9.17474	84	876	9.99509	124	9.17965	85	0.82035	122	24
37	982	9.17558	83	871	9.99507	153	9.18051	85	0.81949	6.5992	23
38	15011	9.17641	83	867	9.99505	183	9.18136	85	0.81864	863	22
39	040	9.17724	83	863	9.99503	213	9.18221	85	0.81779	734	21
40	15069	9.17807	83	98858	9.99501	15243	9.18306	85	0.81694	6.5606	20
41	097	9.17890	83	854	9.99499	272	9.18391	84	0.81609	478	19
42	126	9.17973	82	849	9.99497	302	9.18475	85	0.81525	350	18
43	155	9.18055	82	845	9.99495	332	9.18560	85	0.81440	223	17
44	184	9.18137	83	841	9.99494	362	9.18644	84	0.81356	097	16
45	15212	9.18220	82	98836	9.99492	15391	9.18728	84	0.81272	6.4971	15
46	241	9.18302	81	832	9.99490	421	9.18812	84	0.81188	846	14
47	270	9.18383	82	827	9.99488	451	9.18896	83	0.81104	721	13
48	299	9.18465	82	823	9.99486	481	9.18979	84	0.81021	596	12
49	327	9.18547	81	818	9.99484	511	9.19063	83	0.80937	472	11
50	15356	9.18628	81	98814	9.99482	15540	9.19146	83	0.80854	6.4348	10
51	385	9.18709	81	809	9.99480	570	9.19229	83	0.80771	225	9
52	414	9.18790	81	805	9.99478	600	9.19312	83	0.80688	103	8
53	442	9.18871	81	800	9.99476	630	9.19395	83	0.80605	6.3980	7
54	471	9.18952	81	796	9.99474	660	9.19478	83	0.80522	859	6
55	15500	9.19033	80	98791	9.99472	15689	9.19561	82	0.80439	6.3737	5
56	529	9.19113	80	787	9.99470	719	9.19643	82	0.80357	617	4
57	557	9.19193	80	782	9.99468	749	9.19725	82	0.80275	496	3
58	586	9.19273	80	778	9.99466	779	9.19807	82	0.80193	376	2
59	615	9.19353	80	773	9.99464	809	9.19889	82	0.80111	257	1
60	643	9.19433	80	769	9.99462	838	9.19971	82	0.80029	138	0
	Nat. Cos Log.	d.	Nat. Sin Log.	Nat. Cot Log.	c.d.	Log. Tan Nat.					

'	Nat. Sin Log.		d.	Nat. Cos Log.		Nat. Tan Log.		c.d.	Log. Cot Nat.		
0	15643	9.19433	80	98769	9.99462	15838	9.19971	82	0.80029	6.3138	60
1	672	9.19513	79	764	9.99460	868	9.20053	81	0.79947	019	59
2	701	9.19592	80	760	9.99458	898	9.20134	82	0.79866	6.2901	58
3	730	9.19672	79	755	9.99456	928	9.20216	81	0.79784	783	57
4	758	9.19751	79	751	9.99454	958	9.20297	81	0.79703	666	56
5	15787	9.19830	79	98746	9.99452	15988	9.20378	81	0.79622	6.2549	55
6	816	9.19909	79	741	9.99450	16017	9.20459	81	0.79541	432	54
7	845	9.19988	79	737	9.99448	047	9.20540	81	0.79460	316	53
8	873	9.20067	78	732	9.99446	077	9.20621	80	0.79379	200	52
9	902	9.20145	78	728	9.99444	107	9.20701	80	0.79299	085	51
10	15931	9.20223	78	98723	9.99442	16137	9.20782	81	0.79218	6.1970	50
11	959	9.20302	78	718	9.99440	167	9.20862	80	0.79138	856	49
12	988	9.20380	78	714	9.99438	196	9.20942	80	0.79058	742	48
13	16017	9.20458	77	709	9.99436	226	9.21022	80	0.78978	623	47
14	046	9.20535	77	704	9.99434	256	9.21102	80	0.78898	515	46
15	16074	9.20613	78	98700	9.99432	16286	9.21182	79	0.78818	6.1402	45
16	103	9.20691	77	695	9.99429	316	9.21261	80	0.78739	290	44
17	132	9.20768	77	690	9.99427	346	9.21341	79	0.78659	178	43
18	160	9.20845	77	686	9.99425	376	9.21420	79	0.78580	066	42
19	189	9.20922	77	681	9.99423	405	9.21499	79	0.78501	6.0955	41
20	16218	9.20999	77	98676	9.99421	16435	9.21578	79	0.78422	6.0842	40
21	246	9.21076	77	671	9.99419	465	9.21657	79	0.78343	734	39
22	275	9.21153	76	667	9.99417	495	9.21736	78	0.78264	624	38
23	304	9.21229	76	662	9.99415	525	9.21814	78	0.78186	514	37
24	333	9.21306	77	657	9.99413	555	9.21893	79	0.78107	405	36
25	16361	9.21382	76	98652	9.99411	16585	9.21971	78	0.78029	6.0296	35
26	390	9.21458	76	648	9.99409	615	9.22049	78	0.77951	188	34
27	419	9.21534	76	643	9.99407	645	9.22127	78	0.77873	080	33
28	447	9.21610	76	638	9.99404	674	9.22205	78	0.77795	5.9972	32
29	476	9.21685	76	633	9.99402	704	9.22283	78	0.77717	865	31
30	16505	9.21761	75	98629	9.99400	16734	9.22361	77	0.77639	5.9758	30
31	533	9.21836	76	624	9.99398	764	9.22438	77	0.77562	651	29
32	562	9.21912	75	619	9.99396	794	9.22516	78	0.77484	545	28
33	591	9.21987	75	614	9.99394	824	9.22593	77	0.77407	439	27
34	620	9.22062	75	609	9.99392	854	9.22670	77	0.77330	333	26
35	16648	9.22137	75	98604	9.99390	16884	9.22747	77	0.77253	5.9228	25
36	677	9.22211	74	600	9.99388	914	9.22824	77	0.77176	124	24
37	706	9.22286	75	595	9.99385	944	9.22901	76	0.77099	019	23
38	734	9.22361	75	590	9.99383	974	9.22977	77	0.77023	5.8915	22
39	763	9.22435	74	585	9.99381	17004	9.23054	76	0.76946	811	21
40	16792	9.22509	74	98580	9.99379	17033	9.23130	76	0.76870	5.8708	20
41	820	9.22583	74	575	9.99377	063	9.23206	77	0.76794	605	19
42	849	9.22657	74	570	9.99375	093	9.23283	76	0.76717	502	18
43	878	9.22731	74	565	9.99372	123	9.23359	76	0.76641	400	17
44	906	9.22805	73	561	9.99370	153	9.23435	75	0.76565	298	16
45	16935	9.22878	73	98556	9.99368	17183	9.23510	75	0.76490	5.8197	15
46	964	9.22952	74	551	9.99366	213	9.23586	76	0.76414	095	14
47	992	9.23025	73	546	9.99364	243	9.23661	75	0.76339	5.7994	13
48	17021	9.23098	73	541	9.99362	273	9.23737	76	0.76263	894	12
49	050	9.23171	73	536	9.99359	303	9.23812	75	0.76188	794	11
50	17078	9.23244	73	98531	9.99357	17333	9.23887	75	0.76113	5.7694	10
51	107	9.23317	73	526	9.99355	363	9.23962	75	0.76038	594	9
52	136	9.23390	72	521	9.99353	393	9.24037	75	0.75963	495	8
53	164	9.23462	73	516	9.99351	423	9.24112	75	0.75888	396	7
54	193	9.23535	72	511	9.99348	453	9.24186	74	0.75814	297	6
55	17222	9.23607	72	98506	9.99346	17483	9.24261	74	0.75739	5.7199	5
56	250	9.23679	73	501	9.99344	513	9.24335	75	0.75665	101	4
57	279	9.23752	71	496	9.99342	543	9.24410	75	0.75590	004	3
58	308	9.23823	72	491	9.99340	573	9.24484	74	0.75516	5.6906	2
59	336	9.23895	72	486	9.99337	603	9.24558	74	0.75442	809	1
60	365	9.23967	72	481	9.99335	633	9.24632	74	0.75368	713	0
	Nat. Cos Log.		d.	Nat. Sin Log.		Nat. Cot Log.		c.d.	Log. Tan Nat.		'

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.		
0	17365	9.23967	72	98481	9.99335	2	17633	9.24632	74	0.75368	5.6713	60
1	393	9.24039	71	476	9.99333	2	663	9.24706	74	0.75394	617	59
2	422	9.24110	71	471	9.99331	3	693	9.24779	73	0.75221	521	58
3	451	9.24181	72	466	9.99328	2	723	9.24853	74	0.75147	425	57
4	479	9.24253	71	461	9.99326	2	753	9.24926	73	0.75074	329	56
5	17508	9.24324	71	98455	9.99324	2	17783	9.25000	74	0.75000	5.6234	55
6	537	9.24395	71	450	9.99322	3	813	9.25073	73	0.74927	140	54
7	565	9.24466	70	445	9.99319	2	843	9.25146	73	0.74854	045	53
8	594	9.24536	70	440	9.99317	2	873	9.25219	73	0.74781	5.5951	52
9	623	9.24607	70	435	9.99315	2	903	9.25292	73	0.74708	857	51
10	17651	9.24677	71	98430	9.99313	3	17933	9.25365	72	0.74635	5.5764	50
11	680	9.24748	71	425	9.99310	2	963	9.25437	73	0.74563	671	49
12	708	9.24818	70	420	9.99308	2	993	9.25510	73	0.74490	578	48
13	737	9.24888	70	414	9.99306	2	18023	9.25582	72	0.74418	485	47
14	766	9.24958	70	409	9.99304	3	053	9.25655	73	0.74345	393	46
15	17794	9.25028	70	98404	9.99301	2	18083	9.25727	72	0.74273	5.5301	45
16	823	9.25098	70	399	9.99299	2	113	9.25799	72	0.74201	209	44
17	852	9.25168	70	394	9.99297	3	143	9.25871	72	0.74129	118	43
18	880	9.25237	69	389	9.99294	2	173	9.25943	72	0.74057	026	42
19	909	9.25307	69	383	9.99292	2	203	9.26015	71	0.73985	5.4936	41
20	17937	9.25377	69	98378	9.99290	2	18233	9.26086	72	0.73914	5.4845	40
21	966	9.25445	69	373	9.99288	2	263	9.26158	72	0.73842	755	39
22	995	9.25514	69	368	9.99285	3	293	9.26229	71	0.73771	665	38
23	18023	9.25583	69	362	9.99283	2	323	9.26301	72	0.73699	575	37
24	052	9.25652	69	357	9.99281	2	353	9.26372	71	0.73628	486	36
25	18081	9.25721	69	98352	9.99278	3	18384	9.26443	71	0.73557	5.4397	35
26	109	9.25790	68	347	9.99276	2	414	9.26514	71	0.73486	308	34
27	138	9.25858	68	341	9.99274	3	444	9.26585	70	0.73415	219	33
28	166	9.25927	68	336	9.99271	2	474	9.26655	71	0.73345	131	32
29	195	9.25995	68	331	9.99269	2	504	9.26726	71	0.73274	043	31
30	18224	9.26063	68	98325	9.99267	3	18534	9.26797	70	0.73203	5.3955	30
31	252	9.26131	68	320	9.99264	2	564	9.26867	70	0.73133	868	29
32	281	9.26199	68	315	9.99262	2	594	9.26937	70	0.73063	781	28
33	309	9.26267	68	310	9.99260	2	624	9.27008	71	0.72992	694	27
34	338	9.26335	68	304	9.99257	3	654	9.27078	70	0.72922	607	26
35	18367	9.26403	67	98299	9.99255	3	18684	9.27148	70	0.72852	5.3521	25
36	395	9.26470	67	294	9.99252	2	714	9.27218	70	0.72782	435	24
37	424	9.26538	68	288	9.99250	2	745	9.27288	69	0.72712	349	23
38	452	9.26605	67	283	9.99248	3	775	9.27357	70	0.72643	263	22
39	481	9.26672	67	277	9.99245	2	805	9.27427	69	0.72573	178	21
40	18509	9.26739	67	98272	9.99243	2	18835	9.27496	70	0.72504	5.3093	20
41	538	9.26806	67	267	9.99241	3	865	9.27566	69	0.72434	008	19
42	567	9.26873	67	261	9.99238	2	895	9.27635	69	0.72365	5.2924	18
43	595	9.26940	67	256	9.99236	3	925	9.27704	69	0.72296	839	17
44	624	9.27007	67	250	9.99233	2	955	9.27773	69	0.72227	755	16
45	18652	9.27073	67	98245	9.99231	2	18986	9.27842	69	0.72158	5.2672	15
46	681	9.27140	66	240	9.99229	3	19016	9.27911	69	0.72089	588	14
47	710	9.27206	66	234	9.99226	2	046	9.27980	69	0.72020	505	13
48	738	9.27273	66	229	9.99224	3	076	9.28049	68	0.71951	422	12
49	767	9.27339	66	223	9.99221	2	106	9.28117	69	0.71883	339	11
50	18795	9.27405	66	98218	9.99219	2	19136	9.28186	68	0.71814	5.2257	10
51	824	9.27471	66	212	9.99217	3	166	9.28254	69	0.71746	174	9
52	852	9.27537	65	207	9.99214	2	197	9.28323	68	0.71677	092	8
53	881	9.27602	66	201	9.99212	3	227	9.28391	68	0.71609	011	7
54	910	9.27668	66	196	9.99209	2	257	9.28459	68	0.71541	5.1929	6
55	18938	9.27734	65	98190	9.99207	3	19287	9.28527	68	0.71473	5.1848	5
56	967	9.27799	65	185	9.99204	2	317	9.28595	67	0.71405	767	4
57	995	9.27864	66	179	9.99202	2	347	9.28662	68	0.71338	686	3
58	19024	9.27930	66	174	9.99200	3	378	9.28730	68	0.71270	606	2
59	052	9.27995	65	168	9.99197	2	408	9.28798	67	0.71202	526	1
60	081	9.28060	65	163	9.99195	2	438	9.28865	67	0.71135	446	0
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.		

'	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat. Tan Log. c.d.	Log. Cot Nat.	'
0	19081 9.28060	65 98163 9.99195	3 19438 9.28865	68 0.71135 5.1446	60
1	109 9.28125	65 157 9.99192	2 468 9.28933	67 0.71007 366	59
2	138 9.28190	65 152 9.99190	2 498 9.29000	67 0.71000 286	58
3	167 9.28254	65 146 9.99187	2 529 9.29067	67 0.70933 207	57
4	195 9.28319	65 140 9.99185	2 559 9.29134	67 0.70866 128	56
5	19224 9.28384	65 98135 9.99182	3 19589 9.29201	67 0.70799 5.1049	55
6	252 9.28448	64 129 9.99180	2 619 9.29268	67 0.70732 5.0970	54
7	281 9.28512	64 124 9.99177	3 649 9.29335	67 0.70665 892	53
8	309 9.28577	65 118 9.99175	2 680 9.29402	67 0.70593 814	52
9	338 9.28641	64 112 9.99172	3 710 9.29468	66 0.70532 730	51
10	19366 9.28705	64 98107 9.99170	2 19740 9.29535	67 0.70465 5.0658	50
11	395 9.28769	64 101 9.99167	3 770 9.29601	66 0.70399 581	49
12	423 9.28833	64 096 9.99165	2 801 9.29668	67 0.70332 504	48
13	452 9.28896	63 090 9.99162	2 831 9.29734	66 0.70266 427	47
14	481 9.28960	64 084 9.99160	3 861 9.29800	66 0.70200 350	46
15	19509 9.29024	63 98079 9.99157	2 19891 9.29866	66 0.70134 5.0273	45
16	538 9.29087	63 073 9.99155	3 921 9.29932	66 0.70068 197	44
17	566 9.29150	63 067 9.99152	2 952 9.29998	66 0.70002 121	43
18	595 9.29214	64 061 9.99150	3 982 9.30064	66 0.69936 045	42
19	623 9.29277	63 056 9.99147	3 20012 9.30130	66 0.69870 4.9969	41
20	19652 9.29340	63 98050 9.99145	2 20042 9.30195	66 0.69805 4.9894	40
21	680 9.29403	63 044 9.99142	3 073 9.30261	66 0.69739 819	39
22	709 9.29466	63 039 9.99140	2 103 9.30326	65 0.69674 744	38
23	737 9.29529	63 033 9.99137	3 133 9.30391	65 0.69609 669	37
24	766 9.29591	62 027 9.99135	2 164 9.30457	66 0.69543 594	36
25	19794 9.29654	63 98021 9.99132	3 20194 9.30522	65 0.69478 4.9520	35
26	823 9.29716	62 016 9.99130	2 224 9.30587	65 0.69413 446	34
27	851 9.29779	63 010 9.99127	3 254 9.30652	65 0.69348 372	33
28	880 9.29841	62 004 9.99124	2 285 9.30717	65 0.69283 298	32
29	908 9.29903	62 97998 9.99122	3 315 9.30782	65 0.69218 225	31
30	19937 9.29966	63 97992 9.99119	2 20345 9.30846	64 0.69154 4.9152	30
31	965 9.30028	62 987 9.99117	3 376 9.30911	65 0.69089 078	29
32	994 9.30090	62 981 9.99114	2 406 9.30975	64 0.69025 006	28
33	20022 9.30151	61 975 9.99112	3 436 9.31040	65 0.68960 4.8933	27
34	051 9.30213	62 969 9.99109	2 466 9.31104	64 0.68896 860	26
35	20079 9.30275	62 97965 9.99106	3 20497 9.31168	64 0.68832 4.8788	25
36	108 9.30336	61 958 9.99104	2 527 9.31233	65 0.68767 716	24
37	136 9.30398	61 952 9.99101	3 557 9.31297	64 0.68703 644	23
38	165 9.30459	61 946 9.99099	2 588 9.31361	64 0.68639 573	22
39	193 9.30521	62 940 9.99096	3 618 9.31425	64 0.68575 501	21
40	20222 9.30582	61 97934 9.99093	2 20648 9.31489	64 0.68511 4.8430	20
41	250 9.30643	61 928 9.99091	3 679 9.31552	63 0.68448 359	19
42	279 9.30704	61 922 9.99088	2 709 9.31616	64 0.68384 288	18
43	307 9.30765	61 916 9.99086	3 739 9.31679	63 0.68321 218	17
44	336 9.30826	61 910 9.99083	2 770 9.31743	64 0.68257 147	16
45	20364 9.30887	62 97905 9.99080	3 20800 9.31806	63 0.68194 4.8077	15
46	393 9.30947	60 899 9.99078	2 830 9.31870	64 0.68130 007	14
47	421 9.31008	61 893 9.99075	3 861 9.31933	63 0.68067 4.7937	13
48	450 9.31068	61 887 9.99072	2 891 9.31996	63 0.68004 867	12
49	478 9.31129	61 881 9.99070	3 921 9.32059	63 0.67941 798	11
50	20507 9.31189	61 97875 9.99067	2 20952 9.32122	63 0.67878 4.7729	10
51	535 9.31250	60 869 9.99064	3 982 9.32185	63 0.67815 659	9
52	563 9.31310	60 863 9.99062	2 21013 9.32248	63 0.67752 591	8
53	592 9.31370	60 857 9.99059	3 043 9.32311	63 0.67689 522	7
54	620 9.31430	60 851 9.99056	2 073 9.32373	62 0.67627 453	6
55	20649 9.31490	60 97845 9.99054	3 21104 9.32436	63 0.67564 4.7385	5
56	677 9.31549	59 839 9.99051	2 134 9.32498	62 0.67502 317	4
57	706 9.31609	60 833 9.99048	3 164 9.32561	63 0.67439 249	3
58	734 9.31669	60 827 9.99046	2 195 9.32623	62 0.67377 181	2
59	763 9.31728	59 821 9.99043	3 225 9.32685	62 0.67315 114	1
60	791 9.31788	60 815 9.99040	2 256 9.32747	62 0.67253 046	0
	Nat. Cos Log. d.	Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.	'

	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.	
0	20791	9.31788	97815	9.99040	21256	9.32747	0.67253	4.7046	60
1	820	9.31847	809	9.99038	286	9.32810	0.67190	4.6979	59
2	848	9.31907	803	9.99035	316	9.32872	0.67128	912	58
3	877	9.31966	797	9.99032	347	9.32933	0.67067	845	57
4	905	9.32025	791	9.99030	377	9.32995	0.67005	779	56
5	20933	9.32084	97784	9.99027	21408	9.33057	0.66943	4.6712	55
6	962	9.32143	778	9.99024	438	9.33119	0.66881	646	54
7	990	9.32202	772	9.99022	469	9.33180	0.66820	580	53
8	21019	9.32261	766	9.99019	499	9.33242	0.66758	514	52
9	047	9.32319	760	9.99016	529	9.33303	0.66697	448	51
10	21076	9.32378	97754	9.99013	21560	9.33365	0.66635	4.6382	50
11	104	9.32437	748	9.99011	590	9.33426	0.66574	317	49
12	132	9.32495	742	9.99008	621	9.33487	0.66513	252	48
13	161	9.32553	735	9.99005	651	9.33548	0.66452	187	47
14	189	9.32612	729	9.99002	682	9.33609	0.66391	122	46
15	21218	9.32670	97723	9.99000	21712	9.33670	0.66330	4.6057	45
16	246	9.32728	717	9.98997	743	9.33731	0.66269	4.5993	44
17	275	9.32786	711	9.98994	773	9.33792	0.66208	928	43
18	303	9.32844	705	9.98991	804	9.33853	0.66147	864	42
19	331	9.32902	698	9.98989	834	9.33913	0.66087	800	41
20	21360	9.32960	97692	9.98986	21864	9.33974	0.66026	4.5736	40
21	388	9.33018	686	9.98983	895	9.34034	0.65966	673	39
22	417	9.33075	680	9.98980	925	9.34095	0.65905	609	38
23	445	9.33133	673	9.98978	956	9.34155	0.65845	546	37
24	474	9.33190	667	9.98975	986	9.34215	0.65785	483	36
25	21502	9.33248	97661	9.98972	22017	9.34276	0.65724	4.5420	35
26	530	9.33305	655	9.98969	047	9.34336	0.65664	357	34
27	559	9.33362	648	9.98967	078	9.34396	0.65604	294	33
28	587	9.33420	642	9.98964	108	9.34456	0.65544	232	32
29	616	9.33477	636	9.98961	139	9.34516	0.65484	169	31
30	21644	9.33534	97630	9.98958	22169	9.34576	0.65424	4.5107	30
31	672	9.33591	623	9.98955	200	9.34635	0.65365	045	29
32	701	9.33647	617	9.98953	231	9.34695	0.65305	4.4983	28
33	729	9.33704	611	9.98950	261	9.34755	0.65245	922	27
34	758	9.33761	604	9.98947	292	9.34814	0.65186	860	26
35	21786	9.33818	97598	9.98944	22322	9.34874	0.65126	4.4799	25
36	814	9.33874	592	9.98941	353	9.34933	0.65067	737	24
37	843	9.33931	585	9.98938	383	9.34992	0.65008	676	23
38	871	9.33987	579	9.98936	414	9.35051	0.64949	615	22
39	899	9.34043	573	9.98933	444	9.35111	0.64889	555	21
40	21928	9.34100	97566	9.98930	22475	9.35170	0.64830	4.4494	20
41	956	9.34156	560	9.98927	505	9.35229	0.64771	434	19
42	985	9.34212	553	9.98924	536	9.35288	0.64712	373	18
43	22013	9.34268	547	9.98921	567	9.35347	0.64653	313	17
44	041	9.34324	541	9.98919	597	9.35405	0.64595	253	16
45	22070	9.34380	97534	9.98916	22628	9.35464	0.64536	4.4194	15
46	098	9.34436	528	9.98913	658	9.35523	0.64477	134	14
47	126	9.34491	521	9.98910	689	9.35581	0.64419	075	13
48	155	9.34547	515	9.98907	719	9.35640	0.64360	015	12
49	183	9.34602	508	9.98904	750	9.35698	0.64302	4.3956	11
50	22212	9.34658	97502	9.98901	22781	9.35757	0.64243	4.3897	10
51	240	9.34713	496	9.98898	811	9.35815	0.64185	838	9
52	268	9.34769	489	9.98896	842	9.35873	0.64127	779	8
53	297	9.34824	483	9.98893	872	9.35931	0.64069	721	7
54	325	9.34879	476	9.98890	903	9.35989	0.64011	662	6
55	22353	9.34934	97470	9.98887	22934	9.36047	0.63953	4.3604	5
56	382	9.34989	463	9.98884	964	9.36105	0.63895	546	4
57	410	9.35044	457	9.98881	995	9.36163	0.63837	488	3
58	438	9.35099	450	9.98878	23026	9.36221	0.63779	430	2
59	467	9.35154	444	9.98875	056	9.36279	0.63721	372	1
60	495	9.35209	437	9.98872	087	9.36336	0.63664	315	0
Nat. Cos		Log. d.	Nat. Sin		Log. d.	Nat. Cot		Log. Tan	Nat.

'	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat. Tan Log. c.d.	Log. Cot Nat.	
0	22495 9.35209	97437 9.98872	23087 9.36336	0.63664 4.3315	60
1	523 9.35203	430 9.98809	117 9.36394	0.63606 257	59
2	552 9.35318	424 9.98807	148 9.36452	0.63548 200	58
3	580 9.35373	417 9.98804	179 9.36509	0.63491 143	57
4	608 9.35427	411 9.98801	209 9.36566	0.63434 086	56
5	22637 9.35481	97404 9.98858	23240 9.36624	0.63376 4.3029	55
6	665 9.35536	398 9.98855	271 9.36681	0.63319 4.2972	54
7	693 9.35590	391 9.98852	301 9.36738	0.63262 916	53
8	722 9.35644	384 9.98849	332 9.36795	0.63205 859	52
9	750 9.35698	378 9.98846	363 9.36852	0.63148 803	51
10	22778 9.35752	97371 9.98843	23393 9.36909	0.63091 4.2747	50
11	807 9.35806	365 9.98840	424 9.36966	0.63034 691	49
12	835 9.35860	358 9.98837	455 9.37023	0.62977 635	48
13	863 9.35914	351 9.98834	485 9.37080	0.62920 580	47
14	892 9.35968	345 9.98831	516 9.37137	0.62863 524	46
15	22920 9.36022	97338 9.98828	23547 9.37193	0.62807 4.2468	45
16	948 9.36075	331 9.98825	578 9.37250	0.62750 413	44
17	977 9.36129	325 9.98822	608 9.37306	0.62694 358	43
18	23005 9.36182	318 9.98819	639 9.37363	0.62637 303	42
19	033 9.36236	311 9.98816	670 9.37419	0.62581 248	41
20	23062 9.36289	97304 9.98813	23700 9.37476	0.62524 4.2193	40
21	090 9.36342	298 9.98810	731 9.37532	0.62468 139	39
22	118 9.36395	291 9.98807	762 9.37588	0.62412 084	38
23	146 9.36449	284 9.98804	793 9.37644	0.62356 030	37
24	175 9.36502	278 9.98801	823 9.37700	0.62300 4.1976	36
25	23203 9.36555	97271 9.98798	23854 9.37755	0.62244 4.1922	35
26	231 9.36608	264 9.98795	885 9.37812	0.62188 868	34
27	260 9.36660	257 9.98792	916 9.37868	0.62132 814	33
28	288 9.36713	251 9.98789	946 9.37924	0.62076 760	32
29	316 9.36766	244 9.98786	977 9.37980	0.62020 706	31
30	23345 9.36819	97237 9.98783	24008 9.38035	0.61965 4.1653	30
31	373 9.36871	230 9.98780	039 9.38091	0.61909 600	29
32	401 9.36924	223 9.98777	069 9.38147	0.61853 547	28
33	429 9.36976	217 9.98774	100 9.38202	0.61798 493	27
34	458 9.37028	210 9.98771	131 9.38257	0.61743 441	26
35	23486 9.37081	97203 9.98768	24162 9.38313	0.61687 4.1388	25
36	514 9.37133	196 9.98765	193 9.38368	0.61632 335	24
37	542 9.37185	189 9.98762	223 9.38423	0.61577 282	23
38	571 9.37237	182 9.98759	254 9.38479	0.61521 230	22
39	599 9.37289	176 9.98756	285 9.38534	0.61466 178	21
40	23627 9.37341	97169 9.98753	24316 9.38589	0.61411 4.1126	20
41	656 9.37393	162 9.98750	347 9.38644	0.61356 074	19
42	684 9.37445	155 9.98746	377 9.38699	0.61301 022	18
43	712 9.37497	148 9.98743	408 9.38754	0.61246 4.0970	17
44	740 9.37549	141 9.98740	439 9.38808	0.61192 918	16
45	23769 9.37600	97134 9.98737	24470 9.38863	0.61137 4.0867	15
46	797 9.37652	127 9.98734	501 9.38918	0.61082 815	14
47	825 9.37703	120 9.98731	532 9.38972	0.61028 764	13
48	853 9.37755	113 9.98728	562 9.39027	0.60973 713	12
49	882 9.37806	106 9.98725	593 9.39082	0.60918 662	11
50	23910 9.37858	97100 9.98722	24624 9.39136	0.60864 4.0611	10
51	938 9.37909	093 9.98719	655 9.39190	0.60810 560	9
52	966 9.37960	086 9.98715	686 9.39245	0.60755 509	8
53	995 9.38011	079 9.98712	717 9.39299	0.60701 459	7
54	24023 9.38062	072 9.98709	747 9.39353	0.60647 408	6
55	24051 9.38113	97065 9.98706	24778 9.39407	0.60593 4.0358	5
56	079 9.38164	058 9.98703	809 9.39461	0.60539 308	4
57	108 9.38215	051 9.98700	840 9.39515	0.60485 257	3
58	136 9.38266	044 9.98697	871 9.39569	0.60431 207	2
59	164 9.38317	037 9.98694	902 9.39623	0.60377 158	1
60	192 9.38368	030 9.98690	933 9.39677	0.60323 108	0
	Nat. Cos Log. d.	Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.	'

	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.	
0	24192 9.38368	50	97030 9.98690	3	24933 9.39677	54	0.60323 4.0108	60
1	220 9.38418	51	023 9.98687	3	964 9.39731	54	0.60260 058	59
2	249 9.38469	50	015 9.98684	3	995 9.39785	54	0.60215 009	58
3	277 9.38519	51	008 9.98681	3	25026 9.39838	53	0.60162 3.9959	57
4	305 9.38570	50	001 9.98678	3	056 9.39892	54	0.60108 910	56
5	24333 9.38620	50	96994 9.98675	3	25087 9.39945	53	0.60055 3.9861	55
6	362 9.38670	51	987 9.98671	4	118 9.39999	54	0.60001 812	54
7	390 9.38721	50	980 9.98668	3	149 9.40052	53	0.59948 763	53
8	418 9.38771	50	973 9.98665	3	180 9.40106	54	0.59894 714	52
9	446 9.38821	50	966 9.98662	3	211 9.40159	53	0.59841 665	51
10	24474 9.38871	50	96959 9.98659	3	25242 9.40212	53	0.59788 3.9617	50
11	503 9.38921	50	952 9.98656	3	273 9.40266	54	0.59734 568	49
12	531 9.38971	50	945 9.98652	4	304 9.40319	53	0.59681 520	48
13	559 9.39021	50	937 9.98649	3	335 9.40372	53	0.59628 471	47
14	587 9.39071	50	930 9.98646	3	366 9.40425	53	0.59575 423	46
15	24615 9.39121	49	96923 9.98643	3	25397 9.40478	53	0.59522 3.9375	45
16	644 9.39170	49	916 9.98640	4	428 9.40531	53	0.59469 327	44
17	672 9.39220	50	909 9.98636	4	459 9.40584	53	0.59416 279	43
18	700 9.39270	49	902 9.98633	3	490 9.40636	52	0.59364 232	42
19	728 9.39319	49	894 9.98630	3	521 9.40689	53	0.59311 184	41
20	24756 9.39369	49	96887 9.98627	3	25552 9.40742	53	0.59258 3.9136	40
21	784 9.39418	49	880 9.98623	3	583 9.40795	52	0.59205 089	39
22	813 9.39467	50	873 9.98620	3	614 9.40847	52	0.59153 042	38
23	841 9.39517	49	866 9.98617	3	645 9.40900	53	0.59100 3.8995	37
24	869 9.39566	49	858 9.98614	3	676 9.40952	52	0.59048 947	36
25	24897 9.39615	49	96851 9.98610	4	25707 9.41005	53	0.58995 3.8900	35
26	925 9.39664	49	844 9.98607	3	738 9.41057	52	0.58943 854	34
27	954 9.39713	49	837 9.98604	3	769 9.41109	52	0.58891 807	33
28	982 9.39762	49	829 9.98601	3	800 9.41161	53	0.58839 760	32
29	25010 9.39811	49	822 9.98597	4	831 9.41214	52	0.58786 714	31
30	25038 9.39860	49	96815 9.98594	3	25862 9.41266	52	0.58734 3.8667	30
31	066 9.39909	49	807 9.98591	3	893 9.41318	52	0.58682 621	29
32	094 9.39958	48	800 9.98588	3	924 9.41370	52	0.58630 575	28
33	122 9.40006	48	793 9.98584	3	955 9.41422	52	0.58578 528	27
34	151 9.40055	48	786 9.98581	3	986 9.41474	52	0.58526 482	26
35	25179 9.40103	49	96778 9.98578	4	26017 9.41526	52	0.58474 3.8436	25
36	207 9.40152	48	771 9.98574	4	048 9.41578	52	0.58422 391	24
37	235 9.40200	48	764 9.98571	3	079 9.41629	52	0.58371 345	23
38	263 9.40249	48	756 9.98568	3	110 9.41681	52	0.58319 299	22
39	291 9.40297	48	749 9.98565	3	141 9.41733	51	0.58267 254	21
40	25320 9.40346	48	96742 9.98561	4	26172 9.41784	52	0.58216 3.8208	20
41	348 9.40394	48	734 9.98558	3	203 9.41836	52	0.58164 163	19
42	376 9.40442	48	727 9.98555	3	235 9.41887	52	0.58113 118	18
43	404 9.40490	48	719 9.98551	3	266 9.41939	51	0.58061 073	17
44	432 9.40538	48	712 9.98548	3	297 9.41990	51	0.58010 026	16
45	25460 9.40586	48	96705 9.98545	4	26328 9.42041	51	0.57959 3.7983	15
46	488 9.40634	48	697 9.98541	4	359 9.42093	51	0.57907 938	14
47	516 9.40682	48	690 9.98538	3	390 9.42144	51	0.57856 893	13
48	545 9.40730	48	682 9.98535	3	421 9.42195	51	0.57805 848	12
49	573 9.40778	48	675 9.98531	4	452 9.42246	51	0.57754 804	11
50	25601 9.40825	47	96667 9.98528	3	26483 9.42297	51	0.57703 3.7760	10
51	629 9.40873	48	660 9.98525	3	515 9.42348	51	0.57652 715	9
52	657 9.40921	48	653 9.98521	4	546 9.42399	51	0.57601 671	8
53	685 9.40968	47	645 9.98518	3	577 9.42450	51	0.57550 627	7
54	713 9.41016	47	638 9.98515	3	608 9.42501	51	0.57499 583	6
55	25741 9.41063	47	96630 9.98511	4	26639 9.42552	51	0.57448 3.7539	5
56	769 9.41111	48	623 9.98508	3	670 9.42603	50	0.57397 495	4
57	798 9.41158	47	615 9.98505	4	701 9.42653	51	0.57347 451	3
58	826 9.41205	47	608 9.98501	4	733 9.42704	51	0.57296 408	2
59	854 9.41252	47	600 9.98498	3	764 9.42755	50	0.57245 364	1
60	882 9.41300	48	593 9.98494	4	795 9.42805	51	0.57195 321	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.	

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.		
0	25882	0.41300	47	96593	0.08404	3	26795	0.42805	51	0.57195	3.7321	60
1	910	0.41347	47	585	0.98491	3	826	0.42850	50	0.57144	277	59
2	938	0.41394	47	578	0.98488	4	857	0.42906	51	0.57094	234	58
3	966	0.41441	47	570	0.98484	3	888	0.42957	50	0.57043	191	57
4	994	0.41488	47	562	0.98481	3	920	0.43007	50	0.56993	148	56
5	26022	0.41535	47	96555	0.98477	4	26951	0.43057	51	0.56943	3.7105	55
6	050	0.41582	46	547	0.98474	3	982	0.43108	50	0.56892	062	54
7	079	0.41628	46	540	0.98471	3	27013	0.43158	50	0.56842	019	53
8	107	0.41675	47	532	0.98467	4	044	0.43208	50	0.56792	3.6976	52
9	135	0.41722	47	524	0.98464	3	076	0.43258	50	0.56742	933	51
10	26163	0.41768	46	96517	0.98460	4	27107	0.43308	50	0.56692	3.6891	50
11	191	0.41815	47	503	0.98457	3	138	0.43358	50	0.56642	848	49
12	219	0.41861	46	502	0.98453	4	169	0.43408	50	0.56592	806	48
13	247	0.41908	47	494	0.98450	3	201	0.43458	50	0.56542	764	47
14	275	0.41954	46	486	0.98447	3	232	0.43508	50	0.56492	722	46
15	26303	0.42001	47	96479	0.98443	4	27263	0.43558	50	0.56442	3.6680	45
16	331	0.42047	46	471	0.98440	3	294	0.43607	49	0.56393	638	44
17	359	0.42093	47	463	0.98436	4	326	0.43657	50	0.56343	596	43
18	387	0.42140	46	456	0.98433	3	357	0.43707	50	0.56293	554	42
19	415	0.42186	46	448	0.98429	4	388	0.43756	50	0.56244	512	41
20	26443	0.42232	46	96440	0.98426	3	27419	0.43806	49	0.56194	3.6470	40
21	471	0.42278	46	433	0.98422	4	451	0.43855	49	0.56145	429	39
22	500	0.42324	46	425	0.98419	3	482	0.43905	50	0.56095	387	38
23	528	0.42370	46	417	0.98415	4	513	0.43954	49	0.56046	346	37
24	556	0.42416	46	410	0.98412	3	545	0.44004	50	0.55996	305	36
25	26584	0.42461	45	96402	0.98409	4	27576	0.44053	49	0.55947	3.6264	35
26	612	0.42507	46	394	0.98405	3	607	0.44102	49	0.55898	222	34
27	640	0.42553	46	386	0.98402	3	638	0.44151	49	0.55849	181	33
28	668	0.42599	45	379	0.98398	4	670	0.44201	50	0.55799	140	32
29	696	0.42644	46	371	0.98395	3	701	0.44250	49	0.55750	100	31
30	26724	0.42690	45	96363	0.98391	4	27732	0.44299	49	0.55701	3.6059	30
31	752	0.42735	46	355	0.98388	3	764	0.44348	49	0.55652	018	29
32	780	0.42781	46	347	0.98384	4	795	0.44397	49	0.55603	3.5978	28
33	808	0.42826	46	340	0.98381	3	826	0.44446	49	0.55554	937	27
34	836	0.42872	45	332	0.98377	4	858	0.44495	49	0.55505	897	26
35	26864	0.42917	45	96324	0.98373	3	27889	0.44544	48	0.55456	3.5856	25
36	892	0.42962	46	316	0.98370	4	921	0.44592	48	0.55408	816	24
37	920	0.43008	45	308	0.98366	3	952	0.44641	49	0.55359	776	23
38	948	0.43053	45	301	0.98363	3	983	0.44690	49	0.55310	736	22
39	976	0.43098	45	293	0.98359	4	28015	0.44738	48	0.55262	696	21
40	27004	0.43143	45	96285	0.98356	3	28046	0.44787	49	0.55213	3.5656	20
41	032	0.43188	45	277	0.98352	4	077	0.44836	49	0.55164	616	19
42	060	0.43233	45	269	0.98349	3	109	0.44884	48	0.55116	576	18
43	088	0.43278	45	261	0.98345	3	140	0.44933	48	0.55067	536	17
44	116	0.43323	44	253	0.98342	3	172	0.44981	48	0.55019	497	16
45	27144	0.43367	44	96246	0.98338	4	28203	0.45029	49	0.54971	3.5457	15
46	172	0.43412	45	238	0.98334	3	234	0.45078	48	0.54922	418	14
47	200	0.43457	45	230	0.98331	3	266	0.45126	48	0.54874	379	13
48	228	0.43502	44	222	0.98327	3	297	0.45174	48	0.54826	339	12
49	256	0.43546	45	214	0.98324	3	329	0.45222	48	0.54778	300	11
50	27284	0.43591	44	96206	0.98320	4	28360	0.45271	49	0.54729	3.5261	10
51	312	0.43635	45	198	0.98317	3	391	0.45319	48	0.54681	222	9
52	340	0.43680	44	190	0.98313	4	423	0.45367	48	0.54633	183	8
53	368	0.43724	45	182	0.98309	3	454	0.45415	48	0.54585	144	7
54	396	0.43769	44	174	0.98306	3	486	0.45463	48	0.54537	105	6
55	27424	0.43813	44	96166	0.98302	4	28517	0.45511	48	0.54489	3.5067	5
56	452	0.43857	44	158	0.98299	3	549	0.45559	47	0.54441	028	4
57	480	0.43901	44	150	0.98295	4	580	0.45606	48	0.54394	3.4989	3
58	508	0.43946	45	142	0.98291	3	612	0.45654	48	0.54346	951	2
59	536	0.43990	44	134	0.98288	3	643	0.45702	48	0.54298	912	1
60	564	0.44034	44	126	0.98284	4	675	0.45750	48	0.54250	874	0
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.		

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.	'			
0	27564	9.44034	44	96126	9.98284	3	28675	9.45750	47	0.54250	3.4874	60
1	592	9.44078	44	118	9.98281	4	706	9.45797	48	0.54203	836	59
2	620	9.44122	44	110	9.98277	4	738	9.45845	47	0.54155	798	58
3	648	9.44166	44	102	9.98273	3	769	9.45892	47	0.54108	760	57
4	676	9.44210	43	094	9.98270	4	801	9.45940	48	0.54060	722	56
5	27704	9.44253	44	96086	9.98266	4	28832	9.45987	47	0.54013	3.4684	55
6	731	9.44297	44	078	9.98262	4	864	9.46035	48	0.53965	646	54
7	759	9.44341	44	070	9.98259	3	895	9.46082	47	0.53918	608	53
8	787	9.44385	44	062	9.98255	4	927	9.46130	48	0.53870	570	52
9	815	9.44428	43	054	9.98251	4	958	9.46177	47	0.53823	533	51
10	27843	9.44472	44	96046	9.98248	3	28990	9.46224	47	0.53776	3.4495	50
11	871	9.44516	44	037	9.98244	4	29021	9.46271	48	0.53729	458	49
12	899	9.44559	43	029	9.98240	4	053	9.46319	47	0.53681	420	48
13	927	9.44602	43	021	9.98237	4	084	9.46366	47	0.53634	383	47
14	955	9.44646	44	013	9.98233	4	116	9.46413	47	0.53587	346	46
15	27983	9.44689	43	96005	9.98229	4	29147	9.46460	47	0.53540	3.4308	45
16	28011	9.44733	44	95997	9.98226	3	179	9.46507	47	0.53493	271	44
17	039	9.44776	43	989	9.98222	4	210	9.46554	47	0.53446	234	43
18	067	9.44819	43	981	9.98218	4	242	9.46601	47	0.53399	197	42
19	095	9.44862	43	972	9.98215	3	274	9.46648	47	0.53352	160	41
20	28123	9.44905	43	95964	9.98211	4	29305	9.46694	46	0.53306	3.4124	40
21	150	9.44948	44	956	9.98207	4	337	9.46741	47	0.53259	087	39
22	178	9.44992	44	948	9.98204	3	368	9.46788	47	0.53212	050	38
23	206	9.45035	42	940	9.98200	4	400	9.46835	47	0.53165	014	37
24	234	9.45077	43	931	9.98196	4	432	9.46881	47	0.53119	3.3977	36
25	28262	9.45120	43	95923	9.98192	4	29463	9.46928	46	0.53072	3.3941	35
26	290	9.45163	43	915	9.98189	3	495	9.46975	47	0.53025	904	34
27	318	9.45206	43	907	9.98185	4	526	9.47021	47	0.52979	868	33
28	346	9.45249	43	898	9.98181	4	558	9.47068	47	0.52932	832	32
29	374	9.45292	42	890	9.98177	3	590	9.47114	46	0.52886	796	31
30	28402	9.45334	43	95882	9.98174	3	29621	9.47160	47	0.52840	3.3759	30
31	429	9.45377	43	874	9.98170	4	653	9.47207	47	0.52793	723	29
32	457	9.45419	43	865	9.98166	4	685	9.47253	46	0.52747	687	28
33	485	9.45462	42	857	9.98162	4	716	9.47299	46	0.52701	652	27
34	513	9.45504	42	849	9.98159	3	748	9.47346	47	0.52654	616	26
35	28541	9.45547	43	95841	9.98155	4	29780	9.47392	46	0.52608	3.3580	25
36	569	9.45589	43	832	9.98151	4	811	9.47438	46	0.52562	544	24
37	597	9.45632	43	824	9.98147	3	843	9.47484	46	0.52516	509	23
38	625	9.45674	42	816	9.98144	3	875	9.47530	46	0.52470	473	22
39	652	9.45716	42	807	9.98140	4	906	9.47576	46	0.52424	438	21
40	28680	9.45758	42	95799	9.98136	4	29938	9.47622	46	0.52378	3.3402	20
41	708	9.45801	43	791	9.98132	3	970	9.47668	46	0.52332	367	19
42	736	9.45843	42	782	9.98129	3	30001	9.47714	46	0.52286	332	18
43	764	9.45885	42	774	9.98125	4	033	9.47760	46	0.52240	297	17
44	792	9.45927	42	766	9.98121	4	065	9.47806	46	0.52194	261	16
45	28820	9.45969	42	95757	9.98117	4	30097	9.47852	45	0.52148	3.3226	15
46	847	9.46011	42	749	9.98113	4	128	9.47897	45	0.52103	191	14
47	875	9.46053	42	740	9.98110	3	160	9.47943	46	0.52057	156	13
48	903	9.46095	41	732	9.98106	4	192	9.47989	46	0.52011	122	12
49	931	9.46136	42	724	9.98102	4	224	9.48035	45	0.51965	087	11
50	28959	9.46178	42	95715	9.98098	4	30255	9.48080	46	0.51920	3.3052	10
51	987	9.46220	42	707	9.98094	4	287	9.48126	46	0.51874	017	9
52	29015	9.46262	41	698	9.98090	3	319	9.48171	46	0.51829	3.2983	8
53	042	9.46303	41	690	9.98087	3	351	9.48217	45	0.51783	948	7
54	070	9.46345	41	681	9.98083	4	382	9.48262	45	0.51738	914	6
55	29098	9.46386	42	95673	9.98079	4	30414	9.48307	46	0.51693	3.2879	5
56	126	9.46428	42	664	9.98075	4	446	9.48353	45	0.51647	845	4
57	154	9.46469	41	656	9.98071	4	478	9.48398	45	0.51602	811	3
58	182	9.46511	41	647	9.98067	4	509	9.48443	46	0.51557	777	2
59	209	9.46552	41	639	9.98063	3	541	9.48489	46	0.51511	743	1
60	237	9.46594	42	630	9.98060	3	573	9.48534	45	0.51466	709	0
	Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	'

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.				
0	29237	9.46594		95630	9.98060		30573	9.48534		0.51466	3.2709	60
1	265	9.46035	41	622	9.98056	4	605	9.48579	45	0.51421	675	59
2	293	9.46076	41	613	9.98052	4	637	9.48624	45	0.51376	641	58
3	321	9.46717	41	605	9.98048	4	669	9.48669	45	0.51331	607	57
4	348	9.46758	42	596	9.98044	4	700	9.48714	45	0.51286	573	56
5	29376	9.46800	41	95588	9.98040	4	30732	9.48759	45	0.51241	3.2539	55
6	404	9.46841	41	579	9.98036	4	764	9.48804	45	0.51196	506	54
7	432	9.46882	41	571	9.98032	4	796	9.48849	45	0.51151	472	53
8	460	9.46923	41	562	9.98029	3	828	9.48894	45	0.51106	438	52
9	487	9.46964	41	554	9.98025	4	860	9.48939	45	0.51061	405	51
10	29515	9.47005	40	95545	9.98021	4	30891	9.48984	45	0.51016	3.2371	50
11	543	9.47045	41	536	9.98017	4	923	9.49029	44	0.50971	338	49
12	571	9.47086	41	528	9.98013	4	955	9.49073	44	0.50927	305	48
13	599	9.47127	41	519	9.98009	4	987	9.49118	45	0.50882	272	47
14	626	9.47168	41	511	9.98005	4	31019	9.49163	45	0.50837	238	46
15	29654	9.47209	40	95502	9.98001	4	31051	9.49207	44	0.50793	3.2205	45
16	682	9.47249	41	493	9.97997	4	083	9.49252	45	0.50748	172	44
17	710	9.47290	41	485	9.97993	4	115	9.49296	44	0.50704	139	43
18	737	9.47330	40	476	9.97989	4	147	9.49341	45	0.50659	106	42
19	765	9.47371	40	467	9.97986	3	178	9.49385	44	0.50615	073	41
20	29793	9.47411	41	95459	9.97982	4	31210	9.49430	45	0.50570	3.2041	40
21	821	9.47452	41	450	9.97978	4	242	9.49474	44	0.50526	008	39
22	849	9.47492	41	441	9.97974	4	274	9.49519	45	0.50481	3.1975	38
23	876	9.47533	41	433	9.97970	4	306	9.49563	44	0.50437	943	37
24	904	9.47573	40	424	9.97966	4	338	9.49607	44	0.50393	910	36
25	29932	9.47613	41	95415	9.97962	4	31370	9.49652	45	0.50348	3.1878	35
26	960	9.47654	41	407	9.97958	4	402	9.49696	44	0.50304	845	34
27	987	9.47694	40	398	9.97954	4	434	9.49740	44	0.50260	813	33
28	30015	9.47734	40	389	9.97950	4	466	9.49784	44	0.50216	780	32
29	043	9.47774	40	380	9.97946	4	498	9.49828	44	0.50172	748	31
30	30071	9.47814	40	95372	9.97942	4	31530	9.49872	44	0.50128	3.1716	30
31	098	9.47854	40	363	9.97938	4	562	9.49916	44	0.50084	684	29
32	126	9.47894	40	354	9.97934	4	594	9.49960	44	0.50040	652	28
33	154	9.47934	40	345	9.97930	4	626	9.50004	44	0.49996	620	27
34	182	9.47974	40	337	9.97926	4	658	9.50048	44	0.49952	588	26
35	30209	9.48014	40	95328	9.97922	4	31690	9.50092	44	0.49908	3.1556	25
36	237	9.48054	40	319	9.97918	4	722	9.50136	44	0.49864	524	24
37	265	9.48094	39	310	9.97914	4	754	9.50180	44	0.49820	492	23
38	292	9.48133	40	301	9.97910	4	786	9.50223	43	0.49777	460	22
39	320	9.48173	40	293	9.97906	4	818	9.50267	44	0.49733	429	21
40	30348	9.48213	39	95284	9.97902	4	31850	9.50311	44	0.49689	3.1397	20
41	376	9.48252	40	275	9.97898	4	882	9.50355	43	0.49645	366	19
42	403	9.48292	40	266	9.97894	4	914	9.50398	43	0.49602	334	18
43	431	9.48332	40	257	9.97890	4	946	9.50442	44	0.49558	303	17
44	459	9.48371	39	248	9.97886	4	978	9.50485	43	0.49515	271	16
45	30486	9.48411	39	95240	9.97882	4	32010	9.50529	44	0.49471	3.1240	15
46	514	9.48450	40	231	9.97878	4	042	9.50572	43	0.49428	209	14
47	542	9.48490	39	222	9.97874	4	074	9.50616	44	0.49384	178	13
48	570	9.48529	39	213	9.97870	4	106	9.50659	43	0.49341	146	12
49	597	9.48568	39	204	9.97866	4	139	9.50703	44	0.49297	115	11
50	30625	9.48607	39	95195	9.97861	5	32171	9.50746	43	0.49254	3.1084	10
51	653	9.48647	40	186	9.97857	4	203	9.50789	44	0.49211	053	9
52	680	9.48686	39	177	9.97853	4	235	9.50833	43	0.49167	022	8
53	708	9.48725	39	168	9.97849	4	267	9.50876	43	0.49124	3.0991	7
54	736	9.48764	39	159	9.97845	4	299	9.50919	43	0.49081	961	6
55	30763	9.48803	39	95150	9.97841	4	32331	9.50962	43	0.49038	3.0930	5
56	791	9.48842	39	142	9.97837	4	363	9.51005	43	0.48995	899	4
57	819	9.48881	39	133	9.97833	4	396	9.51048	43	0.48952	868	3
58	846	9.48920	39	124	9.97829	4	428	9.51092	44	0.48908	838	2
59	874	9.48959	39	115	9.97825	4	460	9.51135	43	0.48865	807	1
60	902	9.48998	39	106	9.97821	4	492	9.51178	43	0.48822	777	0
	Nat. Cos	Log. d.	Nat. Sin	Log. d.	Nat. Cot	Log. c.d.	Log. Tan	Nat.				

	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	30902	0.48998	39	95106	0.97821	4	32492	0.51178	43	0.48822	3.0777	60
1	929	0.49037	39	097	0.97817	5	524	0.51221	43	0.48779	749	59
2	957	0.49076	39	088	0.97812	4	556	0.51264	42	0.48736	716	58
3	985	0.49115	39	079	0.97808	4	588	0.51306	42	0.48694	686	57
4	31012	0.49153	38	070	0.97804	4	621	0.51349	43	0.48651	655	56
5	31040	0.49192	39	95061	0.97800	4	32653	0.51392	43	0.48608	3.0625	55
6	068	0.49231	39	052	0.97796	4	685	0.51435	43	0.48565	595	54
7	095	0.49260	38	043	0.97792	4	717	0.51478	43	0.48522	565	53
8	123	0.49308	39	033	0.97788	4	749	0.51520	42	0.48480	535	52
9	151	0.49347	39	024	0.97784	4	782	0.51563	43	0.48437	505	51
10	31178	0.49385	38	95015	0.97779	5	32814	0.51606	43	0.48394	3.0475	50
11	206	0.49424	38	006	0.97775	4	846	0.51648	42	0.48352	445	49
12	233	0.49462	38	94997	0.97771	4	878	0.51691	43	0.48309	415	48
13	261	0.49500	39	988	0.97767	4	911	0.51734	42	0.48266	385	47
14	289	0.49539	39	979	0.97763	4	943	0.51776	42	0.48224	356	46
15	31316	0.49577	38	94970	0.97759	4	32975	0.51819	43	0.48181	3.0326	45
16	344	0.49615	39	961	0.97754	5	33007	0.51861	42	0.48139	296	44
17	372	0.49654	39	952	0.97750	4	040	0.51903	42	0.48097	267	43
18	399	0.49692	38	943	0.97746	4	072	0.51946	43	0.48054	237	42
19	427	0.49730	38	933	0.97742	4	104	0.51988	42	0.48012	208	41
20	31454	0.49768	38	94924	0.97738	4	33136	0.52031	43	0.47969	3.0178	40
21	482	0.49806	38	915	0.97734	4	169	0.52073	42	0.47927	149	39
22	510	0.49844	38	906	0.97729	4	201	0.52115	42	0.47885	120	38
23	537	0.49882	38	897	0.97725	4	233	0.52157	42	0.47843	90	37
24	565	0.49920	38	888	0.97721	4	266	0.52200	43	0.47800	661	36
25	31593	0.49958	38	94878	0.97717	4	33298	0.52242	42	0.47758	3.0032	35
26	620	0.49996	38	869	0.97713	4	330	0.52284	42	0.47716	003	34
27	648	0.50034	38	860	0.97708	5	363	0.52326	42	0.47674	2.9974	33
28	675	0.50072	38	851	0.97704	4	395	0.52368	42	0.47632	945	32
29	703	0.50110	38	842	0.97700	4	427	0.52410	42	0.47590	916	31
30	31730	0.50148	38	94832	0.97696	4	33460	0.52452	42	0.47548	2.9887	30
31	758	0.50185	37	823	0.97691	5	492	0.52494	42	0.47506	858	29
32	786	0.50223	38	814	0.97687	4	524	0.52536	42	0.47464	829	28
33	813	0.50261	38	805	0.97683	4	557	0.52578	42	0.47422	800	27
34	841	0.50298	37	795	0.97679	4	589	0.52620	42	0.47380	772	26
35	31868	0.50336	38	94786	0.97674	5	33621	0.52661	41	0.47339	2.9743	25
36	896	0.50374	38	777	0.97670	4	654	0.52703	42	0.47297	714	24
37	923	0.50411	38	768	0.97666	4	686	0.52745	42	0.47255	686	23
38	951	0.50449	37	758	0.97662	5	718	0.52787	42	0.47213	657	22
39	979	0.50486	37	749	0.97657	5	751	0.52829	42	0.47171	629	21
40	32006	0.50523	38	94740	0.97653	4	33783	0.52870	41	0.47130	2.9600	20
41	034	0.50561	37	730	0.97649	4	816	0.52912	42	0.47088	572	19
42	061	0.50598	37	721	0.97645	4	848	0.52953	42	0.47047	544	18
43	089	0.50635	38	712	0.97640	5	881	0.52995	42	0.47005	515	17
44	116	0.50673	37	702	0.97636	4	913	0.53037	42	0.46963	487	16
45	32144	0.50710	37	94693	0.97632	4	33945	0.53078	41	0.46922	2.9459	15
46	171	0.50747	37	684	0.97628	4	978	0.53120	42	0.46880	431	14
47	199	0.50784	37	674	0.97623	4	34010	0.53161	41	0.46839	403	13
48	227	0.50821	37	665	0.97619	4	043	0.53202	41	0.46798	375	12
49	254	0.50858	38	656	0.97615	5	075	0.53244	42	0.46756	347	11
50	32282	0.50896	37	94646	0.97610	5	34108	0.53285	41	0.46715	2.9319	10
51	309	0.50933	37	637	0.97606	4	140	0.53327	42	0.46673	291	9
52	337	0.50970	37	627	0.97602	4	173	0.53368	41	0.46632	263	8
53	364	0.51007	37	618	0.97597	5	205	0.53409	41	0.46591	235	7
54	392	0.51043	36	609	0.97593	4	238	0.53450	41	0.46550	208	6
55	32419	0.51080	37	94599	0.97589	4	34270	0.53492	42	0.46508	2.9180	5
56	447	0.51117	37	590	0.97584	5	303	0.53533	41	0.46467	152	4
57	474	0.51154	37	580	0.97580	4	335	0.53574	41	0.46426	125	3
58	502	0.51191	37	571	0.97576	4	368	0.53615	41	0.46385	97	2
59	529	0.51227	36	561	0.97571	5	400	0.53656	41	0.46344	070	1
60	557	0.51264	37	552	0.97567	4	433	0.53697	41	0.46303	042	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.					

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.				
0	32557	9.51264	37	94552	9.97507	4	34433	9.53697	4I	0.46303	2.9042	60
1	584	9.51301	37	542	9.97503	4	405	9.53738	4I	0.46262	015	59
2	612	9.51338	36	533	9.97558	5	498	9.53779	4I	0.46221	2.8987	58
3	639	9.51374	37	523	9.97554	4	530	9.53820	4I	0.46180	960	57
4	667	9.51411	36	514	9.97550	4	503	9.53861	4I	0.46139	933	56
5	32694	9.51447	36	94504	9.97545	5	34596	9.53902	4I	0.46098	2.8905	55
6	722	9.51484	36	495	9.97541	4	628	9.53943	4I	0.46057	878	54
7	749	9.51520	36	485	9.97536	5	661	9.53984	4I	0.46016	851	53
8	777	9.51557	36	476	9.97532	4	693	9.54025	4I	0.45975	824	52
9	804	9.51593	36	466	9.97528	4	726	9.54065	4I	0.45935	797	51
10	32832	9.51629	36	94457	9.97523	5	34758	9.54106	4I	0.45894	2.8770	50
11	859	9.51666	36	447	9.97519	4	791	9.54147	4I	0.45853	743	49
12	887	9.51702	36	438	9.97515	4	824	9.54187	4I	0.45813	716	48
13	914	9.51738	36	428	9.97510	5	856	9.54228	4I	0.45772	689	47
14	942	9.51774	37	418	9.97506	4	889	9.54269	4I	0.45731	662	46
15	32969	9.51811	36	94409	9.97501	5	34922	9.54309	4I	0.45691	2.8636	45
16	997	9.51847	36	399	9.97497	4	954	9.54350	4I	0.45650	609	44
17	33024	9.51883	36	390	9.97492	5	987	9.54390	4I	0.45610	582	43
18	051	9.51919	36	380	9.97488	4	35020	9.54431	4I	0.45569	556	42
19	079	9.51955	36	370	9.97484	4	052	9.54471	4I	0.45529	529	41
20	33106	9.51991	36	94361	9.97479	5	35085	9.54512	4I	0.45488	2.8502	40
21	134	9.52027	36	351	9.97475	4	118	9.54552	4I	0.45448	476	39
22	161	9.52063	36	342	9.97470	5	150	9.54593	4I	0.45407	449	38
23	189	9.52099	36	332	9.97466	4	183	9.54633	4I	0.45367	423	37
24	216	9.52135	36	322	9.97461	5	216	9.54673	4I	0.45327	397	36
25	33244	9.52171	36	94313	9.97457	4	35248	9.54714	4I	0.45286	2.8370	35
26	271	9.52207	36	303	9.97453	4	281	9.54754	4I	0.45246	344	34
27	298	9.52242	35	293	9.97448	5	314	9.54794	4I	0.45206	318	33
28	326	9.52278	36	284	9.97444	4	346	9.54835	4I	0.45165	291	32
29	353	9.52314	36	274	9.97439	5	379	9.54875	4I	0.45125	265	31
30	33381	9.52350	35	94264	9.97435	4	35412	9.54915	4I	0.45085	2.8239	30
31	408	9.52385	35	254	9.97430	5	445	9.54955	4I	0.45045	213	29
32	436	9.52421	35	245	9.97426	4	477	9.54995	4I	0.45005	187	28
33	463	9.52456	35	235	9.97421	5	510	9.55035	4I	0.44965	161	27
34	490	9.52492	36	225	9.97417	4	543	9.55075	4I	0.44925	135	26
35	33518	9.52527	35	94215	9.97412	5	35576	9.55115	4I	0.44885	2.8109	25
36	545	9.52563	35	206	9.97408	4	608	9.55155	4I	0.44845	083	24
37	573	9.52598	35	196	9.97403	5	641	9.55195	4I	0.44805	057	23
38	600	9.52634	35	186	9.97399	4	674	9.55235	4I	0.44765	032	22
39	627	9.52669	35	176	9.97394	5	707	9.55275	4I	0.44725	006	21
40	33655	9.52705	35	94167	9.97390	4	35740	9.55315	4I	0.44685	2.7980	20
41	682	9.52740	35	157	9.97385	5	772	9.55355	4I	0.44645	955	19
42	710	9.52775	35	147	9.97381	4	805	9.55395	4I	0.44605	929	18
43	737	9.52811	35	137	9.97376	5	838	9.55434	4I	0.44566	903	17
44	764	9.52846	35	127	9.97372	4	871	9.55474	4I	0.44526	878	16
45	33792	9.52881	35	94118	9.97367	5	35904	9.55514	4I	0.44486	2.7852	15
46	819	9.52916	35	108	9.97363	4	937	9.55554	4I	0.44446	827	14
47	846	9.52951	35	098	9.97358	5	969	9.55593	4I	0.44407	801	13
48	874	9.52986	35	088	9.97353	4	36002	9.55633	4I	0.44367	776	12
49	901	9.53021	35	078	9.97349	5	035	9.55673	4I	0.44327	751	11
50	33929	9.53056	35	94068	9.97344	4	36068	9.55712	4I	0.44288	2.7725	10
51	956	9.53092	36	058	9.97340	5	101	9.55752	4I	0.44248	700	9
52	983	9.53126	34	049	9.97335	4	134	9.55791	4I	0.44209	675	8
53	34011	9.53161	35	039	9.97331	5	167	9.55831	4I	0.44169	650	7
54	038	9.53196	35	029	9.97326	4	199	9.55870	4I	0.44130	625	6
55	34065	9.53231	35	94019	9.97322	5	36232	9.55910	4I	0.44090	2.7600	5
56	093	9.53266	35	009	9.97317	4	265	9.55949	4I	0.44051	575	4
57	120	9.53301	35	93999	9.97312	5	298	9.55989	4I	0.44011	550	3
58	147	9.53336	35	989	9.97308	4	331	9.56028	4I	0.43972	525	2
59	175	9.53370	34	979	9.97303	5	364	9.56067	4I	0.43933	500	1
60	202	9.53405	35	969	9.97299	4	397	9.56107	4I	0.43893	475	0
	Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	'

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.				
0	34202	9.53405	35	93969	9.97299	5	36397	9.56107	39	0.43893	2.7475	60
1	229	9.53440	35	959	9.97294	5	430	9.56146	39	0.43854	450	59
2	257	9.53475	34	949	9.97289	4	403	9.56185	39	0.43815	425	58
3	284	9.53509	34	939	9.97285	5	496	9.56224	40	0.43776	400	57
4	311	9.53544	35	929	9.97280	5	529	9.56264	39	0.43736	376	56
5	34339	9.53578	35	93919	9.97276	4	36562	9.56303	39	0.43697	2.7351	55
6	366	9.53613	35	909	9.97271	5	595	9.56342	39	0.43658	326	54
7	393	9.53647	34	899	9.97266	5	628	9.56381	39	0.43619	302	53
8	421	9.53682	35	889	9.97262	4	661	9.56420	39	0.43580	277	52
9	448	9.53716	34	879	9.97257	5	694	9.56459	39	0.43541	253	51
10	34475	9.53751	35	93869	9.97252	5	36727	9.56498	39	0.43502	2.7228	50
11	503	9.53785	34	859	9.97248	4	760	9.56537	39	0.43463	204	49
12	530	9.53819	35	849	9.97243	5	793	9.56576	39	0.43424	179	48
13	557	9.53854	34	839	9.97238	5	826	9.56615	39	0.43385	155	47
14	584	9.53888	34	829	9.97234	4	859	9.56654	39	0.43346	130	46
15	34612	9.53922	35	93819	9.97229	5	36892	9.56693	39	0.43307	2.7106	45
16	639	9.53957	34	809	9.97224	5	925	9.56732	39	0.43268	082	44
17	666	9.53991	34	799	9.97220	4	958	9.56771	39	0.43229	058	43
18	694	9.54025	34	789	9.97215	5	991	9.56810	39	0.43190	034	42
19	721	9.54059	34	779	9.97210	4	37024	9.56849	38	0.43151	009	41
20	34748	9.54093	34	93769	9.97206	5	37057	9.56888	39	0.43113	2.6985	40
21	775	9.54127	34	759	9.97201	5	090	9.56926	39	0.43074	961	39
22	803	9.54161	34	748	9.97196	5	123	9.56965	39	0.43035	937	38
23	830	9.54195	34	738	9.97192	4	157	9.57004	39	0.42996	913	37
24	857	9.54229	34	728	9.97187	5	190	9.57042	39	0.42958	889	36
25	34884	9.54263	34	93718	9.97182	4	37223	9.57081	38	0.42919	2.6865	35
26	912	9.54297	34	708	9.97178	5	256	9.57120	39	0.42880	841	34
27	939	9.54331	34	698	9.97173	5	289	9.57158	39	0.42842	818	33
28	966	9.54365	34	688	9.97168	5	322	9.57197	39	0.42803	794	32
29	993	9.54399	34	677	9.97163	5	355	9.57235	39	0.42765	770	31
30	35021	9.54433	33	93667	9.97159	4	37388	9.57274	38	0.42726	2.6746	30
31	048	9.54466	33	657	9.97154	5	422	9.57312	39	0.42688	723	29
32	075	9.54500	34	647	9.97149	5	455	9.57351	39	0.42649	699	28
33	102	9.54534	33	637	9.97145	5	488	9.57389	39	0.42611	675	27
34	130	9.54567	34	626	9.97140	5	521	9.57428	39	0.42572	652	26
35	35157	9.54601	34	93616	9.97135	5	37554	9.57466	38	0.42534	2.6628	25
36	184	9.54635	33	606	9.97130	4	588	9.57504	39	0.42496	605	24
37	211	9.54668	33	596	9.97126	5	621	9.57543	39	0.42457	581	23
38	239	9.54702	33	585	9.97121	5	654	9.57581	38	0.42419	558	22
39	266	9.54735	33	575	9.97116	5	687	9.57619	38	0.42381	534	21
40	35293	9.54769	34	93565	9.97111	4	37720	9.57658	39	0.42342	2.6511	20
41	320	9.54802	34	555	9.97107	5	754	9.57696	38	0.42304	488	19
42	347	9.54836	34	544	9.97102	5	787	9.57734	38	0.42266	464	18
43	375	9.54869	33	534	9.97097	5	820	9.57772	38	0.42228	441	17
44	402	9.54903	33	524	9.97092	5	853	9.57810	38	0.42190	418	16
45	35429	9.54936	33	93514	9.97087	4	37887	9.57849	39	0.42151	2.6395	15
46	456	9.54969	33	503	9.97083	5	920	9.57887	38	0.42113	371	14
47	484	9.55003	34	493	9.97078	5	953	9.57925	38	0.42075	348	13
48	511	9.55036	34	483	9.97073	5	986	9.57963	38	0.42037	325	12
49	538	9.55069	33	472	9.97068	5	38020	9.58001	38	0.41999	302	11
50	35565	9.55102	34	93462	9.97063	4	38053	9.58039	38	0.41961	2.6279	10
51	592	9.55136	34	452	9.97059	5	086	9.58077	38	0.41923	256	9
52	619	9.55169	33	441	9.97054	5	120	9.58115	38	0.41885	233	8
53	647	9.55202	33	431	9.97049	5	153	9.58153	38	0.41847	210	7
54	674	9.55235	33	420	9.97044	5	186	9.58191	38	0.41809	187	6
55	36701	9.55268	33	93410	9.97039	4	38220	9.58229	38	0.41771	2.6165	5
56	728	9.55301	33	400	9.97035	5	253	9.58267	37	0.41733	142	4
57	755	9.55334	33	389	9.97030	5	286	9.58304	37	0.41696	119	3
58	782	9.55367	33	379	9.97025	5	320	9.58342	38	0.41658	096	2
59	810	9.55400	33	368	9.97020	5	353	9.58380	38	0.41620	074	1
60	837	9.55433	33	358	9.97015	5	386	9.58418	38	0.41582	051	0
	Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	'

'	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	35837	9.55433	33	93358	9.97015	5	38386	9.58418	37	0.41582	2.6051	60
1	864	9.55406	33	348	9.97010	5	420	9.58455	37	0.41545	028	59
2	891	9.55490	33	337	9.97005	5	453	9.58493	38	0.41507	006	58
3	918	9.55532	33	327	9.97001	4	487	9.58531	38	0.41469	2.5983	57
4	945	9.55504	32	316	9.96996	5	520	9.58599	38	0.41431	961	56
5	35973	9.55597	33	93306	9.96991	5	38553	9.58606	38	0.41394	2.5938	55
6	36000	9.55630	33	295	9.96986	5	587	9.58644	38	0.41356	916	54
7	027	9.55663	32	285	9.96981	5	620	9.58681	37	0.41319	893	53
8	054	9.55695	32	274	9.96976	5	654	9.58719	38	0.41281	871	52
9	081	9.55728	33	264	9.96971	5	687	9.58757	38	0.41243	848	51
10	36108	9.55761	33	93253	9.96966	5	38721	9.58794	37	0.41206	2.5826	50
11	135	9.55793	32	243	9.96962	4	754	9.58832	37	0.41168	804	49
12	162	9.55826	32	232	9.96957	5	787	9.58869	37	0.41131	782	48
13	190	9.55858	32	222	9.96952	5	821	9.58907	38	0.41093	759	47
14	217	9.55891	33	211	9.96947	5	854	9.58944	37	0.41056	737	46
15	36244	9.55923	32	93201	9.96942	5	38888	9.58981	37	0.41019	2.5715	45
16	271	9.55956	32	190	9.96937	5	921	9.59019	38	0.40981	693	44
17	298	9.55988	32	180	9.96932	5	955	9.59056	38	0.40944	671	43
18	325	9.56021	33	169	9.96927	5	988	9.59094	38	0.40906	649	42
19	352	9.56053	32	159	9.96922	5	39022	9.59131	37	0.40869	627	41
20	36379	9.56085	32	93148	9.96917	5	39055	9.59168	37	0.40832	2.5605	40
21	406	9.56118	33	137	9.96912	5	089	9.59205	38	0.40795	583	39
22	434	9.56150	32	127	9.96907	4	122	9.59243	37	0.40757	561	38
23	461	9.56182	32	116	9.96903	4	156	9.59280	37	0.40720	539	37
24	488	9.56215	32	106	9.96898	5	190	9.59317	37	0.40683	517	36
25	36515	9.56247	32	93095	9.96893	5	39223	9.59354	37	0.40646	2.5495	35
26	542	9.56279	32	084	9.96888	5	257	9.59391	37	0.40609	473	34
27	569	9.56311	32	074	9.96883	5	290	9.59429	38	0.40571	452	33
28	596	9.56343	32	063	9.96878	5	324	9.59466	37	0.40534	430	32
29	623	9.56375	33	052	9.96873	5	357	9.59503	37	0.40497	408	31
30	36650	9.56408	32	93042	9.96868	5	39391	9.59540	37	0.40460	2.5386	30
31	677	9.56440	32	031	9.96863	5	425	9.59577	37	0.40423	365	29
32	704	9.56472	32	020	9.96858	5	458	9.59614	37	0.40386	343	28
33	731	9.56504	32	010	9.96853	5	492	9.59651	37	0.40349	322	27
34	758	9.56536	32	92999	9.96848	5	526	9.59688	37	0.40312	300	26
35	36785	9.56568	31	92988	9.96843	5	39559	9.59725	37	0.40275	2.5279	25
36	812	9.56599	32	978	9.96838	5	593	9.59762	37	0.40238	257	24
37	839	9.56631	32	967	9.96833	5	626	9.59799	37	0.40201	236	23
38	867	9.56663	32	956	9.96828	5	660	9.59835	36	0.40165	214	22
39	894	9.56695	32	945	9.96823	5	694	9.59872	37	0.40128	193	21
40	36921	9.56727	32	92935	9.96818	5	39727	9.59909	37	0.40091	2.5172	20
41	948	9.56759	31	924	9.96813	5	761	9.59946	37	0.40054	150	19
42	975	9.56790	31	913	9.96808	5	795	9.59983	37	0.40017	129	18
43	37002	9.56822	32	902	9.96803	5	829	9.60019	36	0.39981	108	17
44	029	9.56854	32	892	9.96798	5	862	9.60056	37	0.39944	086	16
45	37056	9.56886	32	92881	9.96793	5	39896	9.60093	37	0.39907	2.5065	15
46	083	9.56917	31	870	9.96788	5	930	9.60130	36	0.39870	044	14
47	110	9.56949	31	859	9.96783	5	963	9.60166	37	0.39833	023	13
48	137	9.56980	31	849	9.96778	5	997	9.60203	37	0.39797	002	12
49	164	9.57012	32	838	9.96772	5	40031	9.60240	36	0.39760	2.4981	11
50	37191	9.57044	31	92827	9.96767	5	40065	9.60276	37	0.39724	2.4960	10
51	218	9.57075	32	816	9.96762	5	098	9.60313	36	0.39687	939	9
52	245	9.57107	31	805	9.96757	5	132	9.60349	36	0.39651	918	8
53	272	9.57138	31	794	9.96752	5	166	9.60386	37	0.39614	897	7
54	299	9.57169	32	784	9.96747	5	200	9.60422	36	0.39578	876	6
55	37326	9.57201	31	92773	9.96742	5	40234	9.60459	36	0.39541	2.4855	5
56	353	9.57232	32	762	9.96737	5	267	9.60495	37	0.39505	834	4
57	380	9.57264	32	751	9.96732	5	301	9.60532	37	0.39468	813	3
58	407	9.57295	31	740	9.96727	5	335	9.60568	36	0.39432	792	2
59	434	9.57326	31	729	9.96722	5	369	9.60605	37	0.39395	772	1
60	461	9.57358	32	718	9.96717	5	403	9.60641	36	0.39359	751	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'				

'	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat. Tan Log. c.d.	Log. Cot Nat.	
0	37461 9.57358	3I 92718 9.96717	6 40403 9.60641	36 0.39359 2.4751	60
1	488 9.57389	3I 707 9.96711	36 436 9.60677	36 0.39323 730	59
2	515 9.57420	3I 697 9.96706	37 470 9.60714	37 0.39286 709	58
3	542 9.57451	3I 686 9.96701	36 504 9.60750	36 0.39250 689	57
4	569 9.57482	3I 675 9.96696	36 538 9.60786	36 0.39214 668	56
5	37595 9.57514	3I 92664 9.96601	5 40572 9.60823	37 0.39177 2.4648	55
6	622 9.57545	3I 653 9.96686	36 606 9.60859	36 0.39141 627	54
7	649 9.57576	3I 642 9.96681	36 640 9.60895	36 0.39105 606	53
8	676 9.57607	3I 631 9.96676	36 674 9.60931	36 0.39069 586	52
9	703 9.57638	3I 620 9.96670	36 707 9.60967	36 0.39033 566	51
10	37730 9.57669	3I 92609 9.96665	5 40741 9.61004	37 0.38996 2.4545	50
11	757 9.57700	3I 598 9.96660	36 775 9.61040	36 0.38960 525	49
12	784 9.57731	3I 587 9.96655	36 809 9.61076	36 0.38924 504	48
13	811 9.57762	3I 576 9.96650	36 843 9.61112	36 0.38888 484	47
14	838 9.57793	3I 565 9.96645	36 877 9.61148	36 0.38852 464	46
15	37865 9.57824	3I 92554 9.96640	5 40911 9.61184	36 0.38816 2.4443	45
16	892 9.57855	30 543 9.96634	36 945 9.61220	36 0.38780 423	44
17	919 9.57885	3I 532 9.96629	36 979 9.61256	36 0.38744 403	43
18	946 9.57916	3I 521 9.96624	36 41013 9.61292	36 0.38708 383	42
19	973 9.57947	3I 510 9.96619	36 047 9.61328	36 0.38672 362	41
20	37999 9.57978	30 92499 9.96614	5 41081 9.61364	36 0.38636 2.4342	40
21	38026 9.58008	3I 488 9.96608	36 115 9.61400	36 0.38600 322	39
22	053 9.58039	3I 477 9.96603	36 149 9.61436	36 0.38564 302	38
23	080 9.58070	3I 466 9.96598	36 183 9.61472	36 0.38528 282	37
24	107 9.58101	3I 455 9.96593	36 217 9.61508	36 0.38492 262	36
25	38134 9.58131	3I 92444 9.96588	5 41251 9.61544	35 0.38456 2.4242	35
26	161 9.58162	3I 432 9.96582	36 285 9.61579	35 0.38421 222	34
27	188 9.58192	3I 421 9.96577	36 319 9.61615	36 0.38385 202	33
28	215 9.58223	3I 410 9.96572	36 353 9.61651	36 0.38349 182	32
29	241 9.58253	3I 399 9.96567	36 387 9.61687	36 0.38313 162	31
30	38268 9.58284	3I 92388 9.96562	5 41421 9.61722	35 0.38278 2.4142	30
31	295 9.58314	3I 377 9.96556	36 455 9.61758	36 0.38242 122	29
32	322 9.58345	3I 366 9.96551	36 490 9.61794	36 0.38206 102	28
33	349 9.58375	3I 355 9.96546	36 524 9.61830	36 0.38170 082	27
34	376 9.58406	3I 343 9.96541	36 558 9.61865	36 0.38135 062	26
35	38403 9.58436	3I 92332 9.96535	5 41592 9.61901	35 0.38099 2.4043	25
36	430 9.58467	3I 321 9.96530	36 626 9.61936	35 0.38064 023	24
37	456 9.58497	3I 310 9.96525	36 660 9.61972	36 0.38028 003	23
38	483 9.58527	3I 299 9.96520	36 694 9.62008	36 0.37992 2.3984	22
39	510 9.58557	3I 287 9.96514	36 728 9.62043	36 0.37957 964	21
40	38537 9.58588	3I 92276 9.96509	5 41763 9.62079	35 0.37921 2.3945	20
41	564 9.58618	3I 265 9.96504	36 797 9.62114	35 0.37886 925	19
42	591 9.58648	3I 254 9.96498	36 831 9.62150	35 0.37850 906	18
43	617 9.58678	3I 243 9.96493	35 865 9.62185	35 0.37815 886	17
44	644 9.58709	3I 231 9.96488	36 899 9.62221	36 0.37779 867	16
45	38671 9.58739	3I 92220 9.96483	5 41933 9.62256	35 0.37744 2.3847	15
46	698 9.58769	3I 209 9.96477	36 968 9.62292	36 0.37708 828	14
47	725 9.58799	3I 198 9.96472	36 42002 9.62327	35 0.37673 808	13
48	752 9.58829	3I 186 9.96467	36 036 9.62362	36 0.37638 789	12
49	778 9.58859	3I 175 9.96461	36 070 9.62398	36 0.37602 770	11
50	38805 9.58889	3I 92164 9.96456	5 42105 9.62433	35 0.37567 2.3750	10
51	832 9.58919	3I 152 9.96451	36 139 9.62468	36 0.37532 731	9
52	859 9.58949	3I 141 9.96445	36 173 9.62504	36 0.37496 712	8
53	886 9.58979	3I 130 9.96440	36 207 9.62539	35 0.37461 693	7
54	912 9.59009	3I 119 9.96435	36 242 9.62574	35 0.37426 673	6
55	38939 9.59039	3I 92107 9.96429	5 42276 9.62609	35 0.37391 2.3654	5
56	966 9.59069	3I 096 9.96424	36 310 9.62645	35 0.37355 635	4
57	993 9.59098	29 085 9.96419	36 345 9.62680	35 0.37320 616	3
58	39020 9.59128	3I 073 9.96413	36 379 9.62715	35 0.37285 597	2
59	046 9.59158	3I 062 9.96408	36 413 9.62750	35 0.37250 578	1
60	073 9.59188	3I 050 9.96403	36 447 9.62785	35 0.37215 559	0
	Nat. Cos Log. d.	Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.	'

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat Tan Log. c.d. Log. Cot Nat.						
0	39073	9.59188	30	92050	9.96403	6	42447	9.62785	35	0.37215	2.3559	60	
1	100	9.59218	29	039	9.96397	5	482	9.62820	35	0.37180	539	59	
2	127	9.59247	29	028	9.96392	5	516	9.62855	35	0.37145	520	58	
3	153	9.59277	30	016	9.96387	5	551	9.62890	35	0.37110	501	57	
4	180	9.59307	30	005	9.96381	5	585	9.62926	36	0.37074	483	56	
5	39207	9.59336	29	91994	9.96376	5	42619	9.62961	35	0.37039	2.3464	55	
6	234	9.59366	30	982	9.96370	5	654	9.62996	35	0.37004	445	54	
7	260	9.59396	29	971	9.96365	5	688	9.63031	35	0.36969	426	53	
8	287	9.59425	30	959	9.96360	5	722	9.63066	35	0.36934	407	52	
9	314	9.59455	30	948	9.96354	5	757	9.63101	35	0.36899	388	51	
10	39341	9.59484	29	91936	9.96349	5	42791	9.63135	34	0.36865	2.3369	50	
11	367	9.59514	29	925	9.96343	5	826	9.63170	35	0.36830	351	49	
12	394	9.59543	29	914	9.96338	5	860	9.63205	35	0.36795	332	48	
13	421	9.59573	30	902	9.96333	5	894	9.63240	35	0.36760	313	47	
14	448	9.59602	30	891	9.96327	5	929	9.63275	35	0.36725	294	46	
15	39474	9.59632	29	91879	9.96322	5	42963	9.63310	35	0.36690	2.3276	45	
16	501	9.59661	29	868	9.96316	5	998	9.63345	35	0.36655	257	44	
17	528	9.59690	29	856	9.96311	5	43032	9.63379	34	0.36621	238	43	
18	555	9.59720	30	845	9.96305	5	067	9.63414	35	0.36586	220	42	
19	581	9.59749	29	833	9.96300	5	101	9.63449	35	0.36551	201	41	
20	39608	9.59778	29	91822	9.96294	5	43136	9.63484	35	0.36516	2.3183	40	
21	635	9.59808	30	810	9.96289	5	170	9.63519	35	0.36481	164	39	
22	661	9.59837	29	799	9.96284	5	205	9.63553	35	0.36447	146	38	
23	688	9.59866	29	787	9.96278	5	239	9.63588	35	0.36412	127	37	
24	715	9.59895	29	775	9.96273	6	274	9.63623	35	0.36377	109	36	
25	39741	9.59924	30	91764	9.96267	5	43308	9.63657	34	0.36343	2.3090	35	
26	768	9.59954	29	752	9.96262	5	343	9.63692	35	0.36308	072	34	
27	795	9.59983	29	741	9.96256	5	378	9.63726	34	0.36274	053	33	
28	822	9.60012	29	729	9.96251	5	412	9.63761	35	0.36239	035	32	
29	848	9.60041	29	718	9.96245	5	447	9.63796	35	0.36204	017	31	
30	39875	9.60070	29	91706	9.96240	5	43481	9.63830	34	0.36170	2.2998	30	
31	902	9.60099	29	694	9.96234	5	516	9.63865	35	0.36135	980	29	
32	928	9.60128	29	683	9.96229	5	550	9.63899	34	0.36101	962	28	
33	955	9.60157	29	671	9.96223	5	585	9.63934	35	0.36066	944	27	
34	982	9.60186	29	660	9.96218	5	620	9.63968	34	0.36032	925	26	
35	40008	9.60215	29	91648	9.96212	5	43654	9.64003	35	0.35997	2.2907	25	
36	035	9.60244	29	636	9.96207	5	689	9.64037	34	0.35963	889	24	
37	062	9.60273	29	625	9.96201	5	724	9.64072	35	0.35928	871	23	
38	088	9.60302	29	613	9.96196	5	758	9.64106	34	0.35894	853	22	
39	115	9.60331	28	601	9.96190	5	793	9.64140	34	0.35860	835	21	
40	40141	9.60359	29	91590	9.96185	5	43828	9.64175	35	0.35825	2.2817	20	
41	168	9.60388	29	578	9.96179	6	862	9.64209	34	0.35791	799	19	
42	195	9.60417	29	566	9.96174	6	897	9.64243	34	0.35757	781	18	
43	221	9.60446	28	555	9.96168	6	932	9.64278	35	0.35722	763	17	
44	248	9.60474	29	543	9.96162	6	966	9.64312	34	0.35688	745	16	
45	40275	9.60503	29	91531	9.96157	5	44001	9.64346	34	0.35654	2.2727	15	
46	301	9.60532	29	519	9.96151	5	036	9.64381	35	0.35619	709	14	
47	328	9.60561	28	508	9.96146	5	071	9.64415	34	0.35585	691	13	
48	355	9.60589	28	496	9.96140	5	105	9.64449	34	0.35551	673	12	
49	381	9.60618	28	484	9.96135	5	140	9.64483	34	0.35517	655	11	
50	40408	9.60646	29	91472	9.96129	6	44175	9.64517	35	0.35483	2.2637	10	
51	434	9.60675	29	461	9.96123	6	210	9.64552	35	0.35448	620	9	
52	461	9.60704	28	449	9.96118	5	244	9.64586	34	0.35414	602	8	
53	488	9.60732	28	437	9.96112	5	279	9.64620	34	0.35380	584	7	
54	514	9.60761	28	425	9.96107	5	314	9.64654	34	0.35346	566	6	
55	40541	9.60789	29	91414	9.96101	6	44349	9.64688	34	0.35312	2.2549	5	
56	567	9.60818	28	402	9.96095	5	384	9.64722	34	0.35278	531	4	
57	594	9.60846	28	390	9.96090	5	418	9.64756	34	0.35244	513	3	
58	621	9.60875	28	378	9.96084	5	453	9.64790	34	0.35210	496	2	
59	647	9.60903	28	366	9.96079	5	488	9.64824	34	0.35176	478	1	
60	674	9.60931	28	355	9.96073	5	523	9.64858	34	0.35142	460	0	
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.			'

'	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	40674	9.60931	29	91355	9.96073	6	44523	9.64858	34	0.35142	2.2460	60
1	700	9.60900	28	343	9.96007	5	558	9.64892	34	0.35108	443	59
2	727	9.60988	28	331	9.96062	6	593	9.64926	34	0.35074	425	58
3	753	9.61016	28	319	9.96056	6	627	9.64960	34	0.35040	408	57
4	780	9.61045	29	307	9.96050	5	662	9.64994	34	0.35006	390	56
5	40806	9.61073	28	91295	9.96045	5	44697	9.65028	34	0.34972	2.2373	55
6	833	9.61101	28	283	9.96039	5	732	9.65062	34	0.34938	355	54
7	860	9.61129	28	272	9.96034	6	767	9.65096	34	0.34904	338	53
8	886	9.61158	28	260	9.96028	6	802	9.65130	34	0.34870	320	52
9	913	9.61186	28	248	9.96022	5	837	9.65164	34	0.34836	303	51
10	40939	9.61214	28	91236	9.96017	5	44872	9.65197	34	0.34803	2.2286	50
11	966	9.61242	28	224	9.96011	6	907	9.65231	34	0.34769	268	49
12	992	9.61270	28	212	9.96005	5	942	9.65265	34	0.34735	251	48
13	41019	9.61298	28	200	9.96000	5	977	9.65299	34	0.34701	234	47
14	045	9.61326	28	188	9.95994	6	45012	9.65333	34	0.34667	216	46
15	41072	9.61354	28	91176	9.95988	5	45047	9.65366	34	0.34634	2.2199	45
16	098	9.61382	29	164	9.95982	5	082	9.65400	34	0.34600	182	44
17	125	9.61411	29	152	9.95977	6	117	9.65434	34	0.34566	165	43
18	151	9.61438	27	140	9.95971	5	152	9.65467	34	0.34533	148	42
19	178	9.61466	28	128	9.95965	5	187	9.65501	34	0.34499	130	41
20	41204	9.61494	28	91116	9.95960	5	45222	9.65535	34	0.34465	2.2113	40
21	231	9.61522	28	104	9.95954	6	257	9.65568	34	0.34432	096	39
22	257	9.61550	28	092	9.95948	5	292	9.65602	34	0.34398	079	38
23	284	9.61578	28	080	9.95942	6	327	9.65636	34	0.34364	062	37
24	310	9.61606	28	068	9.95937	5	362	9.65669	34	0.34331	045	36
25	41337	9.61634	28	91056	9.95931	6	45397	9.65703	34	0.34297	2.2028	35
26	363	9.61662	27	044	9.95925	5	432	9.65736	34	0.34264	011	34
27	390	9.61689	27	032	9.95920	5	467	9.65770	34	0.34230	2.1994	33
28	416	9.61717	28	020	9.95914	6	502	9.65803	34	0.34197	977	32
29	443	9.61745	28	008	9.95908	5	538	9.65837	34	0.34163	960	31
30	41466	9.61773	27	90996	9.95902	5	45573	9.65870	34	0.34130	2.1943	30
31	496	9.61800	28	984	9.95897	6	608	9.65904	34	0.34096	926	29
32	522	9.61828	28	972	9.95891	5	643	9.65937	34	0.34063	909	28
33	549	9.61856	27	960	9.95885	6	678	9.65971	34	0.34029	892	27
34	575	9.61883	28	948	9.95879	5	713	9.65904	34	0.33996	876	26
35	41602	9.61911	28	90936	9.95873	5	45748	9.66038	34	0.33962	2.1859	25
36	628	9.61939	27	924	9.95868	6	784	9.66071	34	0.33929	842	24
37	655	9.61966	27	911	9.95862	5	819	9.66104	34	0.33896	825	23
38	681	9.61994	27	899	9.95856	6	854	9.66138	34	0.33862	808	22
39	707	9.62021	28	887	9.95850	5	889	9.66171	34	0.33829	792	21
40	41734	9.62049	27	90875	9.95844	5	45924	9.66204	34	0.33796	2.1775	20
41	760	9.62076	27	863	9.95839	6	960	9.66238	34	0.33762	758	19
42	787	9.62104	27	851	9.95833	5	995	9.66271	34	0.33729	742	18
43	813	9.62131	28	839	9.95827	6	46030	9.66304	34	0.33696	725	17
44	840	9.62159	27	826	9.95821	5	065	9.66337	34	0.33663	708	16
45	41866	9.62186	27	90814	9.95815	5	46101	9.66371	34	0.33629	2.1692	15
46	892	9.62214	28	802	9.95810	6	136	9.66404	34	0.33596	675	14
47	919	9.62241	27	790	9.95804	5	171	9.66437	34	0.33563	659	13
48	945	9.62268	27	778	9.95798	6	206	9.66470	34	0.33530	642	12
49	972	9.62296	27	766	9.95792	5	242	9.66503	34	0.33497	625	11
50	41998	9.62323	27	90753	9.95786	5	46277	9.66537	34	0.33463	2.1609	10
51	42024	9.62350	27	741	9.95780	6	312	9.66570	34	0.33430	592	9
52	051	9.62377	28	729	9.95775	5	348	9.66603	34	0.33397	576	8
53	077	9.62405	27	717	9.95769	6	383	9.66636	34	0.33364	560	7
54	104	9.62432	27	704	9.95763	5	418	9.66669	34	0.33331	543	6
55	42130	9.62459	27	90692	9.95757	6	46454	9.66702	34	0.33298	2.1527	5
56	156	9.62486	27	680	9.95751	5	489	9.66735	34	0.33265	510	4
57	183	9.62513	28	668	9.95745	6	525	9.66768	34	0.33232	494	3
58	209	9.62541	27	655	9.95739	5	560	9.66801	34	0.33199	478	2
59	235	9.62568	27	643	9.95733	6	595	9.66834	34	0.33166	461	1
60	262	9.62595	27	631	9.95728	5	631	9.66867	34	0.33133	445	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'				

°	Nat. Sin Log. d.		Nat. Cos Log. d.		Nat. Tan Log. c.d.		Log. Cot Nat.					
0	42262	0.62505	27	90631	0.95728	6	46631	0.66867	33	0.33133	2.1445	60
1	288	0.62622	27	618	0.95722	6	666	0.66900	33	0.33100	429	59
2	315	0.62649	27	606	0.95716	6	702	0.66933	33	0.33067	413	58
3	341	0.62676	27	594	0.95710	6	737	0.66966	33	0.33034	396	57
4	367	0.62703	27	582	0.95704	6	772	0.66999	33	0.33001	380	56
5	42394	0.62730	27	90569	0.95698	6	46808	0.67032	33	0.32968	2.1364	55
6	420	0.62757	27	557	0.95692	6	843	0.67065	33	0.32935	348	54
7	446	0.62784	27	545	0.95686	6	879	0.67098	33	0.32902	332	53
8	473	0.62811	27	532	0.95680	6	914	0.67131	33	0.32869	315	52
9	499	0.62838	27	520	0.95674	6	950	0.67163	32	0.32837	299	51
10	42525	0.62865	27	90507	0.95668	6	46985	0.67196	33	0.32804	2.1283	50
11	552	0.62892	26	495	0.95663	5	47021	0.67229	33	0.32771	267	49
12	578	0.62918	26	483	0.95657	6	056	0.67262	33	0.32738	251	48
13	604	0.62945	27	470	0.95651	6	092	0.67295	33	0.32705	235	47
14	631	0.62972	27	458	0.95645	6	128	0.67327	32	0.32673	219	46
15	42657	0.62999	27	90446	0.95639	6	47163	0.67360	33	0.32640	2.1203	45
16	683	0.63026	26	433	0.95633	6	199	0.67393	33	0.32607	187	44
17	709	0.63052	27	421	0.95627	6	234	0.67426	33	0.32574	171	43
18	736	0.63079	27	408	0.95621	6	270	0.67458	32	0.32542	155	42
19	762	0.63106	27	396	0.95615	6	305	0.67491	33	0.32509	139	41
20	42788	0.63133	26	90383	0.95609	6	47341	0.67524	33	0.32476	2.1123	40
21	815	0.63159	27	371	0.95603	6	377	0.67556	32	0.32444	107	39
22	841	0.63186	27	358	0.95597	6	412	0.67589	33	0.32411	092	38
23	867	0.63213	26	346	0.95591	6	448	0.67622	33	0.32378	076	37
24	894	0.63239	27	334	0.95585	6	483	0.67654	32	0.32346	060	36
25	42920	0.63266	26	90321	0.95579	6	47519	0.67687	33	0.32313	2.1044	35
26	946	0.63292	27	309	0.95573	6	555	0.67719	32	0.32281	028	34
27	972	0.63319	26	296	0.95567	6	590	0.67752	33	0.32248	013	33
28	999	0.63345	26	284	0.95561	6	626	0.67785	33	0.32215	2.0997	32
29	43025	0.63372	26	271	0.95555	6	662	0.67817	32	0.32183	981	31
30	43051	0.63398	27	90259	0.95549	6	47698	0.67850	33	0.32150	2.0965	30
31	977	0.63425	27	246	0.95543	6	733	0.67882	32	0.32118	950	29
32	104	0.63451	27	233	0.95537	6	769	0.67915	33	0.32085	934	28
33	130	0.63478	26	221	0.95531	6	805	0.67947	32	0.32053	918	27
34	156	0.63504	27	208	0.95525	6	840	0.67980	33	0.32020	903	26
35	43182	0.63531	26	90196	0.95519	6	47876	0.68012	32	0.31988	2.0887	25
36	209	0.63557	26	183	0.95513	6	912	0.68044	32	0.31956	872	24
37	235	0.63583	27	171	0.95507	6	948	0.68077	33	0.31923	856	23
38	261	0.63610	27	158	0.95500	7	984	0.68109	32	0.31891	840	22
39	287	0.63636	26	146	0.95494	6	48019	0.68142	33	0.31858	825	21
40	43313	0.63662	27	90133	0.95488	6	48055	0.68174	32	0.31826	2.0809	20
41	340	0.63689	26	120	0.95482	6	091	0.68206	33	0.31794	794	19
42	366	0.63715	26	108	0.95476	6	127	0.68239	33	0.31761	778	18
43	392	0.63741	26	095	0.95470	6	163	0.68271	32	0.31729	763	17
44	418	0.63767	27	082	0.95464	6	198	0.68303	32	0.31697	748	16
45	43445	0.63794	26	90070	0.95458	6	48234	0.68336	33	0.31664	2.0732	15
46	471	0.63820	26	057	0.95452	6	270	0.68368	32	0.31632	717	14
47	497	0.63846	26	045	0.95446	6	306	0.68400	32	0.31600	701	13
48	523	0.63872	26	032	0.95440	6	342	0.68432	32	0.31568	686	12
49	549	0.63898	26	019	0.95434	6	378	0.68465	33	0.31535	671	11
50	43575	0.63924	26	90007	0.95427	7	48414	0.68497	32	0.31503	2.0655	10
51	602	0.63950	26	89994	0.95421	6	450	0.68529	32	0.31471	640	9
52	628	0.63976	26	981	0.95415	6	486	0.68561	32	0.31439	625	8
53	654	0.64002	26	968	0.95409	6	521	0.68593	32	0.31407	609	7
54	680	0.64028	26	956	0.95403	6	557	0.68626	33	0.31374	594	6
55	43706	0.64054	26	89943	0.95397	6	48593	0.68658	32	0.31342	2.0579	5
56	733	0.64080	26	930	0.95391	6	629	0.68690	32	0.31310	564	4
57	759	0.64106	26	918	0.95384	7	665	0.68722	32	0.31278	549	3
58	785	0.64132	26	905	0.95378	6	701	0.68754	32	0.31246	533	2
59	811	0.64158	26	892	0.95372	6	737	0.68786	32	0.31214	518	1
60	837	0.64184	26	879	0.95366	6	773	0.68818	32	0.31182	503	0
	Nat. Cos Log. d.		Nat. Sin Log. d.		Nat. Cot Log. c.d.		Log. Tan Nat.					

'	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	43837	9.64184	26	89879	9.95366	6	48773	9.68818	32	0.31182	2.0503	60
1	863	9.64210	26	867	9.95360	6	809	9.68850	32	0.31150	488	59
2	869	9.64236	26	854	9.95354	6	845	9.68882	32	0.31118	473	58
3	916	9.64262	26	841	9.95348	6	881	9.68914	32	0.31086	458	57
4	942	9.64288	26	828	9.95341	7	917	9.68946	32	0.31054	443	56
5	43968	9.64313	25	89816	9.95335	6	48953	9.68978	32	0.31022	2.0428	55
6	994	9.64339	26	803	9.95329	6	989	9.69010	32	0.30990	413	54
7	44020	9.64365	26	790	9.95323	6	49026	9.69042	32	0.30958	398	53
8	046	9.64391	26	777	9.95317	7	062	9.69074	32	0.30926	383	52
9	072	9.64417	25	764	9.95310	6	098	9.69106	32	0.30894	368	51
10	44098	9.64442	25	89752	9.95304	6	49134	9.69138	32	0.30862	2.0353	50
11	124	9.64468	26	739	9.95298	6	170	9.69170	32	0.30830	338	49
12	151	9.64494	26	726	9.95292	6	206	9.69202	32	0.30798	323	48
13	177	9.64519	25	713	9.95286	6	242	9.69234	32	0.30766	308	47
14	203	9.64545	26	700	9.95279	7	278	9.69266	32	0.30734	293	46
15	44229	9.64571	25	89687	9.95273	6	49315	9.69298	31	0.30702	2.0278	45
16	255	9.64596	25	674	9.95267	6	351	9.69329	31	0.30671	293	44
17	281	9.64622	25	662	9.95261	7	387	9.69361	32	0.30639	248	43
18	307	9.64647	25	649	9.95254	6	423	9.69393	32	0.30607	233	42
19	333	9.64673	25	636	9.95248	6	459	9.69425	32	0.30575	219	41
20	44359	9.64698	26	89623	9.95242	6	49495	9.69457	31	0.30543	2.0204	40
21	385	9.64724	26	610	9.95236	6	532	9.69488	32	0.30512	189	39
22	411	9.64749	26	597	9.95229	6	568	9.69520	32	0.30480	174	38
23	437	9.64775	25	584	9.95223	6	604	9.69552	32	0.30448	160	37
24	464	9.64800	26	571	9.95217	6	640	9.69584	31	0.30416	145	36
25	44490	9.64826	26	89558	9.95211	7	49677	9.69615	32	0.30385	2.0130	35
26	516	9.64851	26	545	9.95204	6	713	9.69647	32	0.30353	115	34
27	542	9.64877	26	532	9.95198	6	749	9.69679	32	0.30321	101	33
28	568	9.64902	25	519	9.95192	7	786	9.69710	31	0.30290	086	32
29	594	9.64927	26	506	9.95185	6	822	9.69742	32	0.30258	072	31
30	44620	9.64953	25	89493	9.95179	6	49858	9.69774	31	0.30226	2.0057	30
31	646	9.64978	25	480	9.95173	6	894	9.69805	32	0.30195	042	29
32	672	9.65003	26	467	9.95167	7	931	9.69837	31	0.30163	028	28
33	698	9.65029	26	454	9.95160	7	967	9.69868	31	0.30132	013	27
34	724	9.65054	25	441	9.95154	6	50004	9.69900	32	0.30100	1.9999	26
35	44750	9.65079	25	89428	9.95148	6	50040	9.69932	31	0.30068	1.9984	25
36	776	9.65104	26	415	9.95141	7	076	9.69963	32	0.30037	970	24
37	802	9.65130	26	402	9.95135	6	113	9.69995	32	0.30005	955	23
38	828	9.65155	25	389	9.95129	6	149	9.70026	31	0.29974	941	22
39	854	9.65180	25	376	9.95122	7	185	9.70058	32	0.29942	926	21
40	44880	9.65205	25	89363	9.95116	6	50222	9.70089	31	0.29911	1.9912	20
41	906	9.65230	25	350	9.95110	7	258	9.70121	32	0.29879	897	19
42	932	9.65255	26	337	9.95103	6	295	9.70152	31	0.29848	883	18
43	958	9.65281	26	324	9.95097	7	331	9.70184	32	0.29816	868	17
44	984	9.65306	25	311	9.95090	6	368	9.70215	31	0.29785	854	16
45	45010	9.65331	25	89298	9.95084	6	50404	9.70247	32	0.29753	1.9840	15
46	036	9.65356	26	285	9.95078	7	441	9.70278	31	0.29722	825	14
47	062	9.65381	25	272	9.95071	6	477	9.70309	31	0.29691	811	13
48	088	9.65406	25	259	9.95065	6	514	9.70341	32	0.29659	797	12
49	114	9.65431	25	245	9.95059	7	550	9.70372	31	0.29628	782	11
50	45140	9.65456	25	89232	9.95052	6	50587	9.70404	32	0.29596	1.9768	10
51	166	9.65481	25	219	9.95046	7	623	9.70435	31	0.29565	754	9
52	192	9.65506	25	206	9.95039	6	660	9.70466	32	0.29534	740	8
53	218	9.65531	25	193	9.95033	6	696	9.70498	31	0.29502	725	7
54	243	9.65556	24	180	9.95027	7	733	9.70529	31	0.29471	711	6
55	45269	9.65580	25	89167	9.95020	6	50769	9.70560	32	0.29440	1.9697	5
56	295	9.65605	25	153	9.95014	7	806	9.70592	31	0.29408	683	4
57	321	9.65630	25	140	9.95007	6	843	9.70623	31	0.29377	669	3
58	347	9.65655	25	127	9.95001	6	879	9.70654	31	0.29346	654	2
59	373	9.65680	25	114	9.94995	7	916	9.70685	31	0.29315	640	1
60	399	9.65705	25	101	9.94988	7	953	9.70717	32	0.29283	626	0
	Nat. Cos Log.	d.		Nat. Sin Log.	d.		Nat. Cot Log.	c.d.		Log. Tan Nat.		'

'	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat Tan Log. c.d.	Log. Cot Nat.				
0	45399 9.65705	24	89101 9.94988	6	50953 9.70717	31	0.20283 1.9626	60
1	425 9.65729	25	087 9.94982	7	989 9.70748	31	0.20252 612	59
2	451 9.65754	25	074 9.94975	7	51026 9.70779	31	0.20221 598	58
3	477 9.65779	25	061 9.94969	6	063 9.70810	31	0.20190 584	57
4	503 9.65804	25	048 9.94962	7	099 9.70841	31	0.20159 570	56
5	45529 9.65828	24	89035 9.94956	6	51136 9.70873	32	0.20127 1.9556	55
6	554 9.65853	25	021 9.94949	7	173 9.70904	31	0.20096 542	54
7	580 9.65878	25	008 9.94943	6	209 9.70935	31	0.20065 528	53
8	606 9.65902	25	88995 9.94936	7	246 9.70966	31	0.20034 514	52
9	632 9.65927	25	981 9.94930	6	283 9.70997	31	0.20003 500	51
10	45658 9.65952	24	88968 9.94923	7	51319 9.71028	31	0.28972 1.9486	50
11	684 9.65976	25	955 9.94917	6	356 9.71059	31	0.28941 472	49
12	710 9.66001	25	942 9.94911	6	393 9.71090	31	0.28910 458	48
13	736 9.66025	24	928 9.94904	7	430 9.71121	31	0.28879 444	47
14	762 9.66050	25	915 9.94898	6	467 9.71153	32	0.28847 430	46
15	45787 9.66075	25	88902 9.94891	7	51503 9.71184	31	0.28816 1.9416	45
16	813 9.66099	24	888 9.94885	6	540 9.71215	31	0.28785 402	44
17	839 9.66124	25	875 9.94878	7	577 9.71246	31	0.28754 388	43
18	865 9.66148	25	862 9.94871	6	614 9.71277	31	0.28723 375	42
19	891 9.66173	25	848 9.94865	7	651 9.71308	31	0.28692 361	41
20	45917 9.66197	24	88835 9.94858	6	51688 9.71339	31	0.28661 1.9347	40
21	942 9.66221	25	822 9.94852	7	724 9.71370	31	0.28630 333	39
22	968 9.66246	24	808 9.94845	6	761 9.71401	31	0.28599 319	38
23	994 9.66270	24	795 9.94839	7	798 9.71431	30	0.28569 306	37
24	46020 9.66295	25	782 9.94832	6	835 9.71462	31	0.28538 292	36
25	46046 9.66319	24	88768 9.94826	7	51872 9.71493	31	0.28507 1.9278	35
26	072 9.66343	24	755 9.94819	6	909 9.71524	31	0.28476 265	34
27	097 9.66368	25	741 9.94813	7	946 9.71555	31	0.28445 251	33
28	123 9.66392	24	728 9.94806	6	983 9.71586	31	0.28414 237	32
29	149 9.66416	24	715 9.94799	7	52020 9.71617	31	0.28383 223	31
30	46175 9.66441	25	88701 9.94793	6	52057 9.71648	31	0.28352 1.9210	30
31	201 9.66465	24	688 9.94786	7	094 9.71679	30	0.28321 196	29
32	226 9.66489	24	674 9.94780	6	131 9.71709	30	0.28291 183	28
33	252 9.66513	24	661 9.94773	7	168 9.71740	31	0.28260 169	27
34	278 9.66537	24	647 9.94767	6	205 9.71771	31	0.28229 155	26
35	46304 9.66562	25	88634 9.94760	7	52242 9.71802	31	0.28198 1.9142	25
36	330 9.66586	24	620 9.94753	6	279 9.71833	30	0.28167 128	24
37	355 9.66610	24	607 9.94747	7	316 9.71863	30	0.28137 115	23
38	381 9.66634	24	593 9.94740	6	353 9.71894	31	0.28106 101	22
39	407 9.66658	24	580 9.94734	7	390 9.71925	31	0.28075 088	21
40	46433 9.66682	24	88566 9.94727	6	52427 9.71955	30	0.28045 1.9074	20
41	458 9.66706	25	553 9.94720	7	464 9.71986	31	0.28014 061	19
42	484 9.66731	25	539 9.94714	6	501 9.72017	31	0.27983 047	18
43	510 9.66755	24	526 9.94707	7	538 9.72048	31	0.27952 034	17
44	536 9.66779	24	512 9.94700	6	575 9.72078	30	0.27922 020	16
45	46561 9.66803	24	88499 9.94694	7	52613 9.72109	31	0.27891 1.9007	15
46	587 9.66827	24	485 9.94687	6	650 9.72140	30	0.27860 1.8993	14
47	613 9.66851	24	472 9.94680	7	687 9.72170	31	0.27830 980	13
48	639 9.66875	24	458 9.94674	6	724 9.72201	30	0.27799 967	12
49	664 9.66899	24	445 9.94667	7	761 9.72231	30	0.27769 953	11
50	46690 9.66922	23	88431 9.94660	6	52798 9.72262	31	0.27738 1.8940	10
51	716 9.66946	24	417 9.94654	7	836 9.72293	31	0.27707 927	9
52	742 9.66970	24	404 9.94647	6	873 9.72323	30	0.27677 913	8
53	767 9.66994	24	390 9.94640	7	910 9.72354	31	0.27646 900	7
54	793 9.67018	24	377 9.94634	6	947 9.72384	30	0.27616 887	6
55	46819 9.67042	24	88363 9.94627	7	52985 9.72415	31	0.27585 1.8873	5
56	844 9.67066	24	349 9.94620	6	53022 9.72445	30	0.27555 860	4
57	870 9.67090	24	336 9.94614	7	059 9.72476	31	0.27524 847	3
58	896 9.67113	23	322 9.94607	6	096 9.72506	30	0.27494 834	2
59	921 9.67137	24	308 9.94600	7	134 9.72537	31	0.27463 820	1
60	947 9.67161	24	295 9.94593	6	171 9.72567	30	0.27433 807	0
	Nat. Cos Log. d.	Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.				'

	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.		
0	46347	9.67161	24	88295	9.94593	6	53171	9.72567	31	0.27433	1.8807	60
1	973	9.67185	23	281	9.94587	7	208	9.72598	30	0.27402	794	59
2	999	9.67208	24	267	9.94580	7	246	9.72628	31	0.27372	781	58
3	47024	9.67232	24	254	9.94573	6	283	9.72659	30	0.27341	768	57
4	050	9.67256	24	240	9.94567	7	320	9.72689	31	0.27311	755	56
5	47076	9.67280	23	88226	9.94560	7	53358	9.72720	30	0.27280	1.8741	55
6	101	9.67303	24	213	9.94553	7	395	9.72750	31	0.27250	728	54
7	127	9.67327	23	199	9.94546	6	432	9.72780	30	0.27220	715	53
8	153	9.67350	24	185	9.94540	7	470	9.72811	31	0.27189	702	52
9	178	9.67374	24	172	9.94533	7	507	9.72841	30	0.27159	689	51
10	47204	9.67398	23	88158	9.94526	7	53545	9.72872	31	0.27128	1.8676	50
11	229	9.67421	24	144	9.94519	6	582	9.72902	30	0.27098	663	49
12	255	9.67445	23	130	9.94513	7	620	9.72932	31	0.27068	650	48
13	281	9.67468	24	117	9.94506	7	657	9.72963	30	0.27037	637	47
14	306	9.67492	23	103	9.94499	7	694	9.72993	31	0.27007	624	46
15	47332	9.67515	24	88089	9.94492	7	53732	9.73023	30	0.26977	1.8611	45
16	358	9.67539	23	075	9.94485	6	769	9.73054	31	0.26946	598	44
17	383	9.67562	24	062	9.94479	7	807	9.73084	30	0.26916	585	43
18	409	9.67586	23	048	9.94472	7	844	9.73114	31	0.26886	572	42
19	434	9.67609	24	034	9.94465	7	882	9.73144	30	0.26856	559	41
20	47460	9.67633	23	88020	9.94458	7	53920	9.73175	31	0.26825	1.8546	40
21	486	9.67656	24	006	9.94451	6	957	9.73205	30	0.26795	533	39
22	511	9.67680	23	87993	9.94445	7	995	9.73235	31	0.26765	520	38
23	537	9.67703	24	979	9.94438	7	54032	9.73265	30	0.26735	507	37
24	562	9.67726	23	965	9.94431	7	070	9.73295	31	0.26705	495	36
25	47588	9.67750	24	87951	9.94424	7	54107	9.73326	30	0.26674	1.8482	35
26	614	9.67773	23	937	9.94417	7	145	9.73356	31	0.26644	469	34
27	639	9.67796	24	923	9.94410	6	183	9.73386	30	0.26614	456	33
28	665	9.67820	23	909	9.94404	7	220	9.73416	31	0.26584	443	32
29	690	9.67843	24	896	9.94397	7	258	9.73446	30	0.26554	430	31
30	47716	9.67866	23	87882	9.94390	7	54296	9.73476	31	0.26524	1.8418	30
31	741	9.67890	24	868	9.94383	7	333	9.73507	30	0.26493	405	29
32	767	9.67913	23	854	9.94376	7	371	9.73537	31	0.26463	392	28
33	793	9.67936	24	840	9.94369	7	409	9.73567	30	0.26433	379	27
34	818	9.67959	23	826	9.94362	7	446	9.73597	31	0.26403	367	26
35	47844	9.67982	24	87812	9.94355	6	54484	9.73627	30	0.26373	1.8354	25
36	869	9.68006	23	798	9.94349	7	522	9.73657	31	0.26343	341	24
37	895	9.68029	24	784	9.94342	7	560	9.73687	30	0.26313	329	23
38	920	9.68052	23	770	9.94335	7	597	9.73717	31	0.26283	316	22
39	946	9.68075	24	756	9.94328	7	635	9.73747	30	0.26253	303	21
40	47971	9.68098	23	87743	9.94321	7	54673	9.73777	31	0.26223	1.8291	20
41	997	9.68121	24	729	9.94314	7	711	9.73807	30	0.26193	278	19
42	48022	9.68144	23	715	9.94307	7	748	9.73837	31	0.26163	265	18
43	048	9.68167	24	701	9.94300	7	786	9.73867	30	0.26133	253	17
44	073	9.68190	23	687	9.94293	7	824	9.73897	31	0.26103	240	16
45	48099	9.68213	24	87673	9.94286	7	54862	9.73927	30	0.26073	1.8228	15
46	124	9.68237	23	659	9.94279	6	900	9.73957	31	0.26043	215	14
47	150	9.68260	24	645	9.94273	7	938	9.73987	30	0.26013	202	13
48	175	9.68283	23	631	9.94266	7	975	9.74017	31	0.25983	190	12
49	201	9.68305	24	617	9.94259	7	55013	9.74047	30	0.25953	177	11
50	48226	9.68328	23	87603	9.94252	7	55051	9.74077	31	0.25923	1.8165	10
51	252	9.68351	24	589	9.94245	7	089	9.74107	30	0.25893	152	9
52	277	9.68374	23	575	9.94238	7	127	9.74137	31	0.25863	140	8
53	303	9.68397	24	561	9.94231	7	165	9.74166	30	0.25833	127	7
54	328	9.68420	23	546	9.94224	7	203	9.74196	31	0.25804	115	6
55	48354	9.68443	24	87532	9.94217	7	55241	9.74226	30	0.25774	1.8103	5
56	379	9.68466	23	518	9.94210	7	279	9.74256	31	0.25744	090	4
57	405	9.68489	24	504	9.94203	7	317	9.74286	30	0.25714	078	3
58	430	9.68512	23	490	9.94196	7	355	9.74316	31	0.25684	065	2
59	456	9.68534	24	476	9.94189	7	393	9.74345	30	0.25655	053	1
60	481	9.68557	23	462	9.94182	7	431	9.74375	31	0.25625	040	0
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.		

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.			
0	48481	9.68557	23	87462	9.94182	7	55431	9.74375	30	0.25625	1.8040	60	
1	506	9.68580	23	448	9.94175	7	469	9.74405	30	0.25595	028	59	
2	532	9.68603	23	434	9.94168	7	507	9.74435	30	0.25565	016	58	
3	557	9.68625	23	420	9.94161	7	545	9.74465	30	0.25535	003	57	
4	583	9.68648	23	406	9.94154	7	583	9.74494	29	0.25506	1.7991	56	
5	48608	9.68671	23	87391	9.94147	7	55621	9.74524	30	0.25476	1.7979	55	
6	634	9.68694	23	377	9.94140	7	659	9.74554	30	0.25446	966	54	
7	659	9.68710	23	363	9.94133	7	697	9.74583	29	0.25417	954	53	
8	684	9.68739	23	349	9.94126	7	736	9.74613	30	0.25387	942	52	
9	710	9.68762	23	335	9.94119	7	774	9.74643	30	0.25357	930	51	
10	48735	9.68784	23	87321	9.94112	7	55812	9.74673	29	0.25327	1.7917	50	
11	761	9.68807	23	306	9.94105	7	850	9.74702	30	0.25298	905	49	
12	786	9.68829	23	292	9.94098	8	888	9.74732	30	0.25268	893	48	
13	811	9.68852	23	278	9.94090	7	926	9.74762	30	0.25238	881	47	
14	837	9.68875	23	264	9.94083	7	964	9.74791	29	0.25209	868	46	
15	48862	9.68897	23	87250	9.94076	7	56003	9.74821	30	0.25179	1.7856	45	
16	888	9.68920	23	235	9.94069	7	041	9.74851	30	0.25149	844	44	
17	913	9.68942	23	221	9.94062	7	079	9.74880	29	0.25120	832	43	
18	938	9.68965	23	207	9.94055	7	117	9.74910	30	0.25090	820	42	
19	964	9.68987	23	193	9.94048	7	156	9.74939	29	0.25061	808	41	
20	48989	9.69010	23	87178	9.94041	7	56194	9.74969	30	0.25031	1.7796	40	
21	49014	9.69032	23	164	9.94034	7	232	9.74998	29	0.25002	783	39	
22	040	9.69055	23	150	9.94027	7	270	9.75028	30	0.24972	771	38	
23	065	9.69077	23	136	9.94020	7	309	9.75058	30	0.24942	759	37	
24	090	9.69100	22	121	9.94012	8	347	9.75087	29	0.24913	747	36	
25	49116	9.69122	22	87107	9.94005	7	56385	9.75117	30	0.24883	1.7735	35	
26	141	9.69144	23	093	9.93998	7	424	9.75146	29	0.24854	723	34	
27	166	9.69167	23	079	9.93991	7	462	9.75176	30	0.24824	711	33	
28	192	9.69189	23	064	9.93984	7	501	9.75205	29	0.24795	699	32	
29	217	9.69212	23	050	9.93977	7	539	9.75235	30	0.24765	687	31	
30	49242	9.69234	22	87036	9.93970	7	56577	9.75264	29	0.24736	1.7675	30	
31	268	9.69256	23	021	9.93963	8	616	9.75294	30	0.24706	663	29	
32	293	9.69279	23	007	9.93955	7	654	9.75323	29	0.24677	651	28	
33	318	9.69301	22	86993	9.93948	7	693	9.75353	30	0.24647	639	27	
34	344	9.69323	22	978	9.93941	7	731	9.75382	29	0.24618	627	26	
35	49369	9.69345	23	86664	9.93934	7	56769	9.75411	30	0.24589	1.7615	25	
36	394	9.69368	23	949	9.93927	7	808	9.75441	30	0.24559	603	24	
37	419	9.69390	22	935	9.93920	8	846	9.75470	29	0.24530	591	23	
38	445	9.69412	22	921	9.93912	7	885	9.75500	30	0.24500	579	22	
39	470	9.69434	22	906	9.93905	7	923	9.75529	29	0.24471	567	21	
40	49495	9.69456	23	86892	9.93898	7	56962	9.75558	30	0.24442	1.7556	20	
41	521	9.69479	23	878	9.93891	7	57000	9.75588	29	0.24412	544	19	
42	546	9.69501	22	863	9.93884	8	039	9.75617	30	0.24383	532	18	
43	571	9.69523	22	849	9.93876	7	078	9.75647	30	0.24353	520	17	
44	596	9.69545	22	834	9.93869	7	116	9.75676	29	0.24324	508	16	
45	49622	9.69567	22	86820	9.93862	7	57155	9.75705	30	0.24295	1.7496	15	
46	647	9.69589	22	805	9.93855	8	193	9.75735	29	0.24265	485	14	
47	672	9.69611	22	791	9.93847	7	232	9.75764	30	0.24236	473	13	
48	697	9.69633	22	777	9.93840	7	271	9.75793	29	0.24207	461	12	
49	723	9.69655	22	762	9.93833	7	309	9.75822	30	0.24178	449	11	
50	49748	9.69677	22	86748	9.93826	7	57348	9.75852	29	0.24148	1.7437	10	
51	773	9.69699	22	733	9.93819	8	386	9.75881	30	0.24119	426	9	
52	798	9.69721	22	719	9.93811	7	425	9.75910	29	0.24090	414	8	
53	824	9.69743	22	704	9.93804	7	464	9.75939	30	0.24061	402	7	
54	849	9.69765	22	690	9.93797	8	503	9.75969	29	0.24031	391	6	
55	49874	9.69787	22	86675	9.93789	7	57541	9.75998	30	0.24002	1.7379	5	
56	899	9.69809	22	661	9.93782	7	580	9.76027	29	0.23973	367	4	
57	924	9.69831	22	646	9.93775	7	619	9.76056	30	0.23944	355	3	
58	950	9.69853	22	632	9.93768	8	657	9.76086	29	0.23914	344	2	
59	975	9.69875	22	617	9.93760	7	696	9.76115	30	0.23885	332	1	
60	50000	9.69897	22	603	9.93753	7	735	9.76144	29	0.23856	321	0	
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.			'

	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.				
0	50000	9.69897	22	86603	9.93753	7	57735	9.76144	29	0.23856	1.7321	60
1	025	9.69919	22	588	9.93746	7	774	9.76173	29	0.23827	309	59
2	050	9.69941	22	573	9.93738	7	813	9.76202	29	0.23798	297	58
3	076	9.69963	21	559	9.93731	7	851	9.76231	29	0.23769	286	57
4	101	9.69984	21	544	9.93724	7	890	9.76261	30	0.23739	274	56
5	50126	9.70006	22	86530	9.93717	7	57929	9.76290	29	0.23710	1.7262	55
6	151	9.70028	22	515	9.93709	7	968	9.76319	29	0.23681	251	54
7	176	9.70050	22	501	9.93702	7	58007	9.76348	29	0.23652	239	53
8	201	9.70072	21	486	9.93695	7	046	9.76377	29	0.23623	228	52
9	227	9.70093	21	471	9.93687	8	085	9.76406	29	0.23594	216	51
10	50252	9.70115	22	86457	9.93680	7	58124	9.76435	29	0.23565	1.7205	50
11	277	9.70137	22	442	9.93673	7	162	9.76464	29	0.23536	193	49
12	302	9.70159	21	427	9.93665	8	201	9.76493	29	0.23507	182	48
13	327	9.70180	22	413	9.93658	8	240	9.76522	29	0.23478	170	47
14	352	9.70202	22	398	9.93650	7	279	9.76551	29	0.23449	159	46
15	50377	9.70224	21	86384	9.93643	7	58318	9.76580	29	0.23420	1.7147	45
16	403	9.70245	22	369	9.93636	8	357	9.76609	30	0.23391	130	44
17	428	9.70267	21	354	9.93628	8	396	9.76639	29	0.23361	124	43
18	453	9.70288	21	340	9.93621	7	435	9.76668	29	0.23332	113	42
19	478	9.70310	22	325	9.93614	7	474	9.76697	29	0.23303	102	41
20	50503	9.70332	22	86310	9.93606	8	58513	9.76725	28	0.23275	1.7090	40
21	528	9.70353	21	295	9.93599	8	552	9.76754	29	0.23246	079	39
22	553	9.70375	21	281	9.93591	7	591	9.76783	29	0.23217	067	38
23	578	9.70396	21	266	9.93584	7	631	9.76812	29	0.23188	056	37
24	603	9.70418	21	251	9.93577	8	670	9.76841	29	0.23159	045	36
25	50628	9.70439	22	86237	9.93569	7	58709	9.76870	29	0.23130	1.7033	35
26	654	9.70461	22	222	9.93562	7	748	9.76899	29	0.23101	022	34
27	679	9.70482	22	207	9.93554	7	787	9.76928	29	0.23072	011	33
28	704	9.70504	21	192	9.93547	8	826	9.76957	29	0.23043	1.6999	32
29	729	9.70525	21	178	9.93539	8	865	9.76986	29	0.23014	988	31
30	50754	9.70547	21	86163	9.93532	7	58905	9.77015	29	0.22985	1.6977	30
31	779	9.70568	21	148	9.93525	7	944	9.77044	29	0.22956	905	29
32	804	9.70590	21	133	9.93517	8	983	9.77073	28	0.22927	954	28
33	829	9.70611	22	119	9.93510	7	59022	9.77101	29	0.22899	943	27
34	854	9.70633	21	104	9.93502	8	061	9.77130	29	0.22870	932	26
35	50879	9.70654	21	86089	9.93495	7	59101	9.77159	29	0.22841	1.6920	25
36	904	9.70675	22	074	9.93487	8	140	9.77188	29	0.22812	909	24
37	929	9.70697	21	059	9.93480	7	179	9.77217	29	0.22783	898	23
38	954	9.70718	21	045	9.93472	7	218	9.77246	28	0.22754	887	22
39	979	9.70739	22	030	9.93465	8	258	9.77274	29	0.22726	875	21
40	51004	9.70761	22	86015	9.93457	7	59297	9.77303	29	0.22697	1.6864	20
41	029	9.70782	21	000	9.93450	8	336	9.77332	29	0.22668	853	19
42	054	9.70803	21	85985	9.93442	7	376	9.77361	29	0.22639	842	18
43	079	9.70824	22	970	9.93435	8	415	9.77390	28	0.22610	831	17
44	104	9.70846	21	956	9.93427	7	454	9.77418	29	0.22582	820	16
45	51129	9.70867	21	85941	9.93420	7	59494	9.77447	29	0.22553	1.6808	15
46	154	9.70888	21	926	9.93412	8	533	9.77476	29	0.22524	797	14
47	179	9.70909	22	911	9.93405	7	573	9.77505	28	0.22495	786	13
48	204	9.70931	21	896	9.93397	8	612	9.77533	29	0.22467	775	12
49	229	9.70952	21	881	9.93390	7	651	9.77562	29	0.22438	764	11
50	51254	9.70973	21	85866	9.93382	8	59691	9.77591	28	0.22409	1.6753	10
51	279	9.70994	21	851	9.93375	7	730	9.77619	29	0.22381	742	9
52	304	9.71015	21	836	9.93367	7	770	9.77648	29	0.22352	731	8
53	329	9.71036	21	821	9.93360	7	809	9.77677	29	0.22323	720	7
54	354	9.71058	22	806	9.93352	8	849	9.77706	28	0.22294	709	6
55	51379	9.71079	21	85792	9.93344	7	59888	9.77734	29	0.22266	1.6698	5
56	404	9.71100	21	777	9.93337	8	928	9.77763	28	0.22237	687	4
57	429	9.71121	21	762	9.93329	7	967	9.77791	29	0.22209	676	3
58	454	9.71142	21	747	9.93322	8	60007	9.77820	29	0.22180	665	2
59	479	9.71163	21	732	9.93314	7	046	9.77849	29	0.22151	654	1
60	504	9.71184	21	717	9.93307	7	086	9.77877	28	0.22123	643	0
	Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	

	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log Cot Nat.		
0	51504	9.71184	21	85717	9.93307	8	60086	9.77877	29	0.22123	1.6643	60
1	529	9.71205	21	702	9.93299	8	126	9.77906	29	0.22094	632	59
2	554	9.71226	21	687	9.93291	7	165	9.77935	28	0.22065	621	58
3	579	9.71247	21	672	9.93284	8	205	9.77963	28	0.22037	610	57
4	604	9.71268	21	657	9.93276	8	245	9.77992	28	0.22008	599	56
5	51628	9.71289	21	85642	9.93269	7	60284	9.78020	29	0.21980	1.6588	55
6	653	9.71310	21	627	9.93261	8	324	9.78049	29	0.21951	577	54
7	678	9.71331	21	612	9.93253	8	364	9.78077	29	0.21923	566	53
8	703	9.71352	21	597	9.93246	7	403	9.78106	28	0.21894	555	52
9	728	9.71373	21	582	9.93238	8	443	9.78135	29	0.21865	545	51
10	51753	9.71393	20	85567	9.93230	8	60483	9.78163	29	0.21837	1.6534	50
11	778	9.71414	21	551	9.93223	7	522	9.78192	28	0.21808	523	49
12	803	9.71435	21	536	9.93215	8	562	9.78220	29	0.21780	512	48
13	828	9.71456	21	521	9.93207	8	602	9.78249	28	0.21751	501	47
14	852	9.71477	21	506	9.93200	7	642	9.78277	28	0.21723	490	46
15	51877	9.71498	21	85491	9.93192	8	60681	9.78306	29	0.21694	1.6479	45
16	902	9.71519	20	476	9.93184	8	721	9.78334	28	0.21666	469	44
17	927	9.71539	21	461	9.93177	7	761	9.78363	29	0.21637	458	43
18	952	9.71560	21	446	9.93169	8	801	9.78391	28	0.21609	447	42
19	977	9.71581	21	431	9.93161	8	841	9.78419	28	0.21581	436	41
20	52002	9.71602	21	85416	9.93154	7	60881	9.78448	29	0.21552	1.6426	40
21	026	9.71622	20	401	9.93146	8	921	9.78476	28	0.21524	415	39
22	051	9.71643	21	385	9.93138	8	960	9.78505	29	0.21495	404	38
23	076	9.71664	21	370	9.93131	8	61000	9.78533	28	0.21467	393	37
24	101	9.71685	20	355	9.93123	8	040	9.78562	29	0.21438	383	36
25	52126	9.71705	21	85340	9.93115	8	61080	9.78590	28	0.21410	1.6372	35
26	151	9.71726	21	325	9.93108	7	120	9.78618	29	0.21382	361	34
27	175	9.71747	20	310	9.93100	8	160	9.78647	28	0.21353	351	33
28	200	9.71767	21	294	9.93092	8	200	9.78675	29	0.21325	340	32
29	225	9.71788	21	279	9.93084	8	240	9.78704	29	0.21296	329	31
30	52250	9.71809	20	85264	9.93077	7	61280	9.78732	28	0.21268	1.6319	30
31	275	9.71829	21	249	9.93069	8	320	9.78760	29	0.21240	308	29
32	299	9.71850	21	234	9.93061	8	360	9.78789	29	0.21211	297	28
33	324	9.71870	21	218	9.93053	8	400	9.78817	28	0.21183	287	27
34	349	9.71891	21	203	9.93046	7	440	9.78845	28	0.21155	276	26
35	52374	9.71911	21	85188	9.93038	8	61480	9.78874	29	0.21126	1.6265	25
36	399	9.71932	20	173	9.93030	8	520	9.78902	28	0.21098	255	24
37	423	9.71952	21	157	9.93022	8	561	9.78930	29	0.21070	244	23
38	448	9.71973	21	142	9.93014	8	601	9.78959	29	0.21041	234	22
39	473	9.71994	21	127	9.93007	7	641	9.78987	28	0.21013	223	21
40	52498	9.72014	20	85112	9.92999	8	61681	9.79015	28	0.20985	1.6212	20
41	522	9.72034	21	096	9.92991	8	721	9.79043	29	0.20957	202	19
42	547	9.72055	21	081	9.92983	8	761	9.79072	29	0.20928	191	18
43	572	9.72075	21	066	9.92976	7	801	9.79100	28	0.20900	181	17
44	597	9.72096	20	051	9.92968	8	842	9.79128	28	0.20872	170	16
45	52621	9.72116	21	85053	9.92960	8	61882	9.79156	29	0.20844	1.6160	15
46	646	9.72137	21	020	9.92952	8	922	9.79185	28	0.20815	149	14
47	671	9.72157	20	005	9.92944	8	962	9.79213	28	0.20787	139	13
48	696	9.72177	21	84989	9.92936	7	62003	9.79241	28	0.20759	128	12
49	720	9.72198	20	974	9.92929	8	043	9.79269	28	0.20731	118	11
50	52745	9.72218	20	84959	9.92921	8	62083	9.79297	29	0.20703	1.6107	10
51	770	9.72238	21	943	9.92913	8	124	9.79326	29	0.20674	097	9
52	794	9.72259	21	928	9.92905	8	164	9.79354	28	0.20646	087	8
53	819	9.72279	20	913	9.92897	8	204	9.79382	28	0.20618	076	7
54	844	9.72299	21	897	9.92889	8	245	9.79410	28	0.20590	066	6
55	52869	9.72320	20	84882	9.92881	7	62285	9.79438	28	0.20562	1.6055	5
56	893	9.72340	20	866	9.92874	8	325	9.79466	29	0.20534	045	4
57	918	9.72360	21	851	9.92866	8	366	9.79495	29	0.20505	034	3
58	943	9.72381	21	836	9.92858	8	406	9.79523	28	0.20477	024	2
59	967	9.72401	20	820	9.92850	8	446	9.79551	28	0.20449	014	1
60	992	9.72421	20	805	9.92842	8	487	9.79579	28	0.20421	003	0
Nat. Cos Log. d. Nat. Sin Log. d. Nat. Cot Log. c.d. Log. Tan Nat. /												

'	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.			
0	52992	9.72421	20	84805	9.92842	8	62487 9.79579	28	0.20421 1.6003	60
1	53017	9.72441	20	789	9.92834	8	527 9.79007	28	0.20393 1.5993	59
2	041	9.72461	21	774	9.92826	8	568 9.79035	28	0.20365 983	58
3	066	9.72482	20	759	9.92818	8	608 9.79063	28	0.20337 972	57
4	091	9.72502	20	743	9.92810	8	649 9.79091	28	0.20309 962	56
5	53115	9.72522	20	84728	9.92803	7	62689 9.79719	28	0.20281 1.5952	55
6	140	9.72542	20	712	9.92795	8	730 9.79747	29	0.20253 941	54
7	164	9.72562	20	697	9.92787	8	770 9.79776	28	0.20224 931	53
8	189	9.72582	20	681	9.92779	8	811 9.79804	28	0.20196 921	52
9	214	9.72602	20	666	9.92771	8	852 9.79832	28	0.20168 911	51
10	53238	9.72622	21	84650	9.92763	8	62892 9.79860	28	0.20140 1.5900	50
11	263	9.72643	20	635	9.92755	8	933 9.79888	28	0.20112 890	49
12	288	9.72663	20	619	9.92747	8	973 9.79916	28	0.20084 880	48
13	312	9.72683	20	604	9.92739	8	63014 9.79944	28	0.20056 869	47
14	337	9.72703	20	588	9.92731	8	055 9.79972	28	0.20028 859	46
15	53361	9.72723	20	84573	9.92723	8	63095 9.80000	28	0.20000 1.5849	45
16	386	9.72743	20	557	9.92715	8	136 9.80028	28	0.19972 839	44
17	411	9.72763	20	542	9.92707	8	177 9.80056	28	0.19944 829	43
18	435	9.72783	20	526	9.92699	8	217 9.80084	28	0.19916 818	42
19	460	9.72803	20	511	9.92691	8	258 9.80112	28	0.19888 808	41
20	53484	9.72823	20	84495	9.92683	8	63299 9.80140	28	0.19860 1.5798	40
21	509	9.72843	20	480	9.92675	8	340 9.80168	27	0.19832 788	39
22	534	9.72863	20	464	9.92667	8	380 9.80195	28	0.19805 778	38
23	558	9.72883	19	448	9.92659	8	421 9.80223	28	0.19777 768	37
24	583	9.72902	19	433	9.92651	8	462 9.80251	28	0.19749 757	36
25	53607	9.72922	20	84417	9.92643	8	63503 9.80279	28	0.19721 1.5747	35
26	632	9.72942	20	402	9.92635	8	544 9.80307	28	0.19693 737	34
27	656	9.72962	20	386	9.92627	8	584 9.80335	28	0.19665 727	33
28	681	9.72982	20	370	9.92619	8	625 9.80363	28	0.19637 717	32
29	705	9.73002	20	355	9.92611	8	666 9.80391	28	0.19609 707	31
30	53730	9.73022	19	84339	9.92603	8	63707 9.80419	28	0.19581 1.5697	30
31	754	9.73041	20	324	9.92595	8	748 9.80447	27	0.19553 687	29
32	779	9.73061	20	308	9.92587	8	789 9.80474	27	0.19526 677	28
33	804	9.73081	20	292	9.92579	8	830 9.80502	28	0.19498 667	27
34	828	9.73101	20	277	9.92571	8	871 9.80530	28	0.19470 657	26
35	53853	9.73121	19	84261	9.92563	8	63912 9.80558	28	0.19442 1.5647	25
36	877	9.73140	20	245	9.92555	9	953 9.80586	28	0.19414 637	24
37	902	9.73160	20	230	9.92546	8	994 9.80614	28	0.19386 627	23
38	926	9.73180	20	214	9.92538	8	64035 9.80642	27	0.19358 617	22
39	951	9.73200	19	198	9.92530	8	076 9.80669	28	0.19331 607	21
40	53975	9.73219	20	84182	9.92522	8	64117 9.80697	28	0.19303 1.5597	20
41	54000	9.73239	20	167	9.92514	8	158 9.80725	28	0.19275 587	19
42	024	9.73259	19	151	9.92506	8	199 9.80753	28	0.19247 577	18
43	049	9.73278	20	135	9.92498	8	240 9.80781	28	0.19219 567	17
44	073	9.73298	20	120	9.92490	8	281 9.80808	27	0.19192 557	16
45	54097	9.73318	20	84104	9.92482	9	64322 9.80836	28	0.19164 1.5547	15
46	122	9.73337	19	088	9.92473	8	363 9.80864	28	0.19136 537	14
47	146	9.73357	20	072	9.92465	8	404 9.80892	27	0.19108 527	13
48	171	9.73377	19	057	9.92457	8	446 9.80920	28	0.19081 517	12
49	195	9.73396	20	041	9.92449	8	487 9.80947	28	0.19053 507	11
50	54220	9.73416	19	84025	9.92441	8	64528 9.80975	28	0.19025 1.5497	10
51	244	9.73435	19	009	9.92433	8	569 9.81003	27	0.18997 487	9
52	269	9.73455	19	83994	9.92425	8	610 9.81030	28	0.18970 477	8
53	293	9.73474	20	978	9.92416	9	652 9.81058	28	0.18942 467	7
54	317	9.73494	19	962	9.92408	8	693 9.81086	27	0.18914 457	6
55	54342	9.73513	20	83946	9.92400	8	64734 9.81113	28	0.18887 1.5448	5
56	366	9.73533	20	930	9.92392	8	775 9.81141	28	0.18859 438	4
57	391	9.73552	19	915	9.92384	8	817 9.81169	27	0.18831 428	3
58	415	9.73572	19	899	9.92376	9	858 9.81196	28	0.18804 418	2
59	440	9.73591	20	883	9.92367	8	899 9.81224	28	0.18776 408	1
60	464	9.73611	20	867	9.92359	8	941 9.81252	28	0.18748 399	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.	'		

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.			
0	54464	9.73611	19	83867	9.92359	8	64941	9.81252	27	0.18748	1.5399	60	
1	488	9.73630	20	851	9.92351	8	982	9.81279	28	0.18721	389	59	
2	513	9.73650	19	835	9.92343	8	65024	9.81307	28	0.18693	379	58	
3	537	9.73669	19	819	9.92335	9	065	9.81335	27	0.18665	369	57	
4	561	9.73689	19	804	9.92326	8	106	9.81362	28	0.18638	359	56	
5	54586	9.73708	19	83788	9.92318	8	65148	9.81390	28	0.18610	1.5350	55	
6	610	9.73727	20	772	9.92310	8	189	9.81418	27	0.18582	340	54	
7	635	9.73747	19	756	9.92302	8	231	9.81445	28	0.18555	330	53	
8	659	9.73766	19	740	9.92293	8	272	9.81473	27	0.18527	320	52	
9	683	9.73785	20	724	9.92285	8	314	9.81500	28	0.18500	311	51	
10	54708	9.73805	19	83708	9.92277	8	65355	9.81528	28	0.18472	1.5301	50	
11	732	9.73824	20	692	9.92269	9	397	9.81556	27	0.18444	291	49	
12	756	9.73843	20	676	9.92260	9	438	9.81583	28	0.18417	282	48	
13	781	9.73863	19	660	9.92252	8	480	9.81611	27	0.18389	272	47	
14	805	9.73882	19	645	9.92244	8	521	9.81638	28	0.18362	262	46	
15	54829	9.73901	19	83629	9.92235	9	65563	9.81666	27	0.18334	1.5253	45	
16	854	9.73921	19	613	9.92227	8	604	9.81693	28	0.18307	243	44	
17	878	9.73940	19	597	9.92219	8	646	9.81721	27	0.18279	233	43	
18	902	9.73959	19	581	9.92211	9	688	9.81748	28	0.18252	224	42	
19	927	9.73978	19	565	9.92202	8	729	9.81776	27	0.18224	214	41	
20	54951	9.73997	19	83549	9.92194	8	65771	9.81803	28	0.18197	1.5204	40	
21	975	9.74017	20	533	9.92186	9	813	9.81831	27	0.18169	195	39	
22	999	9.74036	19	517	9.92177	8	854	9.81858	28	0.18142	185	38	
23	55024	9.74055	19	501	9.92169	8	896	9.81886	27	0.18114	175	37	
24	048	9.74074	19	485	9.92161	9	938	9.81913	28	0.18087	166	36	
25	55072	9.74093	20	83469	9.92152	8	65980	9.81941	27	0.18059	1.5156	35	
26	097	9.74113	19	453	9.92144	8	66021	9.81968	28	0.18032	147	34	
27	121	9.74132	19	437	9.92136	9	063	9.81996	27	0.18004	137	33	
28	145	9.74151	19	421	9.92127	8	105	9.82023	28	0.17977	127	32	
29	169	9.74170	19	405	9.92119	8	147	9.82051	27	0.17949	118	31	
30	55194	9.74189	19	83389	9.92111	9	66189	9.82078	28	0.17922	1.5108	30	
31	218	9.74208	19	373	9.92102	8	230	9.82106	27	0.17894	099	29	
32	242	9.74227	19	356	9.92094	8	272	9.82133	28	0.17867	089	28	
33	266	9.74246	19	340	9.92086	9	314	9.82161	27	0.17839	080	27	
34	291	9.74265	19	324	9.92077	8	356	9.82188	28	0.17812	070	26	
35	55315	9.74284	19	83308	9.92069	9	66398	9.82215	27	0.17785	1.5061	25	
36	339	9.74303	19	292	9.92060	8	440	9.82243	28	0.17757	051	24	
37	363	9.74322	19	276	9.92052	8	482	9.82270	27	0.17730	042	23	
38	388	9.74341	19	260	9.92044	9	524	9.82298	28	0.17702	032	22	
39	412	9.74360	19	244	9.92035	8	566	9.82325	27	0.17675	023	21	
40	55436	9.74379	19	83228	9.92027	9	66608	9.82352	28	0.17648	1.5013	20	
41	460	9.74398	19	212	9.92018	8	650	9.82380	27	0.17620	004	19	
42	484	9.74417	19	195	9.92010	8	692	9.82407	28	0.17593	1.4994	18	
43	509	9.74436	19	179	9.92002	8	734	9.82435	27	0.17565	985	17	
44	533	9.74455	19	163	9.91993	8	776	9.82462	28	0.17538	975	16	
45	55557	9.74474	19	83147	9.91985	9	66818	9.82489	27	0.17511	1.4966	15	
46	581	9.74493	19	131	9.91976	8	860	9.82517	28	0.17483	957	14	
47	605	9.74512	19	115	9.91968	8	902	9.82544	27	0.17456	947	13	
48	630	9.74531	18	098	9.91959	9	944	9.82571	28	0.17429	938	12	
49	654	9.74549	19	082	9.91951	8	986	9.82599	27	0.17401	928	11	
50	55678	9.74568	19	83066	9.91942	9	67028	9.82626	28	0.17374	1.4919	10	
51	702	9.74587	19	050	9.91934	8	071	9.82653	27	0.17347	910	9	
52	726	9.74606	19	034	9.91925	8	113	9.82681	28	0.17319	900	8	
53	750	9.74625	19	017	9.91917	9	155	9.82708	27	0.17292	891	7	
54	775	9.74644	18	001	9.91908	8	197	9.82735	28	0.17265	882	6	
55	55799	9.74662	19	82985	9.91900	9	67239	9.82762	27	0.17238	1.4872	5	
56	823	9.74681	19	969	9.91891	8	282	9.82790	28	0.17210	863	4	
57	847	9.74700	19	953	9.91883	8	324	9.82817	27	0.17183	854	3	
58	871	9.74719	18	936	9.91874	8	366	9.82844	28	0.17156	844	2	
59	895	9.74737	19	920	9.91866	9	409	9.82871	27	0.17129	835	1	
60	919	9.74756	19	904	9.91857	9	451	9.82899	28	0.17101	826	0	
	Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.			'

°	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.					
0	55919	9.74756	19	82904	9.91857	8	67451	9.82809	27	0.17101	1.4826	60	
1	943	9.74775	19	887	9.91849	9	493	9.82926	27	0.17074	816	59	
2	968	9.74794	18	871	9.91840	8	536	9.82953	27	0.17047	807	58	
3	992	9.74812	18	855	9.91832	9	578	9.82980	27	0.17020	798	57	
4	56016	9.74831	19	839	9.91823	8	620	9.83008	28	0.16992	788	56	
5	56040	9.74850	18	82822	9.91815	9	67663	9.83035	27	0.16965	1.4779	55	
6	064	9.74868	18	806	9.91806	8	705	9.83062	27	0.16938	770	54	
7	088	9.74887	19	790	9.91798	8	748	9.83089	28	0.16911	761	53	
8	112	9.74906	18	773	9.91789	9	790	9.83117	27	0.16883	751	52	
9	136	9.74924	19	757	9.91781	8	832	9.83144	27	0.16856	742	51	
10	56160	9.74943	18	82741	9.91772	9	67875	9.83171	27	0.16829	1.4733	50	
11	184	9.74961	19	724	9.91763	8	917	9.83198	27	0.16802	724	49	
12	208	9.74980	19	708	9.91755	9	960	9.83225	27	0.16775	715	48	
13	232	9.74999	18	692	9.91746	8	68002	9.83252	27	0.16748	705	47	
14	256	9.75017	18	675	9.91738	9	045	9.83280	28	0.16720	696	46	
15	56280	9.75036	19	82659	9.91729	9	68088	9.83307	27	0.16693	1.4687	45	
16	305	9.75054	18	643	9.91720	8	130	9.83334	27	0.16666	678	44	
17	329	9.75073	19	626	9.91712	8	173	9.83361	27	0.16639	669	43	
18	353	9.75091	18	610	9.91703	9	215	9.83388	27	0.16612	659	42	
19	377	9.75110	18	593	9.91695	8	258	9.83415	27	0.16585	650	41	
20	56401	9.75128	19	82577	9.91686	9	68301	9.83442	28	0.16558	1.4641	40	
21	425	9.75147	18	561	9.91677	8	343	9.83470	27	0.16530	632	39	
22	449	9.75165	19	544	9.91669	9	386	9.83497	27	0.16503	623	38	
23	473	9.75184	18	528	9.91660	8	429	9.83524	27	0.16476	614	37	
24	497	9.75202	19	511	9.91651	8	471	9.83551	27	0.16449	605	36	
25	56521	9.75221	18	82495	9.91643	9	68514	9.83578	27	0.16422	1.4596	35	
26	545	9.75239	18	478	9.91634	9	557	9.83605	27	0.16395	586	34	
27	569	9.75258	18	462	9.91625	8	600	9.83632	27	0.16368	577	33	
28	593	9.75276	19	446	9.91617	9	642	9.83659	27	0.16341	568	32	
29	617	9.75294	18	429	9.91608	8	685	9.83686	27	0.16314	559	31	
30	56641	9.75313	19	82413	9.91599	9	68728	9.83713	27	0.16287	1.4550	30	
31	665	9.75331	18	396	9.91591	8	771	9.83740	28	0.16260	541	29	
32	689	9.75350	18	380	9.91582	9	814	9.83768	27	0.16232	532	28	
33	713	9.75368	18	363	9.91573	8	857	9.83795	27	0.16205	523	27	
34	736	9.75386	19	347	9.91565	9	900	9.83822	27	0.16178	514	26	
35	56760	9.75405	18	82330	9.91556	9	68942	9.83849	27	0.16151	1.4505	25	
36	784	9.75423	18	314	9.91547	8	985	9.83876	27	0.16124	496	24	
37	808	9.75441	18	297	9.91538	9	69028	9.83903	27	0.16097	487	23	
38	832	9.75459	19	281	9.91530	8	071	9.83930	27	0.16070	478	22	
39	856	9.75478	18	264	9.91521	9	114	9.83957	27	0.16043	469	21	
40	56880	9.75496	19	82248	9.91512	8	69157	9.83984	27	0.16016	1.4460	20	
41	904	9.75514	19	231	9.91504	9	200	9.84011	27	0.15989	451	19	
42	928	9.75533	18	214	9.91495	8	243	9.84038	27	0.15962	442	18	
43	952	9.75551	18	198	9.91486	9	286	9.84065	27	0.15935	433	17	
44	976	9.75569	18	181	9.91477	8	329	9.84092	27	0.15908	424	16	
45	57000	9.75587	19	82165	9.91469	9	69372	9.84119	27	0.15881	1.4415	15	
46	024	9.75605	18	148	9.91460	8	416	9.84146	27	0.15854	406	14	
47	047	9.75624	18	132	9.91451	9	459	9.84173	27	0.15827	397	13	
48	071	9.75642	18	115	9.91442	8	502	9.84200	27	0.15800	388	12	
49	095	9.75660	18	098	9.91433	9	545	9.84227	27	0.15773	379	11	
50	57119	9.75678	18	82082	9.91425	8	69588	9.84254	26	0.15746	1.4370	10	
51	143	9.75696	18	065	9.91416	9	631	9.84280	27	0.15720	361	9	
52	167	9.75714	18	048	9.91407	8	675	9.84307	27	0.15693	352	8	
53	191	9.75733	19	032	9.91398	9	718	9.84334	27	0.15666	344	7	
54	215	9.75751	18	015	9.91389	8	761	9.84361	27	0.15639	335	6	
55	57238	9.75769	18	81999	9.91381	9	69804	9.84388	27	0.15612	1.4326	5	
56	262	9.75787	18	982	9.91372	8	847	9.84415	27	0.15585	317	4	
57	286	9.75805	18	965	9.91363	9	891	9.84442	27	0.15558	308	3	
58	310	9.75823	18	949	9.91354	8	934	9.84469	27	0.15531	299	2	
59	334	9.75841	18	932	9.91345	9	977	9.84496	27	0.15504	290	1	
60	358	9.75859	18	915	9.91336	8	70021	9.84523	27	0.15477	281	0	
		Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	°

°	Nat. Sin Log.	d.	Nat. Cos Log.	d.	Nat. Tan Log.	c.d.	Log. Cot Nat.					
0	57358	9.75859	18	81915	9.91336	8	70021	9.84523	27	0.15477	1.4281	60
1	381	9.75877	18	899	9.91328	9	064	9.84550	26	0.15450	273	59
2	405	9.75895	18	882	9.91319	9	107	9.84576	27	0.15424	264	58
3	429	9.75913	18	865	9.91310	9	151	9.84603	27	0.15397	255	57
4	453	9.75931	18	848	9.91301	9	194	9.84630	27	0.15370	246	56
5	57477	9.75949	18	81832	9.91292	9	70238	9.84657	27	0.15343	1.4237	55
6	501	9.75967	18	815	9.91283	9	281	9.84684	27	0.15316	229	54
7	524	9.75985	18	798	9.91274	8	325	9.84711	27	0.15289	220	53
8	548	9.76003	18	782	9.91266	8	368	9.84738	27	0.15262	211	52
9	572	9.76021	18	765	9.91257	9	412	9.84764	26	0.15236	202	51
10	57596	9.76039	18	81748	9.91248	9	70455	9.84791	27	0.15209	1.4193	50
11	619	9.76057	18	731	9.91239	9	499	9.84818	27	0.15182	185	49
12	643	9.76075	18	714	9.91230	9	542	9.84845	27	0.15155	176	48
13	667	9.76093	18	698	9.91221	9	586	9.84872	27	0.15128	167	47
14	691	9.76111	18	681	9.91212	9	629	9.84899	27	0.15101	158	46
15	57715	9.76129	17	81664	9.91203	9	70673	9.84925	26	0.15075	1.4150	45
16	738	9.76146	18	647	9.91194	9	717	9.84952	27	0.15048	141	44
17	762	9.76164	18	631	9.91185	9	760	9.84979	27	0.15021	132	43
18	786	9.76182	18	614	9.91176	9	804	9.85006	27	0.14994	124	42
19	810	9.76200	18	597	9.91167	9	848	9.85033	27	0.14967	115	41
20	57833	9.76218	18	81580	9.91158	9	70891	9.85059	26	0.14941	1.4106	40
21	857	9.76236	17	503	9.91149	8	935	9.85086	27	0.14914	097	39
22	881	9.76253	18	546	9.91141	9	979	9.85113	27	0.14887	089	38
23	904	9.76271	18	530	9.91132	9	71023	9.85140	27	0.14860	080	37
24	928	9.76289	18	513	9.91123	9	066	9.85166	26	0.14834	071	36
25	57952	9.76307	17	81196	9.91114	9	71110	9.85193	27	0.14807	1.4063	35
26	976	9.76324	18	479	9.91105	9	154	9.85220	27	0.14780	054	34
27	999	9.76342	18	462	9.91096	9	198	9.85247	27	0.14753	045	33
28	58023	9.76360	18	445	9.91087	9	242	9.85273	26	0.14727	037	32
29	047	9.76378	17	428	9.91078	9	285	9.85300	27	0.14700	028	31
30	58070	9.76395	18	81412	9.91069	9	71329	9.85327	27	0.14673	1.4019	30
31	094	9.76413	18	395	9.91060	9	373	9.85354	27	0.14646	011	29
32	118	9.76431	18	378	9.91051	9	417	9.85380	26	0.14620	002	28
33	141	9.76448	18	361	9.91042	9	461	9.85407	27	0.14593	1.3994	27
34	165	9.76466	18	344	9.91033	9	505	9.85434	27	0.14566	985	26
35	58189	9.76484	17	81327	9.91023	10	71549	9.85460	26	0.14540	1.3976	25
36	212	9.76501	18	310	9.91014	9	593	9.85487	27	0.14513	968	24
37	236	9.76519	18	293	9.91005	9	637	9.85514	27	0.14486	959	23
38	260	9.76537	17	276	9.90996	9	681	9.85540	26	0.14460	951	22
39	283	9.76554	18	259	9.90987	9	725	9.85567	27	0.14433	942	21
40	58307	9.76572	18	81242	9.90978	9	71769	9.85594	27	0.14406	1.3934	20
41	330	9.76590	17	225	9.90969	9	813	9.85620	26	0.14380	925	19
42	354	9.76607	18	208	9.90960	9	857	9.85647	27	0.14353	916	18
43	378	9.76625	17	191	9.90951	9	901	9.85674	27	0.14326	908	17
44	401	9.76642	18	174	9.90942	9	946	9.85700	26	0.14300	899	16
45	58425	9.76660	17	81157	9.90933	9	71990	9.85727	27	0.14273	1.3891	15
46	449	9.76677	18	140	9.90924	9	72034	9.85754	27	0.14246	882	14
47	472	9.76695	18	123	9.90915	9	078	9.85780	26	0.14220	874	13
48	496	9.76712	17	106	9.90906	9	122	9.85807	27	0.14193	865	12
49	519	9.76730	18	089	9.90896	10	167	9.85834	27	0.14166	857	11
50	58543	9.76747	17	81072	9.90887	9	72211	9.85860	26	0.14140	1.3848	10
51	567	9.76765	18	055	9.90878	9	255	9.85887	27	0.14113	840	9
52	590	9.76782	18	038	9.90869	9	299	9.85913	26	0.14087	831	8
53	614	9.76800	17	021	9.90860	9	344	9.85940	27	0.14060	823	7
54	637	9.76817	18	004	9.90851	9	388	9.85967	27	0.14033	814	6
55	58661	9.76835	17	80987	9.90842	10	72432	9.85993	27	0.14007	1.3806	5
56	684	9.76852	18	970	9.90832	9	477	9.86020	27	0.13980	798	4
57	708	9.76870	17	953	9.90823	9	521	9.86046	26	0.13954	789	3
58	731	9.76887	18	936	9.90814	9	565	9.86073	27	0.13927	781	2
59	755	9.76904	17	919	9.90805	9	610	9.86100	27	0.13900	772	1
60	779	9.76922	18	902	9.90796	9	654	9.86126	26	0.13874	764	0
	Nat. Cos Log.	d.	Nat. Sin Log.	d.	Nat. Cot Log.	c.d.	Log. Tan Nat.					

	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat. Tan Log. c.d.	Log. Cot Nat.	
0	58779 9.76922	17 80902 9.90796	9 72654 9.86126	27 0.13874 1.3764	60
1	802 9.76939	18 885 9.90787	10 699 9.86153	26 0.13847 755	59
2	826 9.76957	17 867 9.90777	9 743 9.86179	27 0.13821 747	58
3	849 9.76974	17 850 9.90768	9 788 9.86206	27 0.13794 739	57
4	873 9.76991	18 833 9.90759	9 832 9.86232	26 0.13768 730	56
5	58896 9.77009	17 80816 9.90750	9 72877 9.86259	27 0.13741 1.3722	55
6	920 9.77026	17 799 9.90741	10 921 9.86285	26 0.13715 713	54
7	943 9.77043	18 782 9.90731	9 966 9.86312	26 0.13688 705	53
8	967 9.77061	17 765 9.90722	9 73010 9.86338	27 0.13662 697	52
9	990 9.77078	17 748 9.90713	9 955 9.86365	27 0.13635 688	51
10	59014 9.77095	17 80730 9.90704	9 73100 9.86392	26 0.13608 1.3680	50
11	037 9.77112	17 713 9.90694	10 144 9.86418	26 0.13582 672	49
12	061 9.77130	18 696 9.90685	9 189 9.86445	27 0.13555 663	48
13	084 9.77147	17 679 9.90676	9 234 9.86471	26 0.13529 655	47
14	108 9.77164	17 662 9.90667	9 278 9.86498	27 0.13502 647	46
15	59131 9.77181	17 80644 9.90657	10 73323 9.86524	26 0.13476 1.3638	45
16	154 9.77199	18 627 9.90648	9 368 9.86551	26 0.13449 630	44
17	178 9.77216	17 610 9.90639	9 413 9.86577	26 0.13423 622	43
18	201 9.77233	17 593 9.90630	10 457 9.86603	27 0.13397 613	42
19	225 9.77250	18 576 9.90620	9 502 9.86630	26 0.13370 605	41
20	59248 9.77268	17 80558 9.90611	9 73547 9.86656	27 0.13344 1.3597	40
21	272 9.77285	17 541 9.90602	10 592 9.86683	26 0.13317 588	39
22	295 9.77302	17 524 9.90592	9 637 9.86709	26 0.13291 580	38
23	318 9.77319	17 507 9.90583	9 681 9.86736	27 0.13264 572	37
24	342 9.77336	17 489 9.90574	9 726 9.86762	26 0.13238 564	36
25	59395 9.77353	17 80472 9.90565	10 73771 9.86789	27 0.13211 1.3555	35
26	389 9.77370	17 455 9.90555	9 816 9.86815	26 0.13185 547	34
27	412 9.77387	18 438 9.90546	9 861 9.86842	26 0.13158 539	33
28	436 9.77405	17 420 9.90537	9 906 9.86868	26 0.13132 531	32
29	459 9.77422	17 403 9.90527	10 951 9.86894	27 0.13106 522	31
30	59482 9.77439	17 80386 9.90518	9 73996 9.86921	26 0.13079 1.3514	30
31	506 9.77456	17 368 9.90509	10 74041 9.86947	27 0.13053 506	29
32	529 9.77473	17 351 9.90499	9 086 9.86974	26 0.13026 498	28
33	552 9.77490	17 334 9.90490	9 131 9.87000	27 0.13000 490	27
34	576 9.77507	17 316 9.90480	10 176 9.87027	26 0.12973 481	26
35	59599 9.77524	17 80299 9.90471	9 74221 9.87053	27 0.12947 1.3473	25
36	622 9.77541	17 282 9.90462	9 267 9.87079	26 0.12921 465	24
37	646 9.77558	17 264 9.90452	10 312 9.87106	27 0.12894 457	23
38	669 9.77575	17 247 9.90443	9 357 9.87132	26 0.12868 449	22
39	693 9.77592	17 230 9.90434	9 402 9.87158	26 0.12842 440	21
40	59716 9.77609	17 80212 9.90424	10 74447 9.87185	27 0.12815 1.3432	20
41	739 9.77626	17 195 9.90415	9 492 9.87211	26 0.12789 424	19
42	763 9.77643	17 178 9.90405	10 538 9.87238	27 0.12762 416	18
43	786 9.77660	17 160 9.90396	9 583 9.87264	26 0.12736 408	17
44	809 9.77677	17 143 9.90386	10 628 9.87290	27 0.12710 400	16
45	59832 9.77694	17 80125 9.90377	9 74674 9.87317	26 0.12683 1.3392	15
46	856 9.77711	17 108 9.90368	10 719 9.87343	27 0.12657 384	14
47	879 9.77728	17 91 9.90358	9 764 9.87369	26 0.12631 375	13
48	902 9.77744	16 073 9.90349	10 810 9.87396	27 0.12604 367	12
49	926 9.77761	17 056 9.90339	9 855 9.87422	26 0.12578 359	11
50	59949 9.77778	17 80038 9.90330	9 74900 9.87448	27 0.12552 1.3351	10
51	972 9.77795	17 021 9.90320	10 946 9.87475	26 0.12525 343	9
52	995 9.77812	17 003 9.90311	9 991 9.87501	26 0.12499 335	8
53	60019 9.77829	17 79986 9.90301	10 75037 9.87527	27 0.12473 327	7
54	042 9.77846	17 968 9.90292	9 082 9.87554	26 0.12446 319	6
55	60065 9.77862	16 79951 9.90282	10 75128 9.87580	27 0.12420 1.3311	5
56	089 9.77879	17 934 9.90273	9 173 9.87606	26 0.12394 303	4
57	112 9.77896	17 916 9.90263	10 219 9.87633	27 0.12367 295	3
58	135 9.77913	17 899 9.90254	9 264 9.87659	26 0.12341 287	2
59	158 9.77930	17 881 9.90244	10 310 9.87685	27 0.12315 278	1
60	182 9.77946	16 864 9.90235	9 355 9.87711	26 0.12289 270	0
	Nat. Cos Log. d.	Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.	

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.				
0	60182	9.77946	17	79864	9.90235	10	75355	9.87711	27	0.12280	1.3270	60
1	205	9.77903	17	846	9.90225	9	401	9.87738	26	0.12262	262	59
2	228	9.77980	17	829	9.90216	10	447	9.87764	26	0.12236	254	58
3	251	9.77997	17	811	9.90206	9	492	9.87790	26	0.12210	246	57
4	274	9.78013	16	793	9.90197	10	538	9.87817	27	0.12183	238	56
5	60298	9.78030	17	79776	9.90187	9	75584	9.87843	26	0.12157	1.3230	55
6	321	9.78047	17	758	9.90178	9	629	9.87869	26	0.12131	222	54
7	344	9.78063	16	741	9.90168	10	675	9.87895	26	0.12105	214	53
8	367	9.78080	17	723	9.90159	9	721	9.87922	27	0.12078	206	52
9	390	9.78097	16	706	9.90149	10	767	9.87948	26	0.12052	198	51
10	60414	9.78113	17	79688	9.90139	9	75812	9.87974	26	0.12026	1.3190	50
11	437	9.78130	17	671	9.90130	10	858	9.88000	26	0.12000	182	49
12	460	9.78147	16	653	9.90120	9	904	9.88027	27	0.11973	175	48
13	483	9.78163	17	635	9.90111	10	950	9.88053	26	0.11947	167	47
14	506	9.78180	17	618	9.90101	10	996	9.88079	26	0.11921	159	46
15	60529	9.78197	16	79600	9.90091	9	76042	9.88105	26	0.11895	1.3151	45
16	553	9.78213	17	583	9.90082	10	088	9.88131	26	0.11869	143	44
17	576	9.78230	16	565	9.90072	9	134	9.88158	27	0.11842	135	43
18	599	9.78246	17	547	9.90063	10	180	9.88184	26	0.11816	127	42
19	622	9.78263	17	530	9.90053	10	226	9.88210	26	0.11790	119	41
20	60645	9.78280	16	79512	9.90043	9	76272	9.88236	26	0.11764	1.3111	40
21	668	9.78296	17	494	9.90034	10	318	9.88262	26	0.11738	103	39
22	691	9.78313	16	477	9.90024	10	364	9.88289	26	0.11711	095	38
23	714	9.78329	17	459	9.90014	9	410	9.88315	26	0.11685	087	37
24	738	9.78346	16	441	9.90005	10	456	9.88341	26	0.11659	079	36
25	60761	9.78362	17	79424	9.89995	9	76502	9.88367	26	0.11633	1.3072	35
26	784	9.78379	16	406	9.89985	9	548	9.88393	27	0.11607	064	34
27	807	9.78395	17	388	9.89976	10	594	9.88420	26	0.11580	056	33
28	830	9.78412	17	371	9.89966	10	640	9.88446	26	0.11554	048	32
29	853	9.78428	16	353	9.89956	9	686	9.88472	26	0.11528	040	31
30	60876	9.78445	16	79335	9.89947	10	76733	9.88498	26	0.11502	1.3032	30
31	899	9.78461	16	318	9.89937	10	779	9.88524	26	0.11476	024	29
32	922	9.78478	17	300	9.89927	9	825	9.88550	26	0.11450	017	28
33	945	9.78494	16	282	9.89918	10	871	9.88577	27	0.11423	009	27
34	968	9.78510	16	264	9.89908	10	918	9.88603	26	0.11397	001	26
35	60991	9.78527	17	79247	9.89898	9	76964	9.88629	26	0.11371	1.2993	25
36	61015	9.78543	16	229	9.89888	10	77010	9.88655	26	0.11345	985	24
37	038	9.78560	17	211	9.89879	9	057	9.88681	26	0.11319	977	23
38	061	9.78576	16	193	9.89869	10	103	9.88707	26	0.11293	970	22
39	084	9.78592	16	176	9.89859	10	149	9.88733	26	0.11267	962	21
40	61107	9.78609	17	79158	9.89849	9	77196	9.88759	27	0.11241	1.2954	20
41	130	9.78625	17	140	9.89840	10	242	9.88786	26	0.11214	946	19
42	153	9.78642	16	122	9.89830	10	289	9.88812	26	0.11188	938	18
43	176	9.78658	16	105	9.89820	10	335	9.88838	26	0.11162	931	17
44	199	9.78674	17	087	9.89810	9	382	9.88864	26	0.11136	923	16
45	61222	9.78691	16	79069	9.89801	10	77428	9.88890	26	0.11110	1.2915	15
46	245	9.78707	16	051	9.89791	10	475	9.88916	26	0.11084	907	14
47	268	9.78723	16	033	9.89781	10	521	9.88942	26	0.11058	900	13
48	291	9.78739	17	016	9.89771	10	568	9.88968	26	0.11032	892	12
49	314	9.78756	16	78998	9.89761	9	615	9.88994	26	0.11006	884	11
50	61337	9.78772	17	78980	9.89752	10	77661	9.89020	26	0.10980	1.2876	10
51	360	9.78788	16	962	9.89742	10	708	9.89046	27	0.10954	869	9
52	383	9.78805	16	944	9.89732	10	754	9.89073	26	0.10927	861	8
53	406	9.78821	16	926	9.89722	10	801	9.89099	26	0.10901	853	7
54	429	9.78837	16	908	9.89712	10	848	9.89125	26	0.10875	846	6
55	61451	9.78853	16	78891	9.89702	9	77895	9.89151	26	0.10849	1.2838	5
56	474	9.78869	17	873	9.89693	10	941	9.89177	26	0.10823	830	4
57	497	9.78886	16	855	9.89683	10	988	9.89203	26	0.10797	822	3
58	520	9.78902	16	837	9.89673	10	78035	9.89229	26	0.10771	815	2
59	543	9.78918	16	819	9.89663	10	082	9.89255	26	0.10745	807	1
60	566	9.78934	16	801	9.89653	10	129	9.89281	26	0.10719	799	0
	Nat. Cos	Log. d.		Nat. Sin	Log. d.		Nat. Cot	Log. c.d.		Log. Tan	Nat.	'

°	Nat. Sin Log. d.		Nat. Cos Log. d.		Nat. Tan Log. c.d.		Log. Cot Nat.					
0	61566	9.78934	16	78801	9.80653	10	78129	9.80281	26	0.10719	1.2799	60
1	589	9.78950	17	783	9.80043	10	175	9.80307	26	0.10693	792	59
2	612	9.78067	17	765	9.80033	9	222	9.80333	26	0.10667	784	58
3	635	9.78083	16	747	9.80024	10	269	9.80359	26	0.10641	776	57
4	658	9.78999	16	729	9.80614	10	316	9.80385	26	0.10615	769	56
5	61681	9.79015	16	78711	9.80604	10	78363	9.80411	26	0.10589	1.2761	55
6	704	9.79031	16	694	9.80594	10	410	9.80437	26	0.10563	753	54
7	726	9.79047	16	676	9.80584	10	457	9.80463	26	0.10537	746	53
8	749	9.79063	16	658	9.80574	10	504	9.80489	26	0.10511	738	52
9	772	9.79079	16	640	9.80564	10	551	9.80515	26	0.10485	731	51
10	61795	9.79095	16	78622	9.80554	10	78598	9.80541	26	0.10459	1.2723	50
11	818	9.79111	17	604	9.80544	10	645	9.80567	26	0.10433	715	49
12	841	9.79128	16	586	9.80534	10	692	9.80593	26	0.10407	708	48
13	864	9.79144	16	568	9.80524	10	739	9.80619	26	0.10381	700	47
14	887	9.79160	16	550	9.80514	10	786	9.80645	26	0.10355	693	46
15	61909	9.79176	16	78532	9.80504	9	78834	9.80671	26	0.10329	1.2685	45
16	932	9.79192	16	514	9.80495	10	881	9.80697	26	0.10303	677	44
17	955	9.79208	16	496	9.80485	10	928	9.80723	26	0.10277	670	43
18	978	9.79224	16	478	9.80475	10	975	9.80749	26	0.10251	662	42
19	62001	9.79240	16	460	9.80465	10	79022	9.80775	26	0.10225	655	41
20	62024	9.79256	16	78442	9.80455	10	79070	9.80801	26	0.10199	1.2647	40
21	046	9.79272	16	424	9.80445	10	117	9.80827	26	0.10173	640	39
22	069	9.79288	16	405	9.80435	10	164	9.80853	26	0.10147	632	38
23	092	9.79304	16	387	9.80425	10	212	9.80879	26	0.10121	624	37
24	115	9.79319	15	369	9.80415	10	259	9.80905	26	0.10095	617	36
25	62138	9.79335	16	78351	9.80405	10	79306	9.80931	26	0.10069	1.2609	35
26	160	9.79351	16	333	9.80395	10	354	9.80957	26	0.10043	602	34
27	183	9.79367	16	315	9.80385	10	401	9.80983	26	0.10017	594	33
28	206	9.79383	16	297	9.80375	11	449	9.90009	26	0.09991	587	32
29	229	9.79399	16	279	9.80365	10	496	9.90035	26	0.09965	579	31
30	62251	9.79415	16	78261	9.80354	10	79544	9.90061	25	0.09939	1.2572	30
31	274	9.79431	16	243	9.80344	10	591	9.90086	26	0.09914	564	29
32	297	9.79447	16	225	9.80334	10	639	9.90112	26	0.09888	557	28
33	320	9.79463	15	206	9.80324	10	686	9.90138	26	0.09862	549	27
34	342	9.79478	16	188	9.80314	10	734	9.90164	26	0.09836	542	26
35	62365	9.79494	16	78170	9.80304	10	79781	9.90190	26	0.09810	1.2534	25
36	388	9.79510	16	152	9.80294	10	829	9.90216	26	0.09784	527	24
37	411	9.79526	16	134	9.80284	10	877	9.90242	26	0.09758	519	23
38	433	9.79542	16	116	9.80274	10	924	9.90268	26	0.09732	512	22
39	456	9.79558	15	98	9.80264	10	972	9.90294	26	0.09706	504	21
40	62479	9.79573	16	78079	9.80254	10	80020	9.90320	26	0.09680	1.2497	20
41	502	9.79589	16	661	9.80244	11	067	9.90346	25	0.09654	489	19
42	524	9.79605	16	643	9.80233	10	115	9.90371	26	0.09629	482	18
43	547	9.79621	15	625	9.80223	10	163	9.90397	26	0.09603	475	17
44	570	9.79636	16	607	9.80213	10	211	9.90423	26	0.09577	467	16
45	62592	9.79652	16	77988	9.80203	10	80258	9.90449	26	0.09551	1.2460	15
46	615	9.79668	16	970	9.80193	10	306	9.90475	26	0.09525	452	14
47	638	9.79684	15	952	9.80183	10	354	9.90501	26	0.09499	445	13
48	660	9.79699	16	934	9.80173	11	402	9.90527	26	0.09473	437	12
49	683	9.79715	16	916	9.80162	10	450	9.90553	25	0.09447	430	11
50	62706	9.79731	15	77897	9.80152	10	80498	9.90578	26	0.09422	1.2423	10
51	728	9.79746	16	879	9.80142	10	546	9.90604	26	0.09396	415	9
52	751	9.79762	16	861	9.80132	10	594	9.90630	26	0.09370	408	8
53	774	9.79778	16	843	9.80122	10	642	9.90656	26	0.09344	401	7
54	796	9.79793	15	824	9.80112	11	690	9.90682	26	0.09318	393	6
55	62819	9.79809	16	77806	9.80101	10	80738	9.90708	26	0.09292	1.2386	5
56	842	9.79825	16	788	9.80091	10	786	9.90734	25	0.09266	378	4
57	864	9.79840	15	769	9.80081	10	834	9.90759	26	0.09241	371	3
58	887	9.79856	16	751	9.80071	11	882	9.90785	26	0.09215	364	2
59	909	9.79872	15	733	9.80060	10	930	9.90811	26	0.09189	356	1
60	932	9.79887	15	715	9.80050	10	978	9.90837	26	0.09163	349	0
Nat. Cos Log. d. Nat. Sin Log. d. Nat. Cot Log. c.d. Log. Tan Nat. °												

	Nat. Sin Log. d.	Nat. Cos Log. d.	Nat. Tan Log. c.d.	Log. Cot Nat.	
0	62932 9.79887	16 77715 9.80050	10 80978 9.90837	26 0.09163 1.2349	60
1	955 9.79903	15 696 9.80040	10 81027 9.90863	26 0.09137 342	59
2	977 9.79918	15 678 9.80030	10 075 9.90889	26 0.09111 334	58
3	63000 9.79934	16 660 9.80020	11 123 9.90914	25 0.09086 327	57
4	022 9.79950	16 641 9.80009	11 171 9.90940	25 0.09060 320	56
5	63045 9.79965	15 77623 9.88999	10 81220 9.90966	26 0.09034 1.2312	55
6	068 9.79981	15 605 9.88980	11 268 9.90992	26 0.09008 305	54
7	090 9.79996	15 586 9.88978	11 316 9.91018	26 0.08982 298	53
8	113 9.80012	16 568 9.88968	10 364 9.91043	25 0.08957 290	52
9	135 9.80027	15 550 9.88958	10 413 9.91069	26 0.08931 283	51
10	63158 9.80043	16 77531 9.88948	11 81461 9.91095	26 0.08905 1.2276	50
11	180 9.80058	15 513 9.88937	11 510 9.91121	26 0.08879 268	49
12	203 9.80074	15 494 9.88927	11 558 9.91147	25 0.08853 261	48
13	225 9.80089	16 476 9.88917	11 606 9.91172	25 0.08828 254	47
14	248 9.80105	15 458 9.88906	10 655 9.91198	26 0.08802 247	46
15	63271 9.80120	16 77439 9.88896	11 81703 9.91224	26 0.08776 1.2239	45
16	293 9.80136	16 421 9.88886	10 752 9.91250	26 0.08750 232	44
17	316 9.80151	15 402 9.88875	11 800 9.91276	26 0.08724 225	43
18	338 9.80166	15 384 9.88865	10 849 9.91301	25 0.08699 218	42
19	361 9.80182	16 366 9.88855	11 898 9.91327	26 0.08673 210	41
20	63383 9.80197	15 77347 9.88844	11 81946 9.91353	26 0.08647 1.2203	40
21	406 9.80213	16 329 9.88834	11 995 9.91379	26 0.08621 196	39
22	428 9.80228	15 310 9.88824	11 82044 9.91404	25 0.08596 189	38
23	451 9.80244	16 292 9.88813	10 092 9.91430	26 0.08570 181	37
24	473 9.80259	15 273 9.88803	11 141 9.91456	26 0.08544 174	36
25	63496 9.80274	16 77255 9.88793	11 82190 9.91482	25 0.08518 1.2167	35
26	518 9.80290	15 236 9.88782	10 238 9.91507	26 0.08493 160	34
27	540 9.80305	15 218 9.88772	11 287 9.91533	26 0.08467 153	33
28	563 9.80320	16 199 9.88761	11 336 9.91559	26 0.08441 145	32
29	585 9.80336	15 181 9.88751	10 385 9.91585	25 0.08415 138	31
30	63608 9.80351	16 77162 9.88741	11 82434 9.91610	25 0.08390 1.2131	30
31	630 9.80366	15 144 9.88730	11 483 9.91636	26 0.08364 124	29
32	653 9.80382	16 125 9.88720	10 531 9.91662	26 0.08338 117	28
33	675 9.80397	15 107 9.88709	11 580 9.91688	25 0.08312 109	27
34	698 9.80412	16 088 9.88699	10 629 9.91713	26 0.08287 102	26
35	63720 9.80428	15 77070 9.88688	11 82678 9.91739	26 0.08261 1.2095	25
36	742 9.80443	15 051 9.88678	10 727 9.91765	26 0.08235 088	24
37	765 9.80458	15 033 9.88668	11 776 9.91791	25 0.08209 081	23
38	787 9.80473	16 014 9.88657	10 825 9.91816	25 0.08184 074	22
39	810 9.80489	15 76996 9.88647	11 874 9.91842	26 0.08158 066	21
40	63832 9.80504	16 76977 9.88636	11 82923 9.91868	25 0.08132 1.2059	20
41	854 9.80519	15 959 9.88626	10 972 9.91893	26 0.08107 052	19
42	877 9.80534	15 940 9.88615	11 83022 9.91919	26 0.08081 045	18
43	899 9.80550	16 921 9.88605	10 071 9.91945	26 0.08055 038	17
44	922 9.80565	15 903 9.88594	11 120 9.91971	26 0.08029 031	16
45	63944 9.80580	16 76884 9.88584	11 83169 9.91996	25 0.08004 1.2024	15
46	966 9.80595	15 866 9.88573	10 218 9.92022	26 0.07978 017	14
47	989 9.80610	15 847 9.88563	11 268 9.92048	25 0.07952 009	13
48	64011 9.80625	16 828 9.88552	10 317 9.92073	26 0.07927 002	12
49	033 9.80641	15 810 9.88542	11 366 9.92099	26 0.07901 1.1995	11
50	64056 9.80656	16 76791 9.88531	11 83415 9.92125	25 0.07875 1.1988	10
51	078 9.80671	15 772 9.88521	10 465 9.92150	26 0.07850 981	9
52	100 9.80686	15 754 9.88510	11 514 9.92176	26 0.07824 974	8
53	123 9.80701	16 735 9.88499	10 564 9.92202	25 0.07798 967	7
54	145 9.80716	15 717 9.88489	11 613 9.92227	26 0.07773 960	6
55	64167 9.80731	16 76698 9.88478	11 83662 9.92253	25 0.07747 1.1953	5
56	190 9.80746	15 679 9.88468	10 712 9.92279	26 0.07721 946	4
57	212 9.80762	15 661 9.88457	11 761 9.92304	25 0.07696 939	3
58	234 9.80777	16 642 9.88447	10 811 9.92330	26 0.07670 932	2
59	256 9.80792	15 623 9.88436	11 860 9.92356	25 0.07644 925	1
60	279 9.80807	16 604 9.88425	10 910 9.92381	26 0.07619 918	0
Nat. Cos Log. d.		Nat. Sin Log. d.	Nat. Cot Log. c.d.	Log. Tan Nat.	

'	Nat. Sin Log. d.		Nat. Cos Log. d.		Nat. Tan Log. c.d.		Log. Cot Nat.					
0	64279	9.80807	15	76604	9.88425	10	83910	9.92381	26	0.07619	I.1918	60
1	301	9.80822	15	586	9.88415	11	960	9.92407	26	0.07593	910	59
2	323	9.80837	15	567	9.88404	10	84009	9.92433	25	0.07567	903	58
3	346	9.80852	15	548	9.88394	11	059	9.92458	25	0.07542	896	57
4	368	9.80867	15	530	9.88383	11	108	9.92484	26	0.07516	889	56
5	64390	9.80882	15	76511	9.88372	10	84158	9.92510	25	0.07490	I.1882	55
6	412	9.80897	15	492	9.88362	11	208	9.92535	25	0.07465	875	54
7	435	9.80912	15	473	9.88351	11	258	9.92561	26	0.07439	868	53
8	457	9.80927	15	455	9.88340	10	307	9.92587	26	0.07413	861	52
9	479	9.80942	15	436	9.88330	11	357	9.92612	25	0.07388	854	51
10	64501	9.80957	15	76417	9.88319	11	84407	9.92638	26	0.07362	I.1847	50
11	524	9.80972	15	398	9.88308	10	457	9.92663	26	0.07337	840	49
12	546	9.80987	15	380	9.88298	11	507	9.92689	26	0.07311	833	48
13	568	9.81002	15	361	9.88287	11	556	9.92715	25	0.07285	826	47
14	590	9.81017	15	342	9.88276	10	606	9.92740	26	0.07260	819	46
15	64612	9.81032	15	76323	9.88266	11	84656	9.92766	26	0.07234	I.1812	45
16	635	9.81047	14	304	9.88255	11	706	9.92792	25	0.07208	806	44
17	657	9.81061	14	286	9.88244	10	756	9.92817	25	0.07183	799	43
18	679	9.81076	15	267	9.88234	11	806	9.92843	26	0.07157	792	42
19	701	9.81091	15	248	9.88223	11	856	9.92868	26	0.07132	785	41
20	64723	9.81106	15	76229	9.88212	11	84906	9.92894	26	0.07106	I.1778	40
21	746	9.81121	15	210	9.88201	10	956	9.92920	25	0.07080	771	39
22	768	9.81136	15	192	9.88191	11	85006	9.92945	26	0.07055	764	38
23	790	9.81151	15	173	9.88180	11	057	9.92971	26	0.07029	757	37
24	812	9.81166	14	154	9.88169	11	107	9.92996	25	0.07004	750	36
25	64834	9.81180	15	76135	9.88158	10	85157	9.93022	26	0.06978	I.1743	35
26	856	9.81195	15	116	9.88148	11	207	9.93048	26	0.06952	736	34
27	878	9.81210	15	097	9.88137	11	257	9.93073	25	0.06927	729	33
28	901	9.81225	15	078	9.88126	11	308	9.93099	25	0.06901	722	32
29	923	9.81240	14	059	9.88115	10	358	9.93124	26	0.06876	715	31
30	64945	9.81254	15	76041	9.88105	11	85408	9.93150	25	0.06850	I.1708	30
31	967	9.81269	15	022	9.88094	11	458	9.93175	25	0.06825	702	29
32	989	9.81284	15	003	9.88083	11	509	9.93201	26	0.06799	695	28
33	65011	9.81299	15	75984	9.88072	11	559	9.93227	25	0.06773	688	27
34	033	9.81314	14	965	9.88061	10	609	9.93252	26	0.06748	681	26
35	65055	9.81328	15	75946	9.88051	11	85660	9.93278	25	0.06722	I.1674	25
36	077	9.81343	15	927	9.88040	11	710	9.93303	26	0.06697	667	24
37	100	9.81358	14	908	9.88029	11	761	9.93329	25	0.06671	660	23
38	122	9.81372	15	889	9.88018	11	811	9.93354	26	0.06646	653	22
39	144	9.81387	15	870	9.88007	11	862	9.93380	26	0.06620	647	21
40	65166	9.81402	15	75851	9.87996	11	85912	9.93406	25	0.06594	I.1640	20
41	188	9.81417	14	832	9.87985	10	963	9.93431	26	0.06569	633	19
42	210	9.81431	15	813	9.87975	11	86014	9.93457	25	0.06543	626	18
43	232	9.81446	15	794	9.87964	11	064	9.93482	26	0.06518	619	17
44	254	9.81461	14	775	9.87953	11	115	9.93508	25	0.06492	612	16
45	65276	9.81475	15	75756	9.87942	11	86166	9.93533	26	0.06467	I.1606	15
46	298	9.81490	15	738	9.87931	11	216	9.93559	25	0.06441	599	14
47	320	9.81505	14	719	9.87920	11	267	9.93584	26	0.06416	592	13
48	342	9.81519	15	700	9.87909	11	318	9.93610	26	0.06390	585	12
49	364	9.81534	15	680	9.87898	11	368	9.93636	25	0.06364	578	11
50	65386	9.81549	14	75661	9.87887	10	86419	9.93661	26	0.06339	I.1571	10
51	408	9.81563	15	642	9.87877	11	470	9.93687	25	0.06313	565	9
52	430	9.81578	14	623	9.87866	11	521	9.93712	26	0.06288	558	8
53	452	9.81592	15	604	9.87855	11	572	9.93738	26	0.06262	551	7
54	474	9.81607	15	585	9.87844	11	623	9.93763	26	0.06237	544	6
55	65496	9.81622	14	75566	9.87833	11	86674	9.93789	25	0.06211	I.1538	5
56	518	9.81636	15	547	9.87822	11	725	9.93814	25	0.06186	531	4
57	540	9.81651	14	528	9.87811	11	776	9.93840	26	0.06160	524	3
58	562	9.81665	15	509	9.87800	11	827	9.93865	26	0.06135	517	2
59	584	9.81680	15	490	9.87789	11	878	9.93891	26	0.06109	510	1
60	606	9.81694	14	471	9.87778	11	929	9.93916	25	0.06084	504	0
Nat. Cos Log. d. Nat. Sin Log. d. Nat. Cot Log. c.d. Log. Tan Nat. '												

'	Nat. Sin Log. d.			Nat. Cos Log. d.			Nat. Tan Log. c.d.			Log. Cot Nat.		
0	65606	9.81694	15	75471	9.87778	II	86929	9.93916	26	0.06084	I.1504	60
1	628	9.81709	14	452	9.87767	II	980	9.93942	25	0.06085	497	59
2	650	9.81723	15	433	9.87756	II	87031	9.93967	26	0.06033	490	58
3	672	9.81738	14	414	9.87745	II	082	9.93993	25	0.06007	483	57
4	694	9.81752	15	395	9.87734	II	133	9.94018	26	0.05982	477	56
5	65716	9.81767	14	75375	9.87723	II	87184	9.94044	25	0.05956	I.1470	55
6	738	9.81781	15	356	9.87712	II	236	9.94069	26	0.05931	463	54
7	759	9.81796	14	337	9.87701	II	287	9.94095	25	0.05905	456	53
8	781	9.81810	15	318	9.87690	II	338	9.94120	26	0.05880	450	52
9	803	9.81825	14	299	9.87679	II	389	9.94146	25	0.05854	443	51
10	65825	9.81839	15	75280	9.87668	II	87441	9.94171	26	0.05829	I.1436	50
11	847	9.81854	14	261	9.87657	II	492	9.94197	25	0.05803	430	49
12	869	9.81868	15	241	9.87646	II	543	9.94222	26	0.05778	423	48
13	891	9.81882	14	222	9.87635	II	595	9.94248	25	0.05752	416	47
14	913	9.81897	15	203	9.87624	II	646	9.94273	26	0.05727	410	46
15	65935	9.81911	14	75184	9.87613	I2	87698	9.94299	25	0.05701	I.1403	45
16	956	9.81926	15	165	9.87601	II	749	9.94324	26	0.05676	396	44
17	978	9.81940	14	146	9.87590	II	801	9.94350	25	0.05650	389	43
18	66000	9.81955	15	126	9.87579	II	852	9.94375	26	0.05625	383	42
19	022	9.81969	14	107	9.87568	II	904	9.94401	25	0.05599	376	41
20	66044	9.81983	15	75088	9.87557	II	87955	9.94426	26	0.05574	I.1369	40
21	066	9.81998	14	069	9.87546	II	88007	9.94452	25	0.05548	363	39
22	088	9.82012	15	050	9.87535	II	059	9.94477	26	0.05523	356	38
23	109	9.82026	14	030	9.87524	II	110	9.94503	25	0.05497	349	37
24	131	9.82041	15	011	9.87513	I2	162	9.94528	26	0.05472	343	36
25	66153	9.82055	14	74992	9.87501	II	88214	9.94554	25	0.05446	I.1336	35
26	175	9.82069	15	973	9.87490	II	265	9.94579	26	0.05421	329	34
27	197	9.82084	14	953	9.87479	II	317	9.94604	25	0.05396	323	33
28	218	9.82098	15	934	9.87468	II	369	9.94630	26	0.05370	316	32
29	240	9.82112	14	915	9.87457	II	421	9.94655	25	0.05345	310	31
30	66262	9.82126	15	74896	9.87446	I2	88473	9.94681	26	0.05319	I.1303	30
31	284	9.82141	14	876	9.87434	II	524	9.94706	25	0.05294	296	29
32	306	9.82155	15	857	9.87423	II	576	9.94732	26	0.05268	290	28
33	327	9.82169	14	833	9.87412	II	628	9.94757	25	0.05243	283	27
34	349	9.82184	15	818	9.87401	II	680	9.94783	26	0.05217	276	26
35	66371	9.82198	14	74799	9.87390	I2	88732	9.94808	25	0.05192	I.1270	25
36	393	9.82212	15	780	9.87378	II	784	9.94834	26	0.05166	263	24
37	414	9.82226	14	760	9.87367	II	836	9.94859	25	0.05141	257	23
38	436	9.82240	15	741	9.87356	II	888	9.94884	26	0.05116	250	22
39	458	9.82255	14	722	9.87345	II	940	9.94910	25	0.05090	243	21
40	66480	9.82269	15	74703	9.87334	I2	88992	9.94935	26	0.05065	I.1237	20
41	501	9.82283	14	683	9.87322	II	89045	9.94961	25	0.05039	230	19
42	523	9.82297	15	664	9.87311	II	097	9.94986	26	0.05014	224	18
43	545	9.82311	14	644	9.87300	I2	149	9.95012	25	0.04988	217	17
44	566	9.82326	15	625	9.87288	II	201	9.95037	26	0.04963	211	16
45	66588	9.82340	14	74606	9.87277	II	89253	9.95062	25	0.04938	I.1204	15
46	610	9.82354	15	586	9.87266	II	306	9.95088	26	0.04912	197	14
47	632	9.82368	14	567	9.87255	I2	358	9.95113	25	0.04887	191	13
48	653	9.82382	15	548	9.87243	II	410	9.95139	26	0.04861	184	12
49	675	9.82396	14	528	9.87232	II	463	9.95164	25	0.04836	178	11
50	66697	9.82410	15	74509	9.87221	I2	89515	9.95190	26	0.04810	I.1171	10
51	718	9.82424	14	489	9.87209	II	567	9.95215	25	0.04785	165	9
52	740	9.82439	15	470	9.87198	II	620	9.95240	26	0.04760	158	8
53	762	9.82453	14	451	9.87187	I2	672	9.95266	25	0.04734	152	7
54	783	9.82467	15	431	9.87175	II	725	9.95291	26	0.04709	145	6
55	66805	9.82481	14	74412	9.87164	II	89777	9.95317	25	0.04683	I.1139	5
56	827	9.82495	15	392	9.87153	I2	830	9.95342	26	0.04658	132	4
57	848	9.82509	14	373	9.87141	II	883	9.95368	25	0.04632	126	3
58	870	9.82523	15	353	9.87130	II	935	9.95393	26	0.04607	119	2
59	891	9.82537	14	334	9.87119	I2	988	9.95418	25	0.04582	113	1
60	913	9.82551	15	314	9.87107	II	90040	9.95444	26	0.04556	106	0
Nat. Cos Log. d.			Nat. Sin Log. d.			Nat. Cot Log. c.d.			Log. Tan Nat.			'

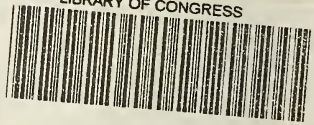
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0	66913	9.82551	74314	9.87107	90040	9.95444	0.04556	I.1106	60
1	935	9.82505	295	9.87096	093	9.95409	0.04531	100	59
2	956	9.82579	276	9.87085	146	9.95495	0.04505	093	58
3	978	9.82593	256	9.87073	199	9.95520	0.04480	087	57
4	999	9.82607	237	9.87062	251	9.95545	0.04455	080	56
5	67021	9.82621	74217	9.87050	90304	9.95571	0.04429	I.1074	55
6	043	9.82635	198	9.87039	357	9.95596	0.04404	067	54
7	064	9.82649	178	9.87028	410	9.95622	0.04378	061	53
8	086	9.82663	159	9.87016	463	9.95647	0.04353	054	52
9	107	9.82677	139	9.87005	516	9.95672	0.04328	048	51
10	67129	9.82691	74120	9.86993	90569	9.95698	0.04302	I.1041	50
11	151	9.82705	100	9.86982	621	9.95723	0.04277	035	49
12	172	9.82719	080	9.86970	674	9.95748	0.04252	028	48
13	194	9.82733	061	9.86959	727	9.95774	0.04226	022	47
14	215	9.82747	041	9.86947	781	9.95799	0.04201	016	46
15	67237	9.82761	74022	9.86936	90834	9.95825	0.04175	I.1009	45
16	258	9.82775	002	9.86924	887	9.95850	0.04150	003	44
17	280	9.82788	73983	9.86913	940	9.95875	0.04125	I.0996	43
18	301	9.82802	963	9.86902	993	9.95901	0.04099	990	42
19	323	9.82816	944	9.86890	91046	9.95926	0.04074	983	41
20	67344	9.82830	73924	9.86879	91099	9.95952	0.04048	I.0977	40
21	366	9.82844	904	9.86867	153	9.95977	0.04023	971	39
22	387	9.82858	885	9.86855	206	9.96002	0.03998	964	38
23	409	9.82872	865	9.86844	259	9.96028	0.03972	958	37
24	430	9.82885	846	9.86832	313	9.96053	0.03947	951	36
25	67452	9.82899	73826	9.86821	91366	9.96078	0.03922	I.0945	35
26	473	9.82913	806	9.86809	419	9.96104	0.03896	939	34
27	495	9.82927	787	9.86798	473	9.96129	0.03871	932	33
28	516	9.82941	767	9.86786	526	9.96155	0.03845	926	32
29	538	9.82955	747	9.86775	580	9.96180	0.03820	919	31
30	67559	9.82968	73728	9.86763	91633	9.96205	0.03795	I.0913	30
31	580	9.82982	708	9.86752	687	9.96231	0.03769	907	29
32	602	9.82996	688	9.86740	740	9.96256	0.03744	900	28
33	623	9.83010	669	9.86728	794	9.96281	0.03719	894	27
34	645	9.83023	649	9.86717	847	9.96307	0.03693	888	26
35	67666	9.83037	73629	9.86705	91901	9.96332	0.03668	I.0881	25
36	688	9.83051	610	9.86694	955	9.96357	0.03643	875	24
37	709	9.83065	590	9.86682	92008	9.96383	0.03617	869	23
38	730	9.83078	570	9.86670	062	9.96408	0.03592	862	22
39	752	9.83092	551	9.86659	116	9.96433	0.03567	856	21
40	67773	9.83106	73531	9.86647	92170	9.96459	0.03541	I.0850	20
41	795	9.83120	511	9.86635	224	9.96484	0.03516	843	19
42	816	9.83133	491	9.86624	277	9.96510	0.03490	837	18
43	837	9.83147	472	9.86612	331	9.96535	0.03465	831	17
44	859	9.83161	452	9.86600	385	9.96560	0.03440	824	16
45	67880	9.83174	73432	9.86589	92439	9.96586	0.03414	I.0818	15
46	901	9.83188	413	9.86577	493	9.96611	0.03389	812	14
47	923	9.83202	393	9.86565	547	9.96636	0.03364	805	13
48	944	9.83215	373	9.86554	601	9.96662	0.03338	799	12
49	965	9.83229	353	9.86542	655	9.96687	0.03313	793	11
50	67987	9.83242	73333	9.86530	92709	9.96712	0.03288	I.0786	10
51	68008	9.83256	314	9.86518	763	9.96738	0.03262	780	9
52	029	9.83270	294	9.86507	817	9.96763	0.03237	774	8
53	051	9.83283	274	9.86495	872	9.96788	0.03212	768	7
54	072	9.83297	254	9.86483	926	9.96814	0.03186	761	6
55	68093	9.83310	73234	9.86472	92980	9.96839	0.03161	I.0755	5
56	115	9.83324	215	9.86460	93034	9.96864	0.03136	749	4
57	136	9.83338	195	9.86448	088	9.96890	0.03110	742	3
58	157	9.83351	175	9.86436	143	9.96915	0.03085	736	2
59	179	9.83365	155	9.86425	197	9.96940	0.03060	730	1
60	200	9.83378	135	9.86413	252	9.96966	0.03034	724	0
	Nat. Cos	Log. d.	Nat. Sin	Log. d.	Nat. Cot	Log. c.d.	Log. Tan	Nat.	

'	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.	'
0	68200	9.83378	73135	9.86413	93252	9.96966	0.03034	1.0724	60
1	221	9.83392	116	9.86401	306	9.96991	0.03009	717	59
2	242	9.83405	096	9.86389	360	9.97016	0.02984	711	58
3	264	9.83419	076	9.86377	415	9.97042	0.02958	705	57
4	285	9.83432	056	9.86366	469	9.97067	0.02933	699	56
5	68306	9.83446	73036	9.86354	93524	9.97092	0.02908	1.0692	55
6	327	9.83459	016	9.86342	578	9.97118	0.02882	686	54
7	349	9.83473	72996	9.86330	633	9.97143	0.02857	680	53
8	370	9.83486	976	9.86318	688	9.97168	0.02832	674	52
9	391	9.83500	957	9.86306	742	9.97193	0.02807	668	51
10	68412	9.83513	72937	9.86295	93797	9.97219	0.02781	1.0661	50
11	434	9.83527	917	9.86283	852	9.97244	0.02756	655	49
12	455	9.83540	897	9.86271	906	9.97269	0.02731	649	48
13	476	9.83554	877	9.86259	961	9.97295	0.02705	643	47
14	497	9.83567	857	9.86247	94016	9.97320	0.02680	637	46
15	68518	9.83581	72837	9.86235	94071	9.97345	0.02655	1.0630	45
16	539	9.83594	817	9.86223	125	9.97371	0.02629	624	44
17	561	9.83608	797	9.86211	180	9.97396	0.02604	618	43
18	582	9.83621	777	9.86200	235	9.97421	0.02579	612	42
19	603	9.83634	757	9.86188	290	9.97447	0.02553	606	41
20	68624	9.83648	72737	9.86176	94345	9.97472	0.02528	1.0599	40
21	645	9.83661	717	9.86164	400	9.97497	0.02503	593	39
22	666	9.83674	697	9.86152	455	9.97523	0.02477	587	38
23	688	9.83688	677	9.86140	510	9.97548	0.02452	581	37
24	709	9.83701	657	9.86128	565	9.97573	0.02427	575	36
25	68730	9.83715	72637	9.86116	94620	9.97598	0.02402	1.0569	35
26	751	9.83728	617	9.86104	676	9.97624	0.02376	562	34
27	772	9.83741	597	9.86092	731	9.97649	0.02351	556	33
28	793	9.83755	577	9.86080	786	9.97674	0.02326	550	32
29	814	9.83763	557	9.86068	841	9.97700	0.02300	544	31
30	68835	9.83781	72537	9.86056	94896	9.97725	0.02275	1.0538	30
31	857	9.83795	517	9.86044	952	9.97750	0.02250	532	29
32	878	9.83803	497	9.86032	95007	9.97776	0.02224	526	28
33	899	9.83821	477	9.86020	062	9.97801	0.02199	519	27
34	920	9.83834	457	9.86008	118	9.97826	0.02174	513	26
35	68941	9.83843	72437	9.85996	95173	9.97851	0.02149	1.0507	25
36	962	9.83861	417	9.85984	229	9.97877	0.02123	501	24
37	983	9.83874	397	9.85972	284	9.97902	0.02098	495	23
38	69004	9.83887	377	9.85960	340	9.97927	0.02073	489	22
39	025	9.83901	357	9.85948	395	9.97953	0.02047	483	21
40	69046	9.83914	72337	9.85936	95451	9.97978	0.02022	1.0477	20
41	067	9.83927	317	9.85924	506	9.98003	0.01997	470	19
42	088	9.83940	297	9.85912	562	9.98029	0.01971	464	18
43	109	9.83954	277	9.85900	618	9.98054	0.01946	458	17
44	130	9.83967	257	9.85888	673	9.98079	0.01921	452	16
45	69151	9.83980	72236	9.85876	95729	9.98104	0.01896	1.0446	15
46	172	9.83993	216	9.85864	785	9.98130	0.01870	440	14
47	193	9.84006	196	9.85851	841	9.98155	0.01845	434	13
48	214	9.84020	176	9.85839	897	9.98180	0.01820	428	12
49	235	9.84033	156	9.85827	952	9.98206	0.01794	422	11
50	69256	9.84046	72136	9.85815	96008	9.98231	0.01769	1.0416	10
51	277	9.84059	116	9.85803	064	9.98256	0.01744	410	9
52	298	9.84072	095	9.85791	120	9.98281	0.01719	404	8
53	319	9.84085	075	9.85779	176	9.98307	0.01693	398	7
54	340	9.84098	055	9.85766	232	9.98332	0.01668	392	6
55	69361	9.84112	72035	9.85754	96288	9.98357	0.01643	1.0385	5
56	382	9.84125	015	9.85742	344	9.98383	0.01617	379	4
57	403	9.84138	71995	9.85730	400	9.98408	0.01592	373	3
58	424	9.84151	974	9.85718	457	9.98433	0.01567	367	2
59	445	9.84164	954	9.85706	513	9.98458	0.01542	361	1
60	466	9.84177	934	9.85693	569	9.98484	0.01516	355	0
	Nat. Cos	Log. d.	Nat. Sin	Log. d.	Nat. Cot	Log. c.d.	Log. Tan	Nat.	'

	Nat. Sin	Log. d.	Nat. Cos	Log. d.	Nat. Tan	Log. c.d.	Log. Cot	Nat.	
0	69466	9.84177	71934	9.85603	96569	9.98484	0.01516	1.0355	60
1	487	9.84190	914	9.85681	625	9.98509	0.01491	349	59
2	508	9.84203	894	9.85660	681	9.98534	0.01466	343	58
3	529	9.84216	873	9.85657	738	9.98560	0.01440	337	57
4	549	9.84229	853	9.85645	794	9.98585	0.01415	331	56
5	69570	9.84242	71833	9.85632	96850	9.98610	0.01390	1.0325	55
6	591	9.84255	813	9.85620	907	9.98635	0.01365	319	54
7	612	9.84269	792	9.85608	963	9.98661	0.01339	313	53
8	633	9.84282	772	9.85596	97020	9.98686	0.01314	307	52
9	654	9.84295	752	9.85583	076	9.98711	0.01289	301	51
10	69675	9.84308	71732	9.85571	97133	9.98737	0.01263	1.0295	50
11	696	9.84321	711	9.85559	189	9.98762	0.01238	289	49
12	717	9.84334	691	9.85547	246	9.98787	0.01213	283	48
13	737	9.84347	671	9.85534	302	9.98812	0.01188	277	47
14	758	9.84360	650	9.85522	359	9.98832	0.01162	271	46
15	69779	9.84373	71630	9.85510	97416	9.98863	0.01137	1.0265	45
16	800	9.84385	610	9.85497	472	9.98888	0.01112	259	44
17	821	9.84398	590	9.85485	529	9.98913	0.01087	253	43
18	842	9.84411	569	9.85473	586	9.98939	0.01061	247	42
19	862	9.84424	549	9.85460	643	9.98964	0.01036	241	41
20	69883	9.84437	71529	9.85448	97700	9.98989	0.01011	1.0235	40
21	904	9.84450	508	9.85436	756	9.99015	0.00985	230	39
22	925	9.84463	488	9.85423	813	9.99040	0.00960	224	38
23	946	9.84476	468	9.85411	870	9.99065	0.00935	218	37
24	966	9.84489	447	9.85399	927	9.99090	0.00910	212	36
25	69987	9.84502	71427	9.85386	97984	9.99116	0.00884	1.0206	35
26	70008	9.84515	407	9.85374	98041	9.99141	0.00859	200	34
27	029	9.84528	386	9.85361	098	9.99166	0.00834	194	33
28	049	9.84540	366	9.85349	155	9.99191	0.00809	188	32
29	070	9.84553	345	9.85337	213	9.99217	0.00783	182	31
30	70091	9.84566	71325	9.85324	98270	9.99242	0.00758	1.0176	30
31	112	9.84579	305	9.85312	327	9.99267	0.00733	170	29
32	132	9.84592	284	9.85299	384	9.99293	0.00707	164	28
33	153	9.84605	264	9.85287	441	9.99318	0.00682	158	27
34	174	9.84618	243	9.85274	499	9.99343	0.00657	152	26
35	70195	9.84630	71223	9.85262	98556	9.99368	0.00632	1.0147	25
36	215	9.84643	203	9.85250	613	9.99394	0.00606	141	24
37	236	9.84656	182	9.85237	671	9.99419	0.00581	135	23
38	257	9.84669	162	9.85225	728	9.99444	0.00555	129	22
39	277	9.84682	141	9.85212	786	9.99469	0.00531	123	21
40	70298	9.84694	71121	9.85200	98843	9.99495	0.00505	1.0117	20
41	319	9.84707	100	9.85187	901	9.99520	0.00480	111	19
42	339	9.84720	080	9.85175	958	9.99545	0.00455	105	18
43	360	9.84733	059	9.85162	99016	9.99570	0.00430	099	17
44	381	9.84745	039	9.85150	073	9.99595	0.00404	094	16
45	70401	9.84758	71019	9.85137	99131	9.99621	0.00379	1.0088	15
46	422	9.84771	70998	9.85125	189	9.99646	0.00354	082	14
47	443	9.84784	978	9.85112	247	9.99672	0.00328	076	13
48	463	9.84796	957	9.85100	304	9.99697	0.00303	070	12
49	484	9.84809	937	9.85087	362	9.99722	0.00278	064	11
50	70505	9.84822	70916	9.85074	99420	9.99747	0.00253	1.0058	10
51	525	9.84835	896	9.85062	478	9.99773	0.00227	052	9
52	546	9.84847	875	9.85049	536	9.99798	0.00202	047	8
53	567	9.84860	855	9.85037	594	9.99823	0.00177	041	7
54	587	9.84873	834	9.85024	652	9.99848	0.00152	035	6
55	70608	9.84885	70813	9.85012	99710	9.99874	0.00126	1.0029	5
56	628	9.84898	793	9.84999	768	9.99899	0.00101	023	4
57	649	9.84911	772	9.84986	826	9.99924	0.00076	017	3
58	670	9.84923	752	9.84974	884	9.99949	0.00051	012	2
59	690	9.84936	731	9.84961	942	9.99975	0.00025	006	1
60	711	9.84949	711	9.84949	10000	0.00000	0.00000	000	0
	Nat. Cos	Log. d.	Nat. Sin	Log. d.	Nat. Cot	Log. c.d.	Log. Tan	Nat.	

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